Quantum chaos in Bose-Hubbard circuits

Chaos, irreversibility and Hamiltonian Hysteresis

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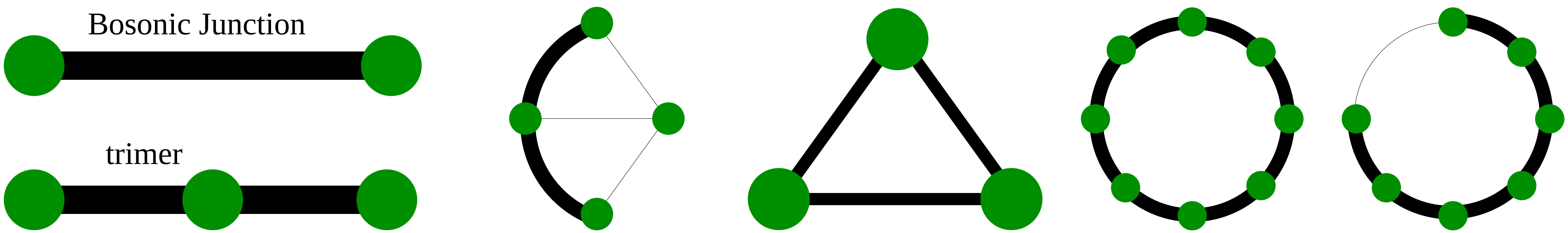
Circuits with condensed bosons are the building blocks for quantum Atomtronics. Such circuits will be used as QUBITs (for quantum computation) or as SQUIDs (for sensing of acceleration or gravitation). We study the feasibility and the design considerations for devices that are described by the Bose-Hubbard Hamiltonian. It is essential to realize that the theory involves "Quantum chaos" considerations.

- The Bose-Hubbard Hamiltonian.
- Relevance of chaos for *Nonlinear Adiabatic Passage*.
- Relevance of chaos for *Hamiltonian Hysteresis*.
- Relevance of chaos for *Metastability and Ergodicity*.



The Bose Hubbard Hamiltonian

The system consists of N bosons in M sites. Optionally we add a gauge-field Φ.

$$\mathcal{H}_{\mathrm{BHH}} = \frac{U}{2}\sum_{j=1}^{M} a_j^\dagger a_j^\dagger a_j a_j - \frac{\Omega}{2}\sum_{j=1}^{M}\left(a_{j+1}^\dagger a_j + a_j^\dagger a_{j+1}\right)$$

$$u \equiv \frac{NU}{\Omega} \qquad \text{[classical, stability, supefluidity, self-trapping]}$$

$$\gamma \equiv \frac{Mu}{N^2} \qquad \text{[quantum, Mott-regime]}$$

Dimer, Many body Landau-Zener dynamics
[semiclassical] Liu, Wu, Niu, et al (PRA 2000 – PRL 2003)
[quantum] Smith-Mannschott, M. Chuchem, M. Hiller, T. Kottos, DC **(PRL 2009)**

Trimer, Adiabatic passage through chaos
[semiclassical] Amit Dey, DC, Amichay Vardi, **(PRL 2018)**
[quantum] Amit Dey, DC, Amichay Vardi, **(PRA 2019)**

Hamiltonian Hysteresis
[dimer] Ralf Burkle, Amichay Vardi, DC, James Anglin, **(arXiv 2019a)**
[trimer] Ralf Burkle, Amichay Vardi, DC, James Anglin, **(arXiv 2019b)**

The standard picture of irreversibility

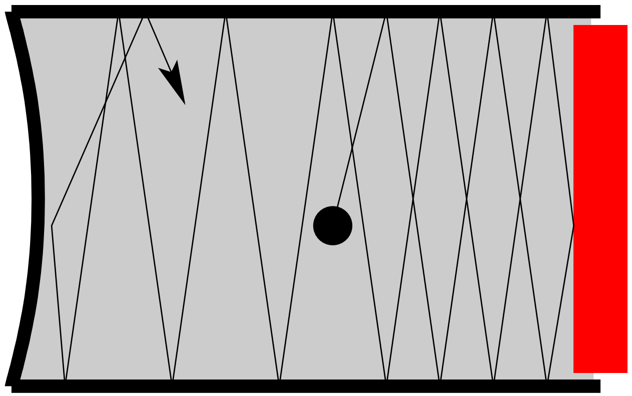
$H[Q, P; x(t)]$

The Ott-Wilkinson-Kubo picture:

$\delta E^2 \ \sim \ 2 D_E \, t$ (diffusion)

$D_E \ \propto \ \dot{x}^2$

$\dot{E} \ \propto \ \dot{x}^2$ (dissipation)



$x(t) \quad = \quad$ piston position

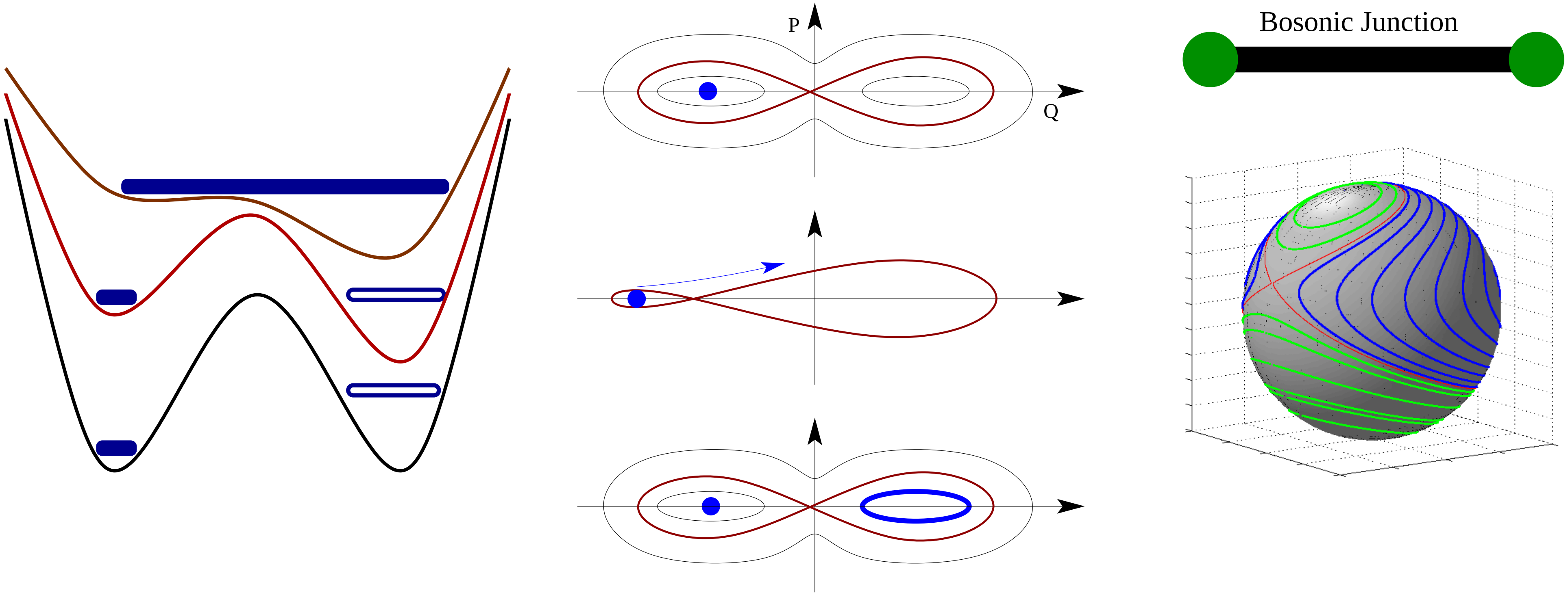
- Chaos - motion of particles (Q, P) inside the box is chaotic
- Quasi static limit - the piston position (x) is varied very slowly

The standard dogma: Irreversibility requires chaos and non-adiabaticity.

