The conductance of mesoscopic rings

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$GIF, $ISF

References

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The conductance of a closed mesoscopic system,
D. Cohen, T. Kottos and H. Schanz, cond-mat/0505295

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Google "Doron Cohen"

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cond-mat archive

“Quantum Chaos”

$\mathcal{H}(Q,P)$ = quantized chaotic system

X = some parameter in the Hamiltonian

Universality on small energy scales ($\propto \hbar^d$)

Fingerprints on larger energy scales ($\propto \hbar$)

- Spectral / eigenstates statistics of $\mathcal{H}(Q,P)$
- Parametric analysis of $\mathcal{H}(Q,P;X)$
- Study of driven systems $\mathcal{H}(Q,P;X(t))$

The question:

How is the $\hbar$ scale reflected in the theory???

The message:

The Kubo formalism should be revised!

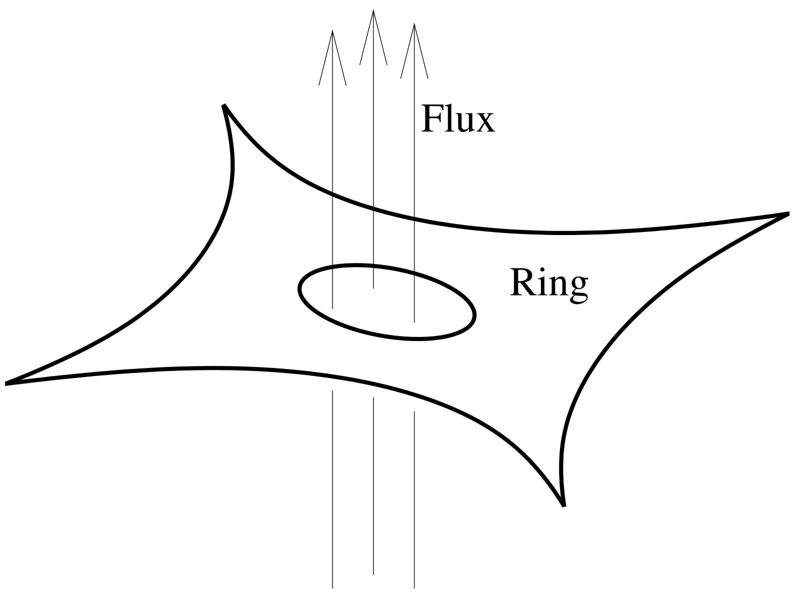
Driven Systems

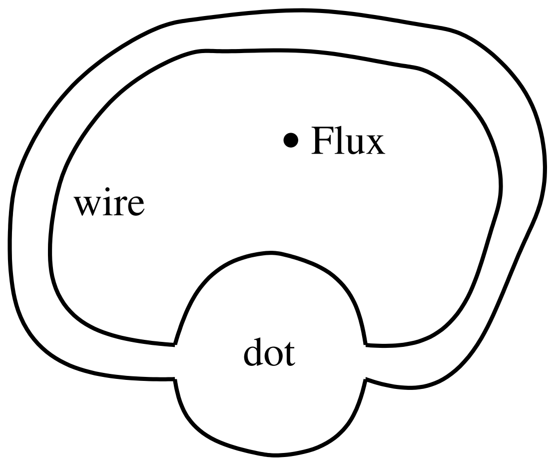
Non interacting "spinless" electrons.

