Energy absorption and the conductance of small mesoscopic rings

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$ISF, \ $GIF, \ $DIP, \ $BSF
Driven Systems

Non interacting “spinless” electrons in a ring.

\[ \mathcal{H}(Q, P; \Phi(t)) \]

\[-\dot{\Phi} = \text{electro motive force} \]
\[ G \dot{\Phi}^2 = \text{rate of energy absorption} \]
Linear Response Theory (LRT)

\[ H = \{ E_n \} - \Phi(t)\{ \mathcal{I}_{nm} \} \]

\[ G = \pi \hbar \sum_{n,m} |\mathcal{I}_{mn}|^2 \delta_T(E_n - E_F) \delta_T(E_m - E_n) \]

\[ G = \pi \hbar (\varrho(E_F))^2 \langle \langle |\mathcal{I}_{mn}|^2 \rangle \rangle \]

applies if

EMF driven transitions \( \ll \) relaxation

otherwise

connected sequences of transitions are essential.

leading to

Semi Linear Response Theory (SLRT)
Semi Linear Response Theory (SLRT)

\[ H = \{ E_n \} - \Phi(t)\{ \mathcal{I}_{nm} \} \]

\[ \frac{dp_n}{dt} = -\sum_m w_{nm}(p_n - p_m) \]

\[ w_{nm} = \text{const} \times g_{nm} \times \text{EMF}^2 \]

Scaled transition rates:

\[ g_{nm} = 2\rho_F^{-3} \frac{|\mathcal{I}_{nm}|^2}{(E_n - E_m)^2} \delta(\Gamma(E_n - E_m)) \]
Semi Linear Response Theory (cont.)

\( g_{nm} = 2 \rho_F^{-3} \frac{|I_{nm}|^2}{(E_n - E_m)^2} \delta \Gamma (E_n - E_m) \)

The SLRT analog of the Kubo formula:

\[ G = \pi \hbar (\rho(E_F))^2 \langle \langle |I_{mn}|^2 \rangle \rangle \]

where

\[ \langle \langle |I_{mn}|^2 \rangle \rangle \equiv \text{inverse resistivity of the network} \]

\[ \langle \langle |I_{mn}|^2 \rangle \rangle_{\text{harmonic}} \ll \langle \langle |I_{mn}|^2 \rangle \rangle \ll \langle \langle |I_{mn}|^2 \rangle \rangle_{\text{algebraic}} \]
Conductance of mesoscopic rings

Naive expectation (assuming $\Gamma > \Delta$):

$$G = \frac{e^2}{2\pi \hbar} M \frac{\ell}{L} + \mathcal{O} \left( \frac{\Delta}{\Gamma} \right)$$

$L =$ perimeter of the ring  
$\ell =$ mean free path

(!) Diffusive ring: $\ell \ll L$

?? Ballistic ring: $\ell \gg L$
Conductance versus disorder

**Weak disorder** (ballistic rings):
Wavefunctions are localized in mode space.
[See numerical results]

**Strong disorder** (Anderson localization):
Wavefunctions are localized in real space.
[SLRT $\sim$ VRH]
Numerical results for a ballistic ring

![Graphs showing conductance and G/M versus γ and 1-g_T]
LRT, SLRT and beyond

\[-\dot{\Phi} = \text{electro motive force}\]
\[G \dot{\Phi}^2 = \text{rate of energy absorption}\]

**Semi linear response theory**


[4] D. Cohen, "From the Kubo formula to variable range hopping" (PRB 2007)

**Beyond (semi) linear response theory**


Beyond (semi) linear response

\[ EMF = V \text{oltage} \]

\[ \delta E_{cl} = eV_{EMF} \]

\[ A(x; t) = \Phi(t)\delta(x - x_0) = \text{the vector potential.} \]

\[ \mathcal{E}(x) = -\frac{1}{c} \dot{\Phi}\delta(x - x_0) = \text{the electric field.} \]

semiclassical regime: \( \delta E_{cl} \gg \Delta \)

Note: \( \delta E_{cl} \gg \Delta \iff V_{EMF} \gg \frac{\hbar \nu_E}{L} \)
Beyond (semi) linear response: regimes

- adiabatic
- diabatic
- semi-classical

Landau-Zener

Non-adiabatic

\( \Phi \)

\( \dot{\Phi} \)

\( E_n \)

\( \Phi(t) \)
Beyond (semi) linear response: other effects

(ordered by degree of relevance):

\[ H = \{ E_n \} - \Phi(t)\{ \mathcal{I}_{nm} \} \]

Beyond FGR:
Silva and Kravtsov
[cond-mat/0611083]

Non-perturbative response:
Cohen and Kottos
[PRL 2000]

Dynamical localization:
Basko, Skvortsov and Kravtsov
[PRL 2003]