Quantum chaos and perturbation theory: from the analysis of wavefunctions to the implications on pumping, dissipation and decoherence

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$GIF, ISF$
The subject of this talk is $\Gamma$

$\Gamma$ is an energy scale (inspired by Wigner).

$\Gamma$ is used as a measure for the size of the perturbation.

$\Gamma$ is used to distinguish between prt and non-prt regimes.

- Regimes in the theory of quantum dynamics [1]
- Regimes in LDOS / wavepacket dynamics [1,2,3]
- Regimes in the study of fidelity [4]
- Regimes in the theory of driven systems [1,5]
- The role of $\Gamma$ in quantum pumping [6,7]
- Regimes in the theory of quantum dissipation [8]

[8] DC + Kottos, PRE(R) 2004
The Hamiltonian

\[ \mathcal{H} = \mathcal{H}(Q, P; x) \]

\( x = \) control parameter

The system is chaotic

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case [1] - LDOS / wavepacket dynamics:

\[ \mathcal{H}_0 = \mathcal{H}(Q, P; x_0) \]

\[ \mathcal{H} = \mathcal{H}(Q, P; x) \]

The Hamiltonians \( \mathcal{H}_0 \) and \( \mathcal{H} \) are equally chaotic.

The size of the perturbation: \( \delta x = x - x_0 \)

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case [2] - Driven chaotic(!) system:

Linear driving: \( x(t) = x_0 + Vt \)

Periodic driving: \( x(t) = A \sin(\Omega t) \)

The size of the perturbation: \( \dot{x} \)
The parameter $\Gamma$ - case [1]

\[ H_{nm} = E_n \delta_{nm} + \delta x B_{nm} \]

$\Delta = \text{mean level spacing}$

$\Delta_b = \text{bandwidth}$

$|B_{nm}| \sim \sigma \text{ for } |E_n - E_m| < \Delta_b$

Assume a small constant perturbation $\delta x$

$\Gamma(\delta x) \approx \left( \frac{\sigma \delta x}{\Delta} \right)^2 \Delta$ \hspace{1cm} [Wigner]

$\Gamma/\Delta$ is the number of levels that are mixed non-perturbatively,

as implied by perturbation theory (to infinite order).

$\Gamma < \Delta \quad \sim \quad \text{1st order perturbation theory}$

$\Gamma < \Delta_b \quad \sim \quad \text{perturbative (Wigner/FGR) regime}$

\textit{else} \quad \sim \quad \text{non-perturbative (semiclassical?) regime}
The parameter $\Gamma$ - case [2]

Re-write the Hamiltonian in the adiabatic basis:

$$\mathcal{H}_{nm} = E_n \delta_{nm} + \dot{x} \frac{i\hbar B_{nm}}{E_n - E_m}$$

Assume a slow variation $\dot{x}$

$$\Gamma(\dot{x}) \approx \left( \frac{\hbar \sigma \dot{x}}{\Delta^2} \right)^{2/3} \Delta$$

$$\Gamma < \Delta \quad \sim \quad \text{adiabatic regime}$$  
$$\Gamma < \Delta_b \quad \sim \quad \text{perturbative (FGR/Kubo) regime}$$  
else \quad \sim \quad \text{non-perturbative (semiclassical?) regime}

Wilkinson and Austin, JPA 1995 -  
An attempt to derive / challenge the Qchaos Kubo

DC, PRL 1999, Annals 2000 -  
Derivation of the Qchaos Kubo with the condition $\Gamma < \Delta_b$

Wilkinson, JPA 2002 -  
Alternate (semiclassical) derivation of the condition
case [3] - Interaction with environment

\[ \mathcal{H}_{\text{total}} = \mathcal{H}_0(x, p) + \mathcal{H}(Q, P; x) \]

After linearization (say \( x_0 = 0 \))

\[ \mathcal{H}_{\text{total}} = \mathcal{H}_0(x, p) + E + xB \]

- interaction with bath (harmonic oscillators)
- interaction with chaos (few freedoms environment)
- interaction with RMT modeled environment

Assume that \( x \) performs motion with amplitude \( A \) and velocity \( V \).

\( \Gamma \) is related to \( (\sigma A)^2 \) and \( (\sigma V)^2 \) as follows:

\[
\frac{\Gamma}{\Delta} = \text{minimum} \left( \left( \frac{\sigma A}{\Delta} \right)^2, \left( \frac{\hbar \sigma}{\Delta^2} V \right)^{2/3} \right)
\]

\[ \Gamma < \Delta \quad \sim \quad \text{Born-Oppenheimer regime} \]

\[ \Gamma < \Delta_b \quad \sim \quad \text{F-V regime (effective bath picture)} \]

else \( \sim \) non-perturbative (semiclassical?) regime

For a two level system

\[ K = \frac{1}{16\pi} \left( \frac{\Gamma}{T} \right) = \text{Kondo Parameter} \]
Driven Systems

Non interacting “spinless” electrons.