But in fact: irreversibility can be observed also for a quasi-static driving of a non-chaotic system.

Irreversibility and Hamiltonian Hysteresis

The twist: separatrix crossing always breaks adiabaticity, even in the quasi static limit.



"How to probe the microscopic onset of irreversibility with ultracold atoms"
Ralf Burkle, Amichay Vardi, DC, James R. Anglin, arXiv (2019a).

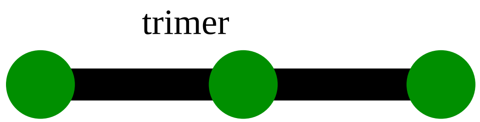
$$H = \frac{x(t)}{2}(\hat{n}_1 - \hat{n}_2) + \frac{U}{2}\sum_{j=1}^{2}\hat{n}_j^2 - \frac{\Omega}{2}\left[a_2^\dagger a_1 + a_1^\dagger a_2\right]$$

Q = occupation difference
P = conjugate phase
$x(t)$ = detuning

Hamiltonian Hysteresis and Chaos

The question arises what are the fingerprints of chaos in Hamiltonian Hysteresis.

Minimal model: The Bose-Hubbard trimer.



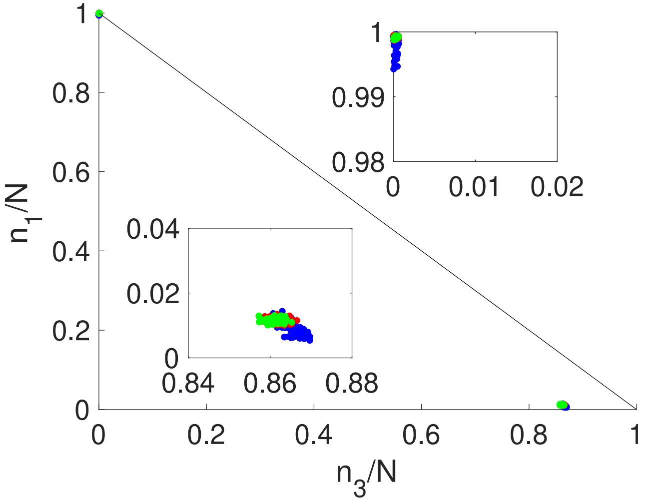
$$H = \frac{x(t)}{2}(\hat{n}_1 - \hat{n}_3) + \frac{U}{2}\sum_{j=1}^{3}\hat{n}_j^2 - \frac{\Omega}{2}\sum_{j=1}^{2}\left[a_{j+1}^{\dagger}a_j + a_j^{\dagger}a_{j+1}\right]$$

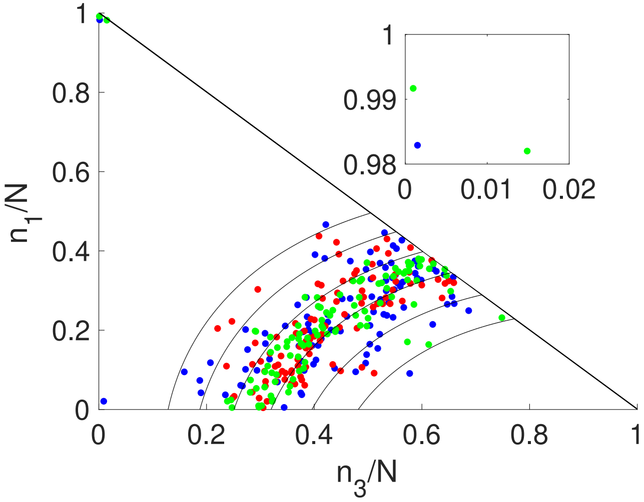
Distinct mechanisms for Hamiltonian Hysteresis:

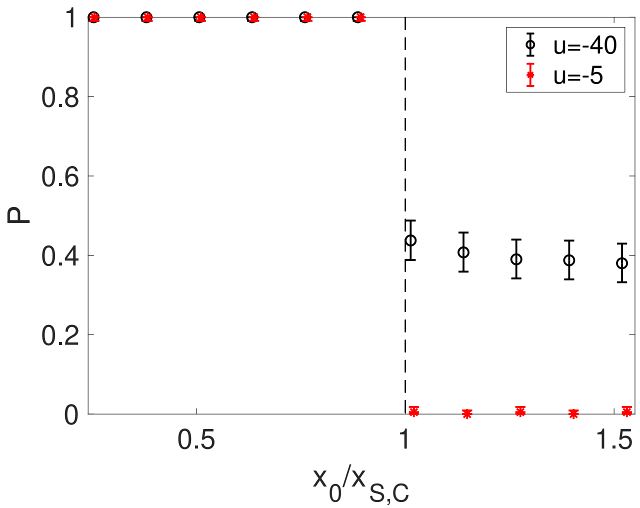
- Irreversible splitting - The high-dimensional version of separatrix crossing.
- Residual irreversibility - Interplay between adiabatic mechanisms (Landau vs Ott)
- Excess irreversibility - diffusion away from the energy surface (Ott-Wilkinson-Kubo picture)

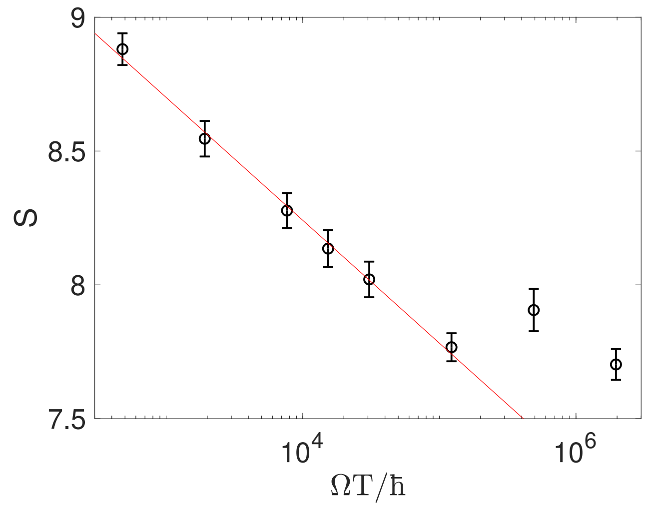
$$\mathcal{W} \;=\; \text{Residual } (\dot{x} \to 0) \;+\; \text{Excess (finite } \dot{x})$$