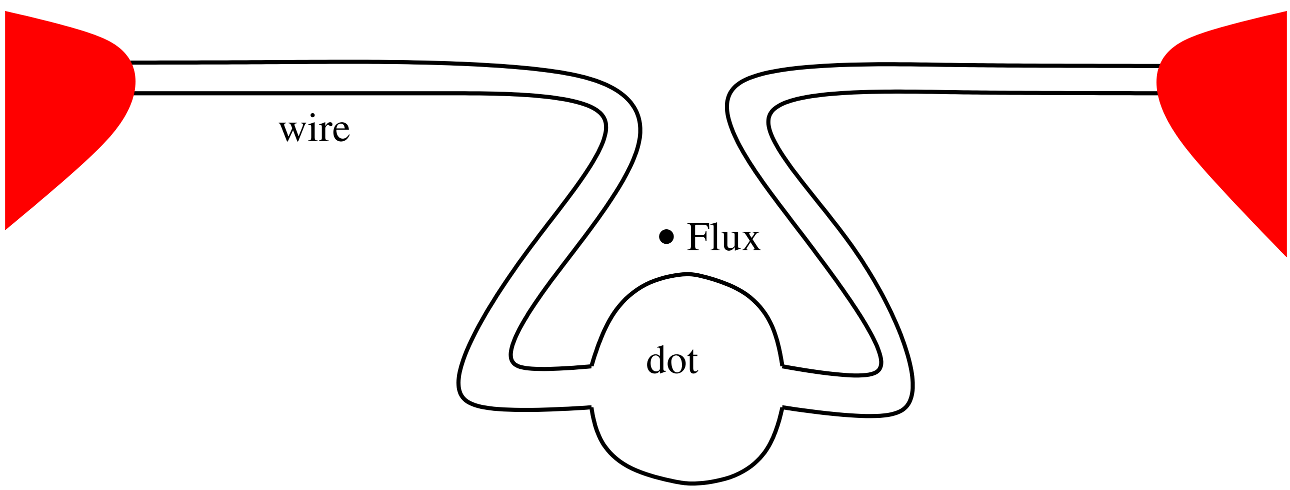
Held by a potential (e.g. AB ring geometry).

$X \quad = \quad$ shape parameter

$\Phi \; = \; (\hbar/e)\phi \; = \;$ magnetic flux







“Ohm law”

For one parameter driving by EMF

$$I = \boldsymbol{G} \times (-\dot{\Phi})$$
$$dQ = -\boldsymbol{G}\, d\Phi$$

For driving by changing another parameter

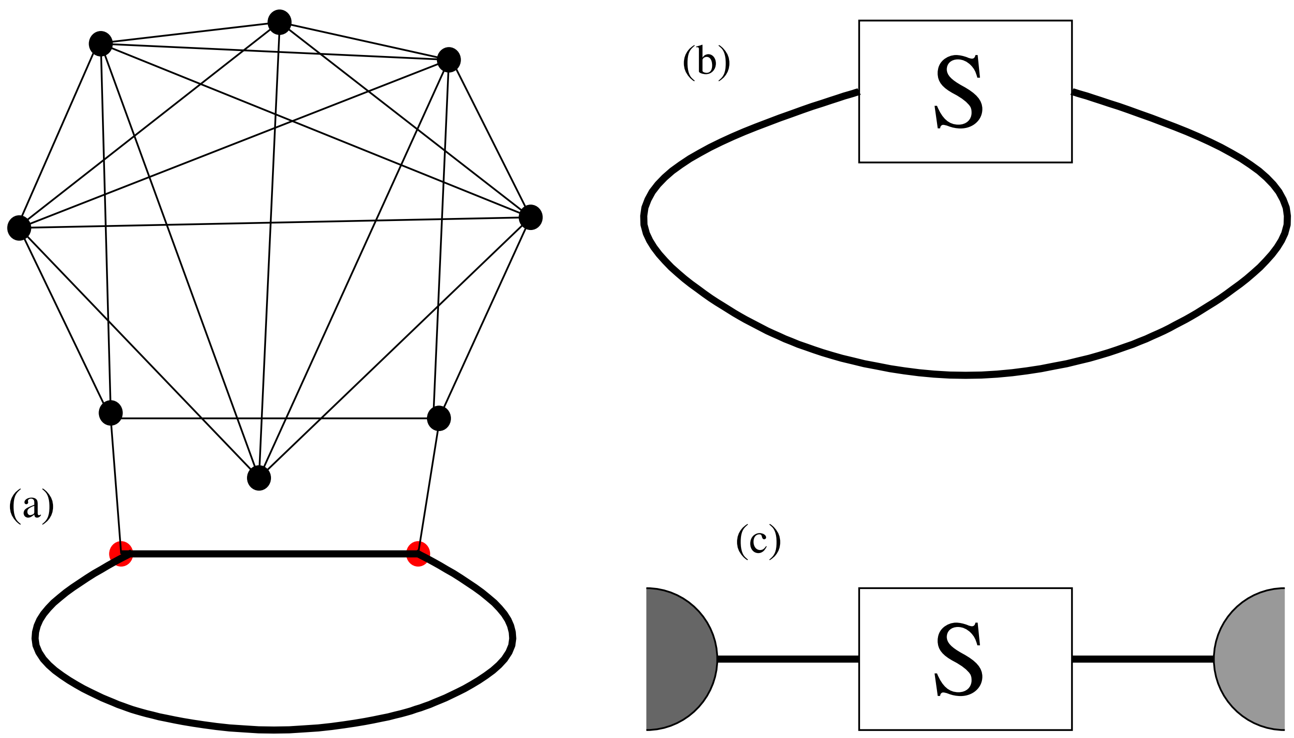
$$I = -\boldsymbol{G}\, \dot{X}$$
$$dQ = -\boldsymbol{G}\, dX$$