Held by a potential (e.g. AB ring geometry).

\[ x_1, x_2 = \text{shape parameters} \]

\[ x_3 = \Phi = (\hbar/e)\phi = \text{magnetic flux} \]
“Ohm law”

For one parameter driving by EMF

\[ I = G^{33} \times (-\dot{x}_3) \]
\[ dQ = -G^{33} \, dx_3 \]

For driving by changing another parameter

\[ I = -G^{31} \, \dot{x}_1 \]
\[ dQ = -G^{31} \, dx_1 \]

For two parameter driving

\[ I = -G^{31} \dot{x}_1 - G^{32} \dot{x}_2 \]
\[ dQ = -G^{31} \, dx_1 - G^{32} \, dx_2 \]
\[ Q = -\oint G \cdot dx \]

and in general

\[ \langle F^k \rangle = -\sum_j G^{kj} \, \dot{x}_j \]
What is the problem?

From Kubo formula we get a formal expression for $G^{kj}$.

Can we trust this expression? Conditions? Quantum chaos!

How to use this expression? The bare Kubo formula gives no dissipation!

To define an energy scale $\Gamma$ Beyond first order perturbation theory!

$\Gamma$ in case of isolated system is due to non-adiabaticity.

$\Gamma$ affects both the dissipative and the non-dissipative (geometric) part of the response.
From Kubo to a “FD relation”

\[ \mathcal{H} = \mathcal{H}(\mathbf{r}, \mathbf{p}; x_1(t), x_2(t), x_3(t)) \]

\[ \mathcal{F}^k = -\frac{\partial \mathcal{H}}{\partial x_k} \]

**Generalized Ohm law:**

\[ \langle \mathcal{F}^k \rangle = -\sum_j G^{kj} \dot{x}_j \]

\[ K^{kj}(\tau) = \frac{i}{\hbar} \langle [\mathcal{F}^k(\tau), \mathcal{F}^j(0)] \rangle \]

\[ C^{kj}(\tau) = \frac{1}{2} \left( \langle \mathcal{F}^k(\tau) \mathcal{F}^j(0) \rangle + cc \right) \]

\[ G^{kj} = \lim_{\omega \to 0} \frac{\text{Im} \chi^{kj}(\omega)}{\omega} = \int_0^\infty K_F^{kj}(\tau) \tau d\tau \]

\[ = g(E_F) \int_0^\infty C^{kj}_{E_F}(\tau) d\tau \]

DC, PRB(R) 2003
BPT versus Geometric magnetism

\[ G^{kj} = g(E_F) \int_0^\infty C_{EF}^{kj}(\tau) d\tau \]

Due to non-adiabaticity

\[ C_{E}^{kj}(\tau) \rightarrow C_{E}^{kj}(\tau) \exp\left(-\frac{\Gamma}{\hbar} t\right) \]

\[ \Gamma = \left( \frac{\hbar \sigma}{\Delta^2} |\dot{x}| \right)^{2/3} \Delta \sim \left( L |\dot{x}| \right)^{2/3} \frac{1}{L} \]

For \( x_3 = 0 \) one obtains \( G^{kj} = B^{kj} \) where

\[ B^{kj} = 2\hbar \sum_n f(E_n) \sum_{m(\neq n)} \frac{\text{Im} \left[ \mathcal{F}_{nm}^k \mathcal{F}_{mn}^j \right]}{(E_m - E_n)^2 + (\Gamma/2)^2} \]

For dot-wire system in the \( L \rightarrow \infty \) limit

we have \( \Delta \ll \Gamma \rightarrow 0 \) leading to

\[ G^{3j} = \frac{e}{2\pi i} \text{trace} \left( P_A \frac{\partial S}{\partial x_j} S^\dagger \right) \quad \text{[BPT]} \]
Simple model systems - networks

\[\begin{align*}
dQ &= (1-g_0) \times \frac{e}{\pi} k_F \times dX \\
dQ &= 1 \times \frac{e}{\pi} k_F \times dX \\
dQ &= \left[ \frac{g_T}{1-g_T} \right] \left[ \frac{1-g_0}{g_0} \right] \times \frac{e}{\pi} k_F \times dX
\end{align*}\]

Adiabatic versus non-adiabatic result

DC, Kottos, Schanz, cond-mat 2004
Speculations...

For the open system, using BPT:

\[ Q \approx 1 - g_0 \]

- Is it due to non-adiabaticity?
- Is it a dissipative contribution?

What about strict adiabatic cycle in a closed system?

Shutenko, Aleiner, Altshuler (2000):

“If the system were closed [and strictly adiabatic], the charge distribution after each period of the perturbation would return to the original distribution, and therefore, the pumped charge would be exactly quantized.”

Not correct!
The $B$ field

$$\vec{B} = (B^{12}, B^{23}, B^{31})$$

$$B^{kj} = \sum_n f(E_n) B_{n}^{kj}$$

$$B_{n}^{kj} = 2\hbar \sum_{m(\neq n)} \frac{\text{Im} \left[ F_{km} F_{jn} \right]}{(E_m - E_n)^2 + (\Gamma/2)^2}$$

This field is divergenceless \text{(for } \Gamma = 0)\text{ [geometric magnetism]}$

A chain of degeneracies:

$$\left( x_1^{(0)}, x_2^{(0)}, \Phi^{(0)} + 2\pi \frac{e}{\hbar} \times \text{integer} \right)$$

The degeneracies are like Dirac monopoles

- The issue of bandwidth
- The effect of screening
The two barrier model

\[ E_n \]

wire states

dot state

\[ \frac{1}{C_1} \quad \frac{1}{C_2} \]

position

\[ X_1 = \frac{1}{2}(C_1 - C_2) = \text{left-right bias} \]
\[ X_2 = -\frac{1}{2}(C_1 + C_2) = \text{dot potential floor} \]