The experiment: inspecting occupation statistics

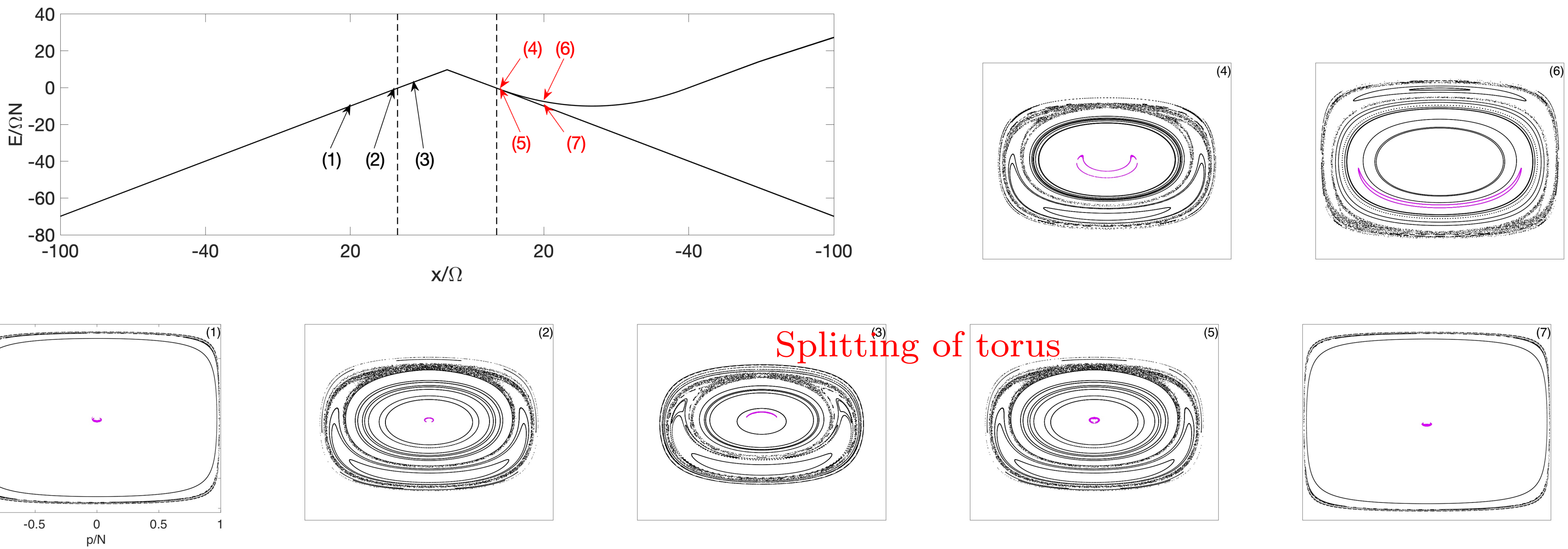








The "irreversible splitting" scenario



The original torus of panels (1-2) is ejected.

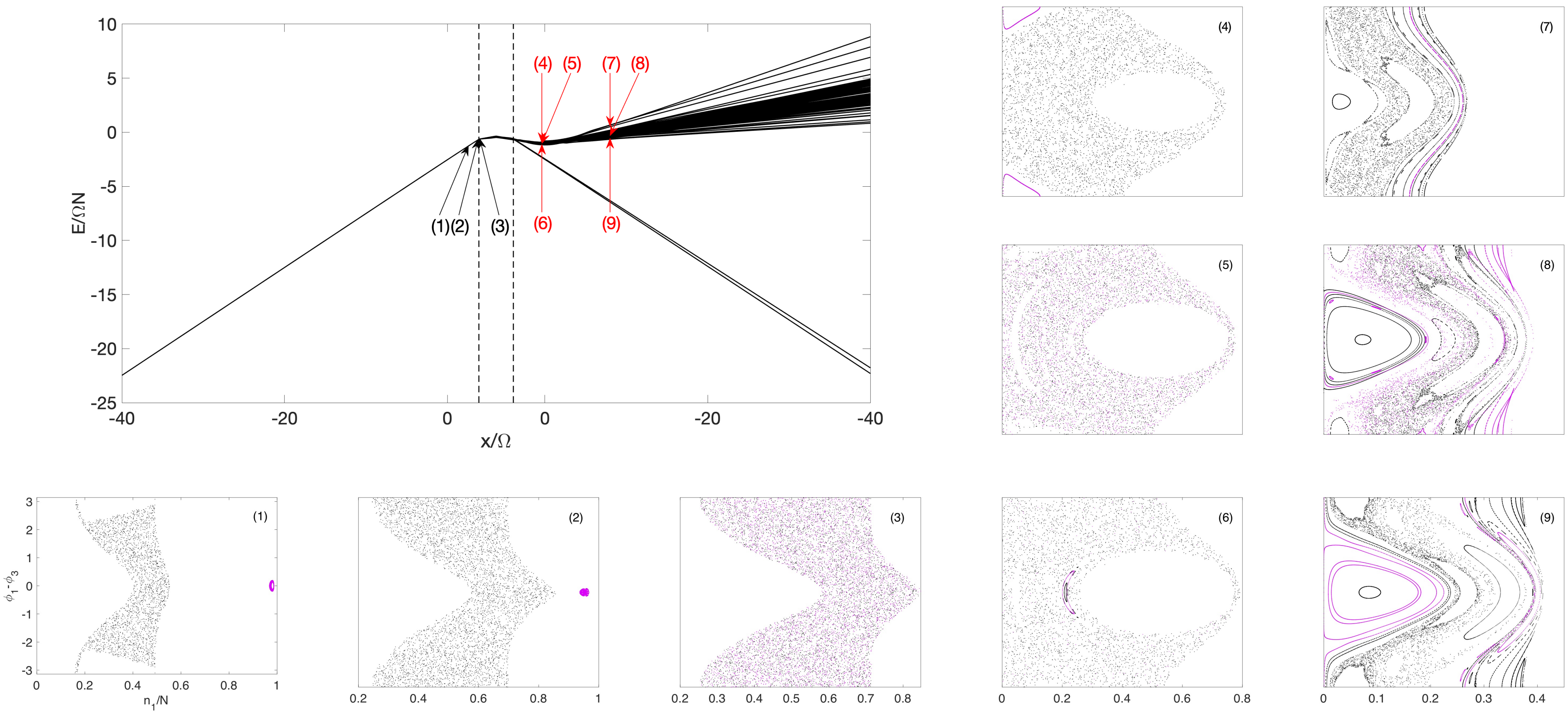
The torus of panel (3) splits into the tori of (4-6) and (5-7).

The torus of (5-7) is the original torus.

The secondary torus: $H[\boldsymbol{I}^1; x_s] = H[\boldsymbol{I}^0; x_s]$

Landau-Einstein adiabaticity: $E(t) \;=\; H[\boldsymbol{I}^k; x(t)], \qquad k = 0, 1$

The "sweep through chaos" scenario



Tori that emerge from the chaotic sea: $H[\boldsymbol{I};x_c] = H[\boldsymbol{I}^0;x_c]$

Landau-Einstein adiabaticity: $E(t) \;=\; H[\boldsymbol{I};x(t)]$

Observations

- Mechanism for irreversibility in the quasi-static limit requires mixed phase-space
- Implied parametric dissipation with rate $\propto \Omega$ instead of $\propto \Omega^2$.

- Sweep through chaos reduces the efficiency of the sweep protocol.
- Slow is bad for efficiency... fast might be better.
- Global phase-space topology is important for efficiency.

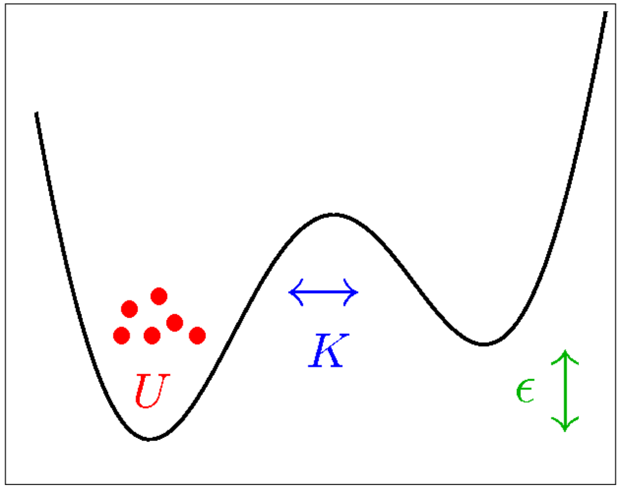
Relation to previous study of BEC dynamical meta-stability.

- Stability: whether a condensate survives for a given x - quench protocol.
- Efficiency: whether a condensate survives for a given $\dot{x}$ - sweep protocol.