Note that $\boldsymbol{G}$ in case of EMF driving is of dissipative rather then of geometric nature.

$$\frac{dE}{dt} = \boldsymbol{G}\, \dot{\Phi}^2$$

The dissipative $\boldsymbol{G}$ reflects the stochastic-like diffusion D_E in energy space.

Problem No.1 - driving by EMF

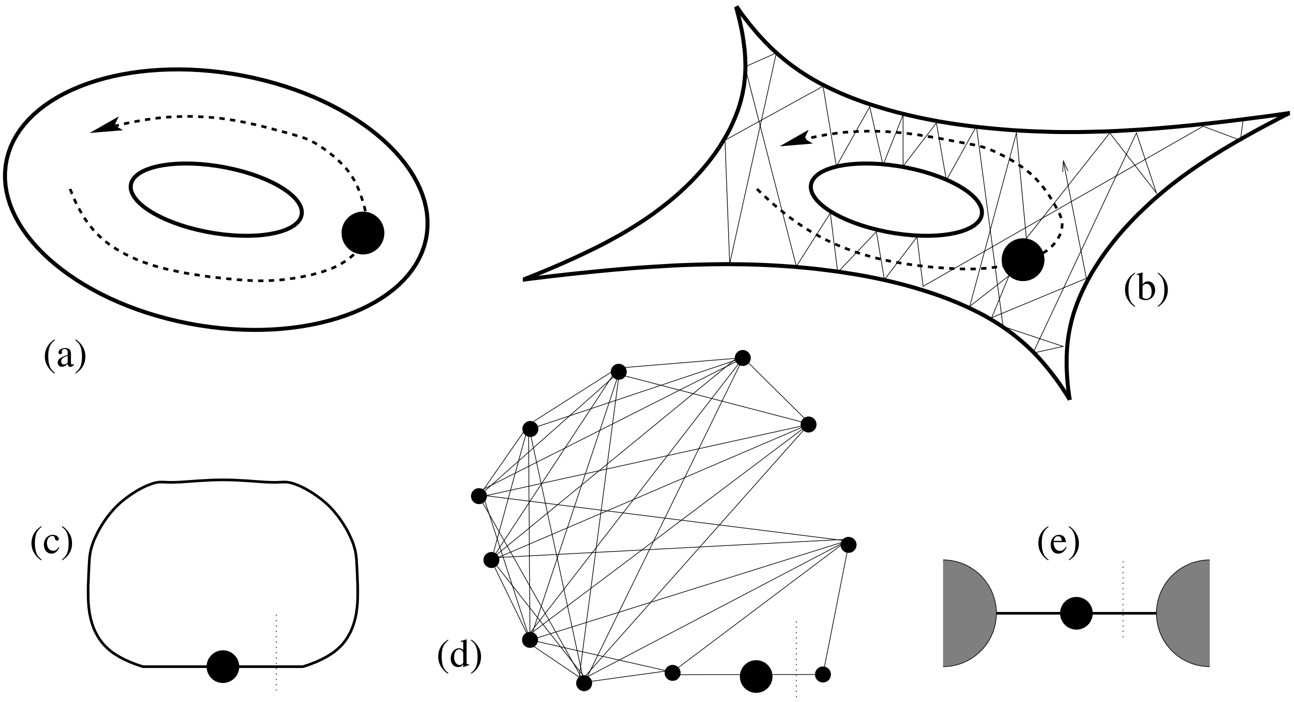


$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar}\; g_{cl}$$