Dictionary:

- BEC = coherent state = Gaussian in phase space.
- Energetic stability = Gaussian is sitting in minimum/maximum of the energy landscape.
- Dynamical stability = Gaussian is sitting on an island formed of tori.

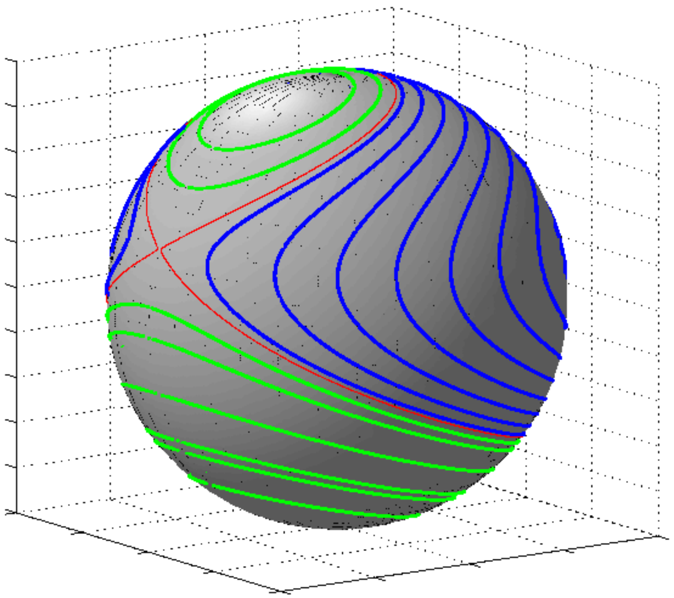
The many body Landau-Zener transition

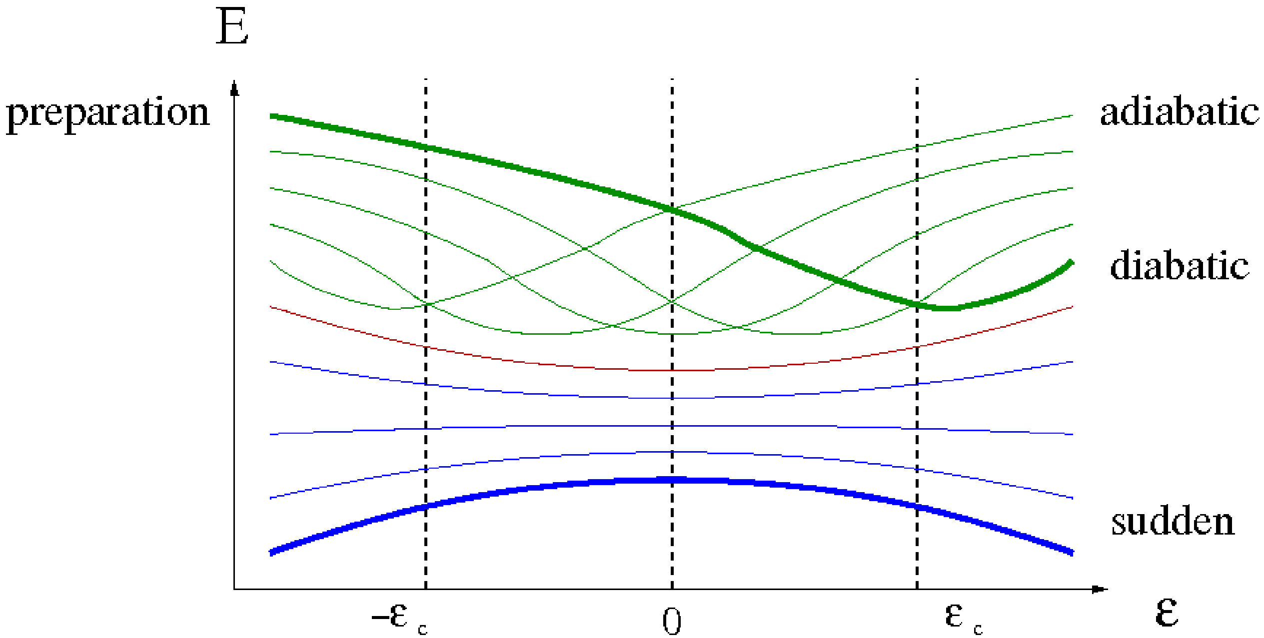


Classical picture: Liu, Wu, Niu, et al (PRA 2000 - PRL 2003)

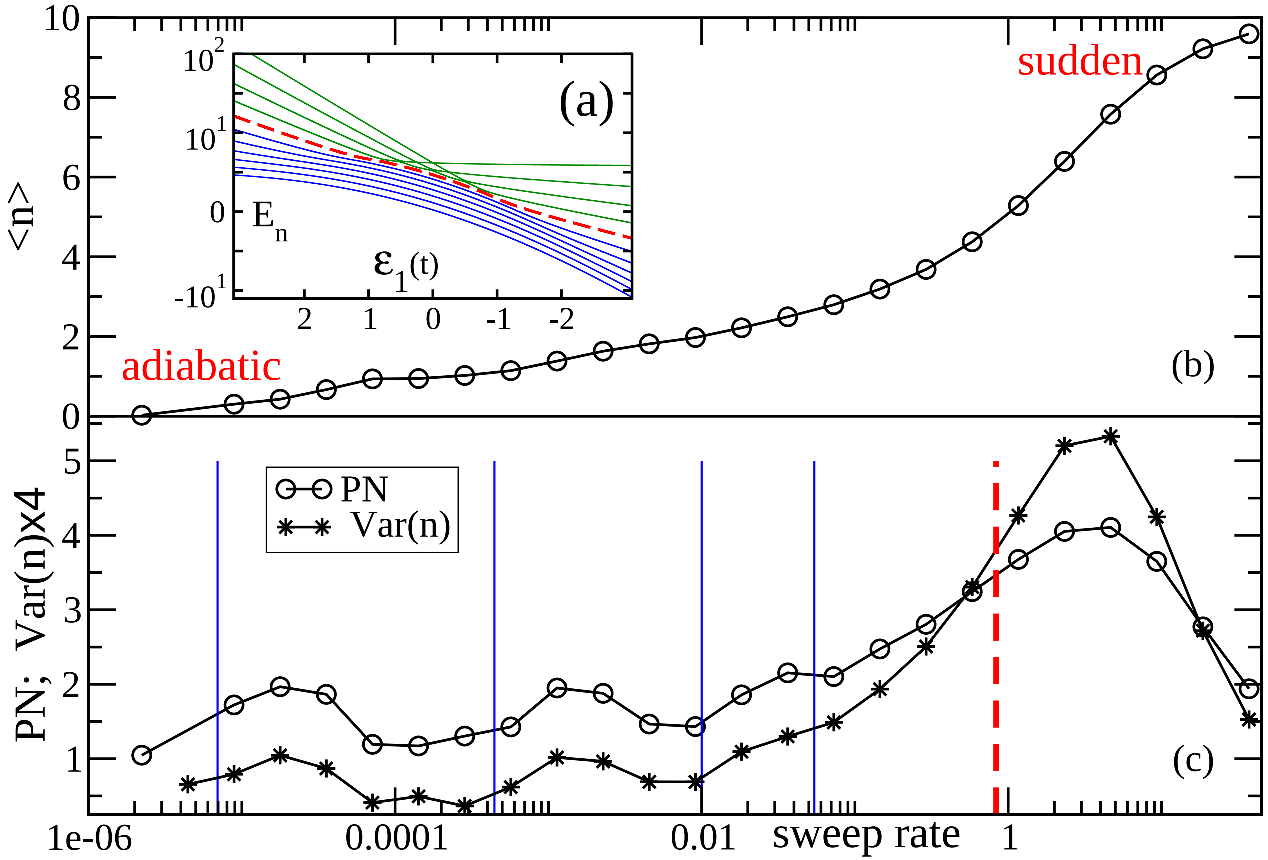
Quantum dynamical scenarios: adiabatic/diabatic/sudden

Smith-Mannschott, Chuchem, Hiller, Kottos, DC (PRL 2009).





Occupation Statistics

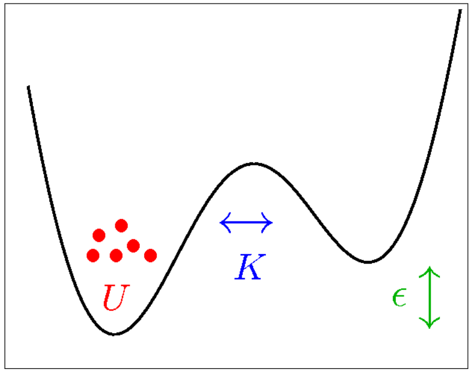


Adiabtic-diabatic (quantum) crossover

Diabatic-sudden (semiclassical) crossover

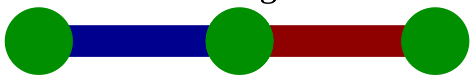
Smith-Mannschott, Chuchem, Hiller, Kottos, DC (PRL 2009).

The *nonlinear adiabatic passage* scenario



Bosonic Junction

STIRAP through chaos

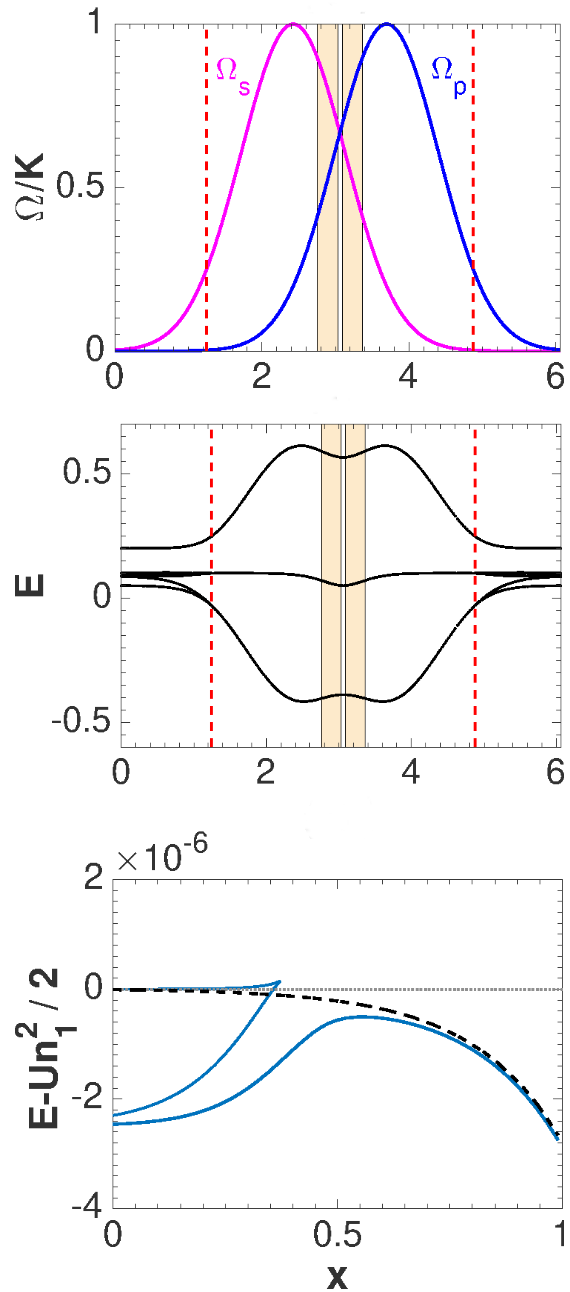


Hamiltonian:

$$\mathcal{H} = \mathcal{E}\hat{n}_2 + \frac{U}{2}\sum_{j=1}^{3}\hat{n}_i^2$$
$$-\frac{1}{2}\left(\Omega_p(x)\hat{a}_0^\dagger\hat{a}_1 + \Omega_s(x)\hat{a}_2^\dagger\hat{a}_0\right)$$

Adiabatic condition:

$$\frac{\xi_s}{t_s} < \dot{x} < \frac{1}{3\pi}\Omega$$



Traditional paradigm:

Liu, Wu, Niu, et al (PRA 2000 – PRL 2003)
Graefe, Korsch, Witthaut (PRA 2006)

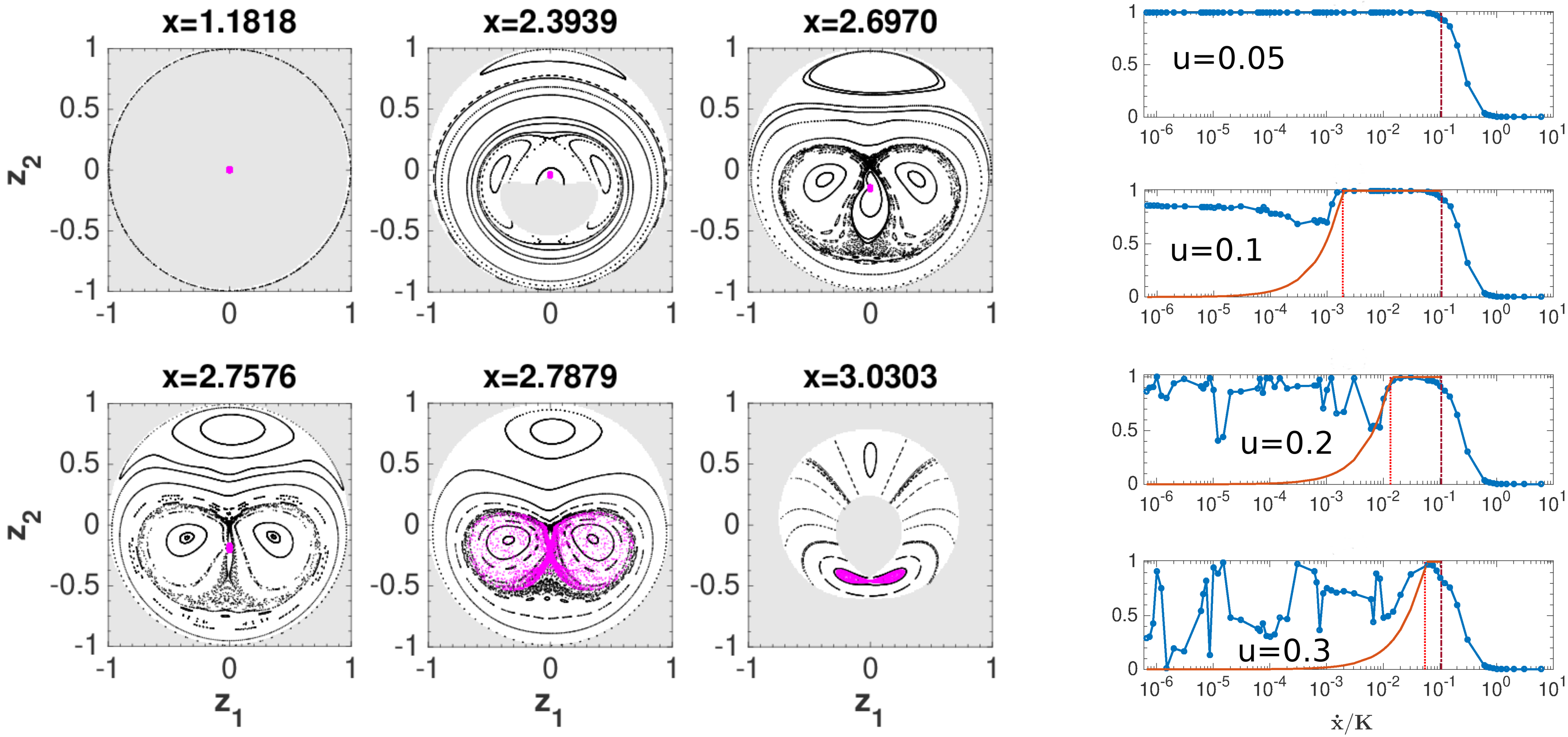
New paradigm:

Amit Dey, DC, AV (PRL 2018)

$$i\begin{pmatrix}\dot{a}_1\\ \dot{a}_0\\ \dot{a}_2\end{pmatrix} = \begin{pmatrix} U\,|a_1|^2 & -\frac{\Omega_p}{0} & 0\\ -\frac{\Omega_p}{2} & \mathcal{E}+u\,|a_0|^2 & -\frac{\Omega_s}{2}\\ 0 & -\frac{\Omega_s}{2} & U\,|a_2|^2\end{pmatrix}\begin{pmatrix}a_1\\ a_0\\ a_2\end{pmatrix}$$

The *passage through chaos* mechanism

The followed stationary-point is chocked by a stochastic-belt.



The Bogoliubov frequencies

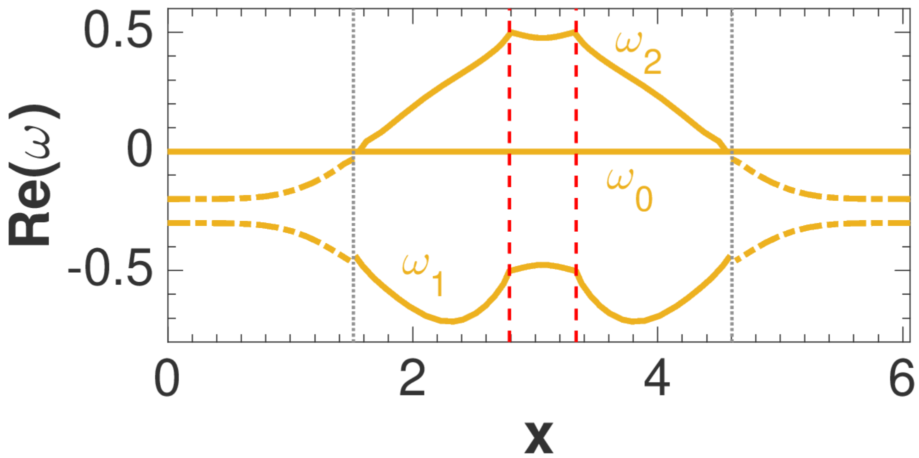
The Bogoliubov procedure brings the Hamiltonian in the vicinity of the SP to a diagonalized form.

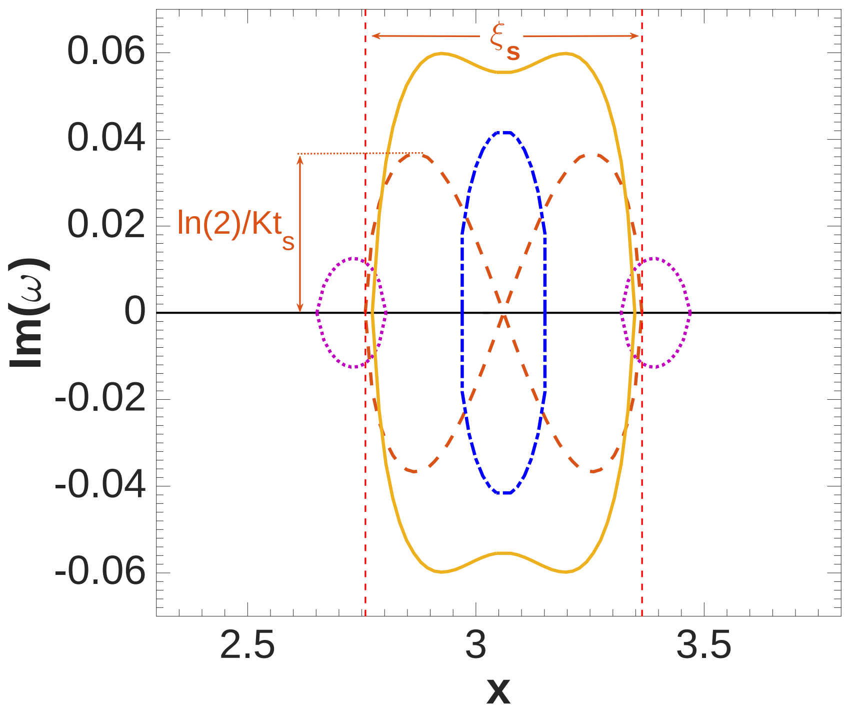
$$H \quad \approx \quad E[\mathrm{SP}] + \sum_q \omega_q c_q^\dagger c_q$$

Equations of motion: $\dot{z} = \mathbb{J}\partial H$

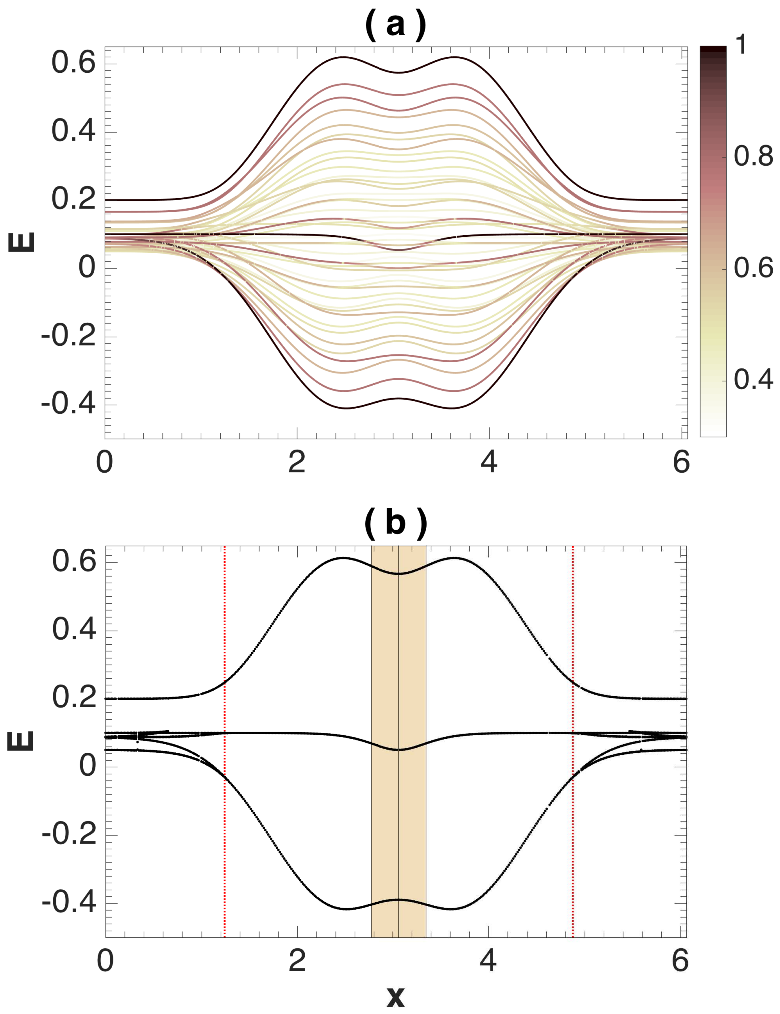
For one dof –
Canonical coordinates: $z = (a, \bar{a})$
Symplectic matrix: $\mathbb{J}$ is the second Pauli matrix.