$$G = \frac{e^2}{2\pi\hbar} \left(\frac{g_{cl}}{1-g_{cl}} \right)$$

[classical]

Problem No.2 - driving by pushing



$$G_{\text{BPT}} \ = -(1-g_0) \times \frac{e}{\pi} k_{\text{F}}$$

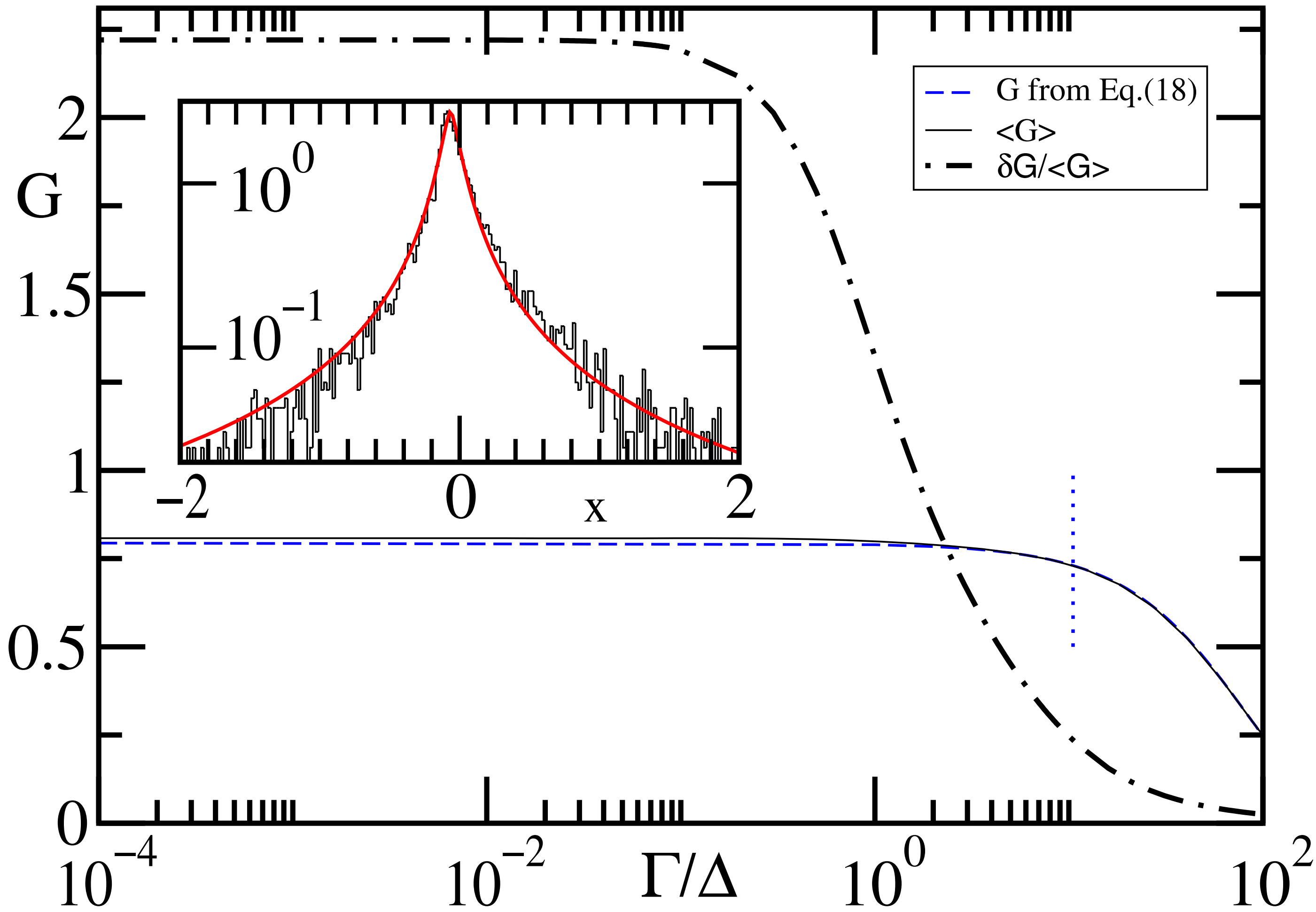
$$G = -\left[\frac{1-g_0}{g_0}\right] \left[\frac{g_T}{1-g_T}\right] \times \frac{e}{\pi} k_{\text{F}}$$