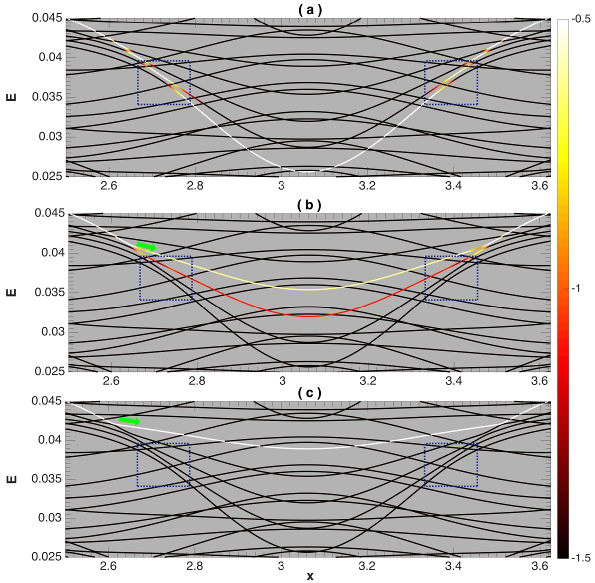
In our case there are 3 dof –
Linarized equations: $\dot{z} = [\mathbb{J}\mathcal{A}]\, z$
Characteristic equation: $\det(\lambda - \mathbb{J}\mathcal{A}) = 0$
Eigenvalues are: $\lambda_{q,\pm} = \pm i\omega_q$ (note the sign issue)
One frequency is zero:
total occupation (N) is conserved.

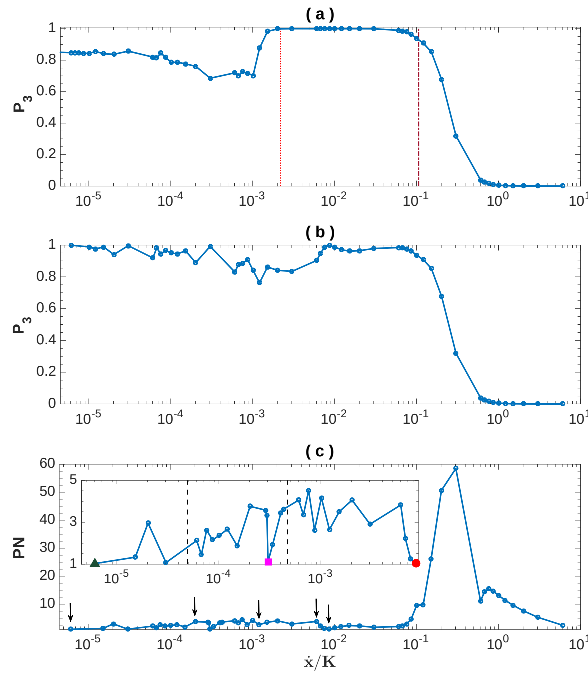




Quantum detours around chaos







Spectra (above): $N = 8$

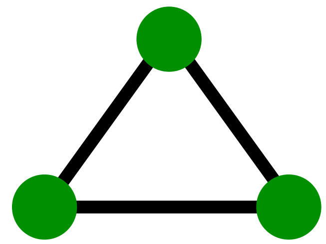
Simulations (right): $N = 30$

Purity: $\gamma = \mathrm{Tr}\left([\rho^{(sp)}]^2\right)$

Quantum Chaos perspective on Metastability and Ergodicity

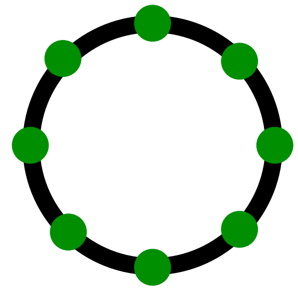
Stability of flow-states (I):

- Landau stability of flow-states ("Landau criterion")
- Bogoliubov perspective of dynamical stability
- KAM perspective of dynamical stability



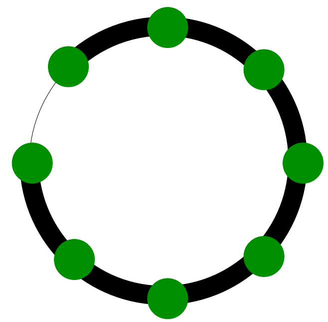
Stability of flow-states (II):

- Considering high dimensional chaos ($M > 3$).
- Web of non-linear resonances.
- Irrelevance of the the familiar Beliaev and Landau damping terms.
- Analysis of the quench scenario.



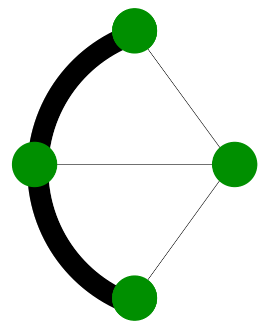
Coherent Rabi oscillations:

- The hallmark of coherence is Rabi oscillation between flow-states.
- Ohmic-bath perspective $\rightsquigarrow \quad \eta = (\pi/\sqrt{\gamma})$
- Feasibility of Rabi oscillation for $M < 6$ devices.
- Feasibility of of chaos-assisted Rabi oscillation.



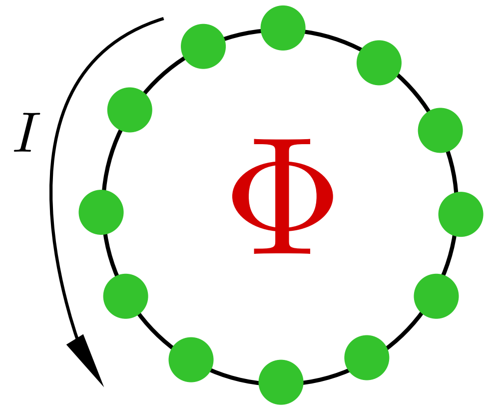
Thermalization:

- Spreading in phase space is similar to Percolation.
- Resistor-Network calculation of the diffusion coefficient.
- Observing regions with Semiclassical Localization.
- Observing regions with Dynamical Localization.



Bose Hubbard Ring

In the rotating reference frame we have a Coriolis force, which is like magntic field $\mathcal{B} = 2\mathsf{m}\Omega$. Hence is is like having flux



$$\Phi = 2\pi R^2 \mathsf{m}\,\Omega$$

$$\mathcal{H} = \sum_{j=1}^{M} \left[\frac{U}{2} a_j^\dagger a_j^\dagger a_j a_j - \frac{K}{2} \left(\mathrm{e}^{i(\Phi/L)} a_{j+1}^\dagger a_j + \mathrm{e}^{-i(\Phi/L)} a_j^\dagger a_{j+1} \right) \right]$$

$$\mathcal{H} = \sum_k \epsilon_k(\Phi)\, b_k^\dagger b_k + \frac{U}{2L} \sum{}' \, b_{k_4}^\dagger b_{k_3}^\dagger b_{k_2} b_{k_1}$$

For L=3 sites, using $b_k = \sqrt{n_k} e^{i\varphi_k}$, and $M = (n_1 - n_2)/2$, and $n = (n_1 + n_2)/2$

$$\mathcal{H}(\varphi, n; \phi, M) = \mathcal{H}^{(0)}(\varphi, n; M) + \left[\mathcal{H}^{(+)} + \mathcal{H}^{(-)} \right]$$