[classical]

Problem No.2 - quantum results

$$\mathcal{H} = \text{network} + \lambda \frac{\hbar^2}{2\text{m}} \delta(x - X)$$

$$g_0 = \frac{1}{1+(\lambda/(2k_F))^2} = \text{transmission}$$



Back to Problem No.1

Recall the classical results

Single mode versions:

$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar} g_{cl}$$

$$G = \frac{e^2}{2\pi\hbar} \left(\frac{g_{cl}}{1 - g_{cl}} \right)$$

Multimode versions:

$$\boldsymbol{g} = \begin{pmatrix} g^R & g^T \\ g^T & g^R \end{pmatrix}$$

$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar} \sum_{n,m} g^T_{nm}$$

$$G = \frac{e^2}{2\pi\hbar} \sum_{nm} \left[2g^T / (1 - g^T + g^R) \right]_{nm}$$

Can we trust the classical result?

We are interested in the case $g_{cl} \sim 1$

The velocity-velocity correlaion time

$$\tau_{cl} \approx \left(\frac{1}{1-g_{cl}}\right) \times \frac{L_0}{v_F}$$

Quantum mechanics introduces

$$t_{\text{Heisenberg}} \approx \mathcal{M}_{\text{modes}} \times \frac{L_0}{v_F}$$

Necessary condition for QCC:

$$\mathcal{M}_{\text{modes}} \quad \gg \quad \left(\frac{1}{1-g_{cl}}\right)$$

Single mode ring??? Ring-Tree model???

Pure quantum results!

New ingredient in the theoy of mesoscopic condctance!

Quantum results

$$g_{cl} \quad = \quad \text{the average Landauer conductance.}$$

The spectroscopic result:

$$G_{\text{spectroscopic}} = \frac{e^2}{2\pi\hbar}\left(\frac{g_{cl}}{1-g_{cl}}\right)$$

in general level statistics is important.

The mesoscopic result:

In the limit $g_{cl} \rightarrow 1$

$G \quad \rightarrow \quad \infty$ [classical]

$G \quad \rightarrow \quad 0$ [quantal]

For the ring-tree model:

$$G \quad = \quad \frac{e^2}{2\pi\hbar}\,(1-g_{cl})^2\, g_{cl}$$

The FD version of the Kubo formula

$$\mathcal{H} = \mathcal{H}(Q, P; X(t))$$

$\mathsf{g}(E)$ = density of states

$$\mathcal{F} = -\frac{\partial \mathcal{H}}{\partial X}$$

$$\mathcal{I} = -ev\delta(x - x_1)$$

$$C_E(\tau) = \langle \mathcal{I}(\tau)\mathcal{F}(0)\rangle_E$$

$$G \;=\; \mathsf{g}(E_F)\int_0^\infty C_{E_F}(\tau)d\tau$$

For a dot-wire geometry there is a limit in which

$$G_{\text{BPT}} \;=\; \frac{e}{2\pi i}\text{trace}\left(P_{\text{A}}\frac{\partial S}{\partial X}S^\dagger\right)$$

[D.C. PRB(R) 2003]

The non-adiabaticity parameter

$$\boldsymbol{G} = \pi\hbar \sum_{n,m} |\mathcal{I}_{mn}|^2 \; \delta_T(E_n - E_F) \; \delta_\Gamma(E_m - E_n)$$

$$C_E(\tau) \quad \mapsto \quad C_E(\tau) \; \mathrm{e}^{-(\Gamma/2\hbar)|\tau|}$$

$$\Gamma \quad = \quad \left(\frac{\hbar\sigma}{\Delta^2}|\dot{X}|\right)^{2/3} \times \; \Delta$$

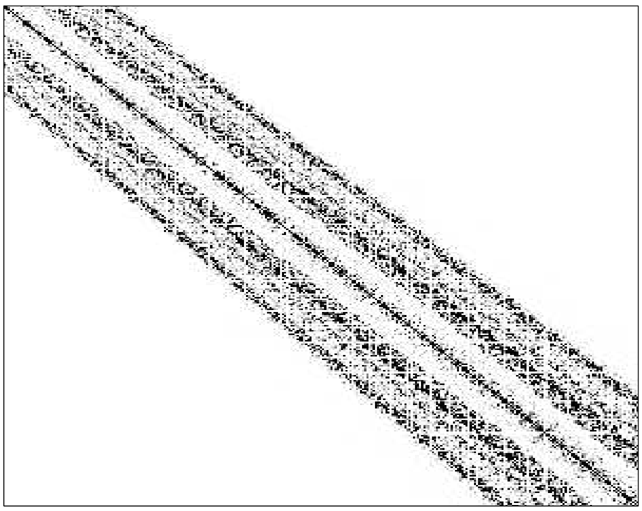
We assume

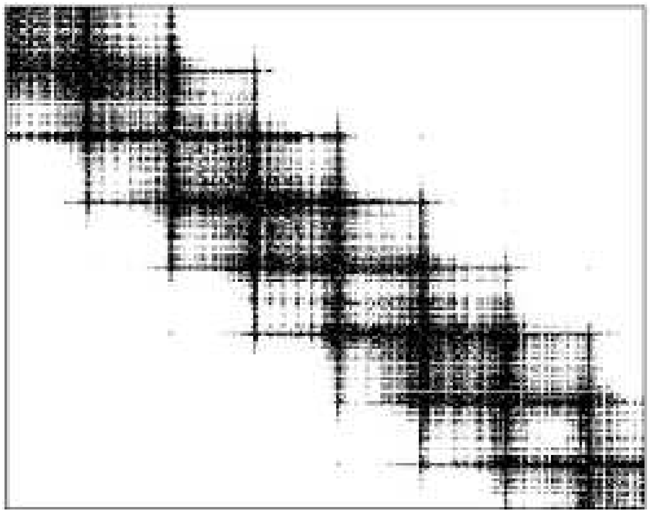
$$\Delta \quad \ll \quad \Gamma \quad \ll \quad ...$$

The calculation....

$$\boldsymbol{G} = \pi\hbar(\mathsf{g}(E_F))^2 \ \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle$$

Let us look on $\mathcal{I}_{nm}$





for structured matrix algebraic average is wrong!

Thereore the classical result is not obtained.

The diffusion picture

RMT modeling of the Hamiltonian:

$$\mathcal{H} = \boldsymbol{E} + \Phi(t)\boldsymbol{B}$$

where

$$\begin{aligned} \boldsymbol{E} &= \{E_n\} \\ \boldsymbol{B} &= \{-\mathcal{I}_{nm}\} \end{aligned}$$

How is the coarse grained diffusion determined?

$$\langle\langle D_E \rangle\rangle = \left[\overline{1/D_E}\right]^{-1}$$