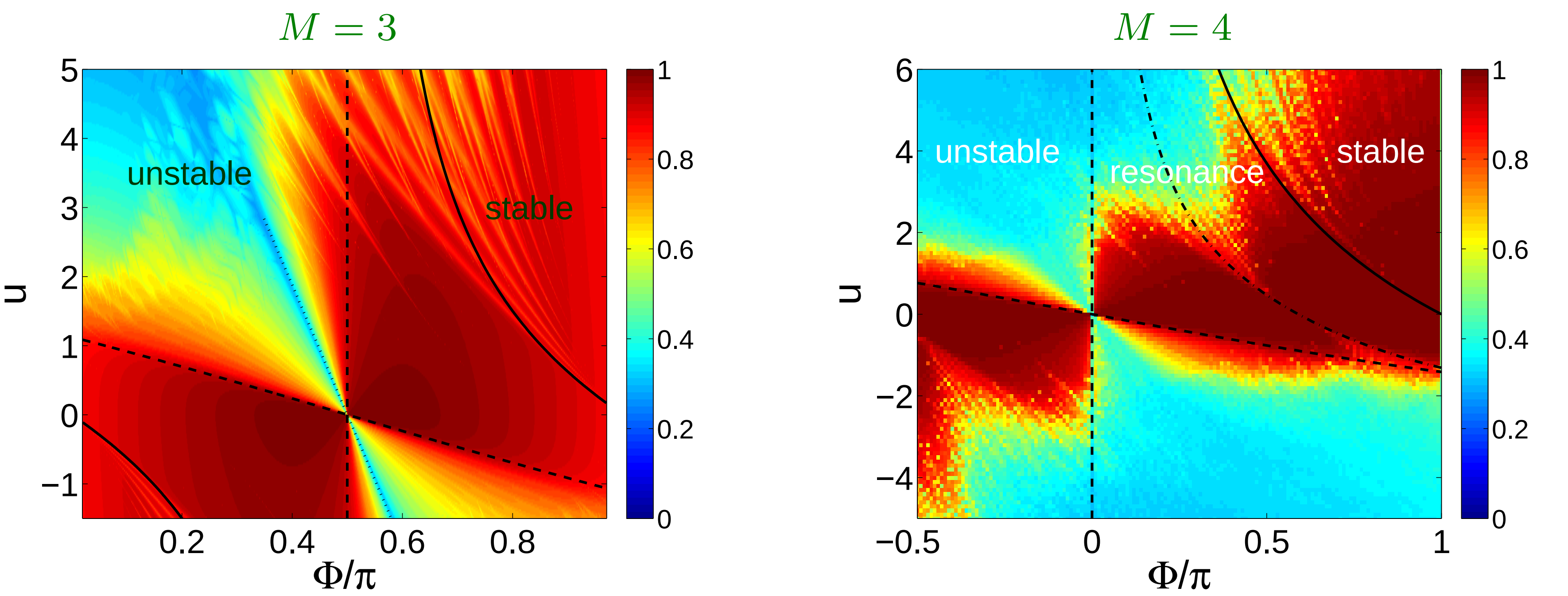
$$\mathcal{H}^{(0)}(\varphi, n; M) = \mathcal{E} n + \mathcal{E}_\perp M - \frac{U}{3} M^2 + \frac{2U}{3} (N - 2n) \left[\frac{3}{4} n + \sqrt{n^2 - M^2} \cos(\varphi) \right]$$

$$\mathcal{H}^{(\pm)} = \frac{2U}{3} \sqrt{(N-2n)(n \pm M)} (n \mp M) \cos\left(\frac{3\phi \mp \varphi}{2} \right)$$

Flow-state stability regime diagram

The I of the maximum current state is imaged as a function of (Φ, u)

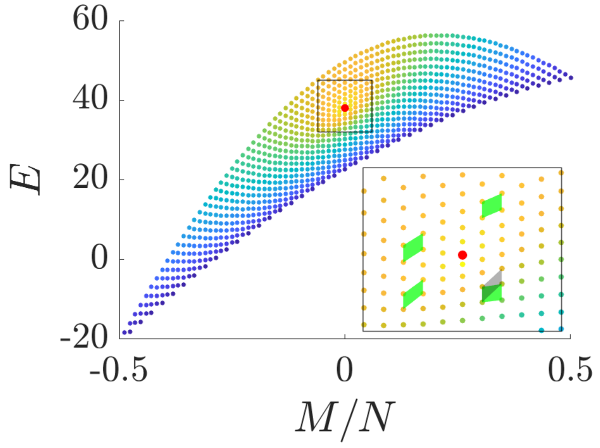
- solid lines = energetic stability borders (Landau)
- dashed lines = dynamical stability borders (Bogoliubov)

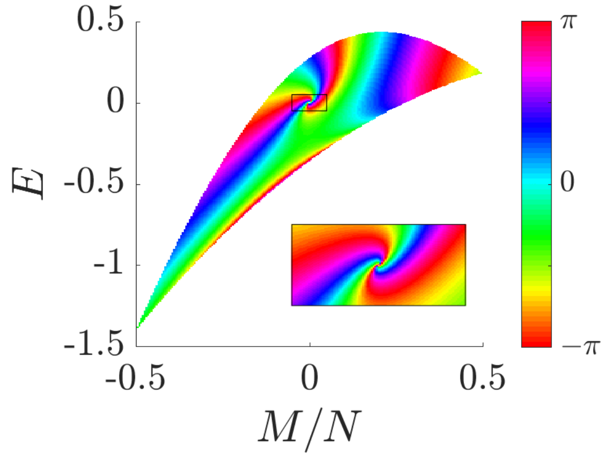


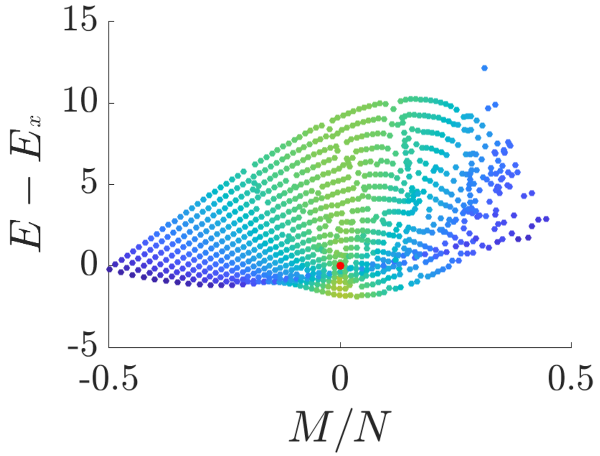
The traditional paradigm associates flow-states with stationary fixed-points in phase space. Consequently the Landau criterion, and more generally the Bogoliubov linear-stability-analysis, are used to determine the viability of superfluidity.

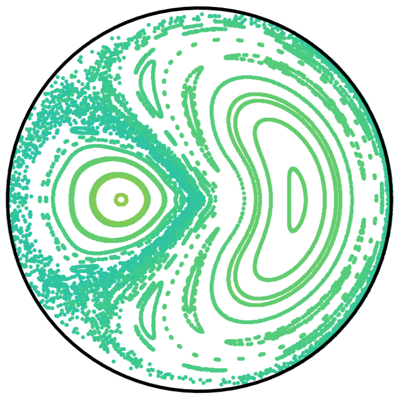
Arwas, Vardi, DC [**Scientific Reports 2015**]

Monodromy, Chaos, and Metastability of superflow









$M = (n_1 - n_2)/2$, constant of motion.
$n = (n_1 + n_2)/2$, conjugate phase φ.

Section energy $E = E[\mathrm{SP}]$
Section coordinates (φ, n)

Regular separatrix $E = E_x(M)$

The swap transition

Arwas, DC [**PRA 2019**]