Quantum chaos and perturbation theory: from the analysis of wavefunctions to the implications on pumping, dissipation and decoherence

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$GIF, $ISF

The subject of this talk is

Γ

Γ is an energy scale (inspired by Wigner).

Γ is used as a measure for the size of the perturbation.

Γ is used to distingushed between prt and non-prt regimes.

- Regimes in the theory of quantum dynamics [1]
- Regimes in LDOS / wavepacket dynamics [1,2,3]
- Regimes in the study of fidelity [4]
- Regimes in the theory of driven systems [1,5]
- The role of Γ in quantum pumping [6,7]
- Regimes in the theory of quantum dissipation [8]

[1] DC, PRL 1999, Annals 2000

[2] DC + Heller, PRL 2000

[3] Mentez + Kottos + Cohen 2004

[4] Jacquod + Adagdeli + Beenakker, PRL 2002

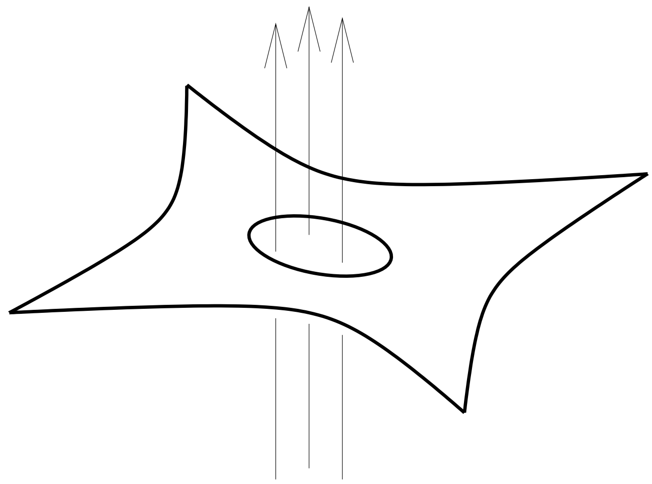
[5] DC + Kottos, PRL 2000 etc.

[6] DC, PRB 2003, PRB(R) 2003

[7] DC + Kottos + Holger 2004

[8] DC + Kottos, PRE(R) 2004

The Hamiltonian



$\mathcal{H} \;=\; \mathcal{H}(Q,P;x)$

$x \;=\;$ control parameter

The system is chaotic

case [1] - LDOS / wavepacket dynamics:

$\mathcal{H}_0 \;=\; \mathcal{H}(Q,P;x_0)$

$\mathcal{H} \;=\; \mathcal{H}(Q,P;x)$

The Hamiltonians $\mathcal{H}_0$ and $\mathcal{H}$ are equaly chaotic.

The size of the perturbation: $\delta x = x - x_0$

case [2] - Driven chaotic(!) system:

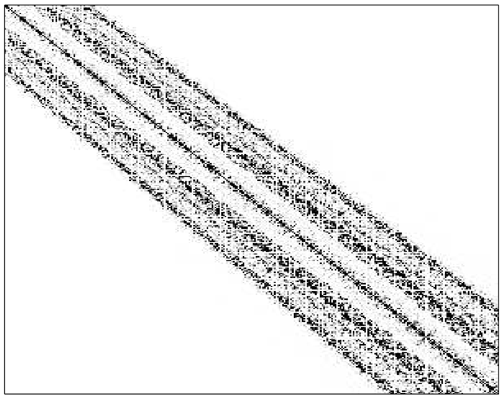
Linear driving: $x(t) = x_0 + Vt$

Periodic driving: $x(t) = A\sin(\Omega t)$

The size of the perturbation: $\dot{x}$

The parameter Γ - case [1]

$$\mathcal{H}_{nm} = E_n \delta_{nm} + \delta x \boldsymbol{B}_{nm}$$



Δ = mean level spacing

Δ_b = bandwidth

$|\boldsymbol{B}_{nm}| \sim \sigma$ for $|E_n - E_m| < \Delta_b$

Assume a small constant perturbation δx

$$\Gamma(\delta x) \approx \left(\frac{\sigma \delta x}{\Delta}\right)^2 \Delta \qquad \text{[Wigner]}$$

Γ/Δ is the number of levels that are mixed non-perturbatively,

as implied by perturbation theory (to infinite order).

$\Gamma < \Delta \quad \rightsquigarrow \quad$ 1st order perturbation theory

$\Gamma < \Delta_b \quad \rightsquigarrow \quad$ perturbative (Wigner/FGR) regime

else $\quad \rightsquigarrow \quad$ non-perturbative (semiclassical?) regime

The parameter Γ - case [2]

Re-write the Hamiltonian in the adiabatic basis:

$$\mathcal{H}_{nm} = E_n \delta_{nm} + \dot{x} \frac{i\hbar \boldsymbol{B}_{nm}}{E_n - E_m}$$

Assume a slow variation $\dot{x}$

$$\Gamma(\dot{x}) \approx \left(\frac{\hbar \sigma \dot{x}}{\Delta^2}\right)^{2/3} \Delta$$

$\Gamma < \Delta$ $\rightsquigarrow$ adiabatic regime

$\Gamma < \Delta_b$ $\rightsquigarrow$ perturbative (FGR/Kubo) regime

else $\rightsquigarrow$ non-perturbative (semiclassical?) regime

Wilkinson and Austin, JPA 1995 -
An attempt to derive / challange the Qchaos Kubo

DC, PRL 1999, Annals 2000 -
Derivation of the Qchaos Kubo with the condition $\Gamma < \Delta_b$

Wilkinson, JPA 2002 -
Alternate (semiclassical) derivation of the condition

case [3] - Interaction with environment

$$\mathcal{H}_{\text{total}} = \mathcal{H}_0(x,p) + \mathcal{H}(Q,P;x)$$

After linearization (say $x_0 = 0$)

$$\mathcal{H}_{\text{total}} = \mathcal{H}_0(x,p) + \boldsymbol{E} + x\boldsymbol{B}$$

- interaction with bath (harmonic oscillators)
- interaction with chaos (few freedoms environment)
- interaction with RMT modeled environment

Assume that x performs motion with amplitude A and velocity V.

Γ is related to $(\sigma A)^2$ and $(\sigma V)^2$ as follows:

$$\frac{\Gamma}{\Delta} = \text{minimum}\left(\left(\frac{\sigma}{\Delta}A\right)^2, \left(\frac{\hbar\sigma}{\Delta^2}V\right)^{2/3}\right)$$

$\Gamma < \Delta \quad \rightsquigarrow \quad$ Born-Oppenheimer regime

$\Gamma < \Delta_b \quad \rightsquigarrow \quad$ F-V regime (effective bath picture)

else $\quad \rightsquigarrow \quad$ non-perturbative (semiclassical?) regime

For a two level system

$$K = \frac{1}{16\pi}\left(\frac{\Gamma}{T}\right) = \text{Kondo Parameter}$$

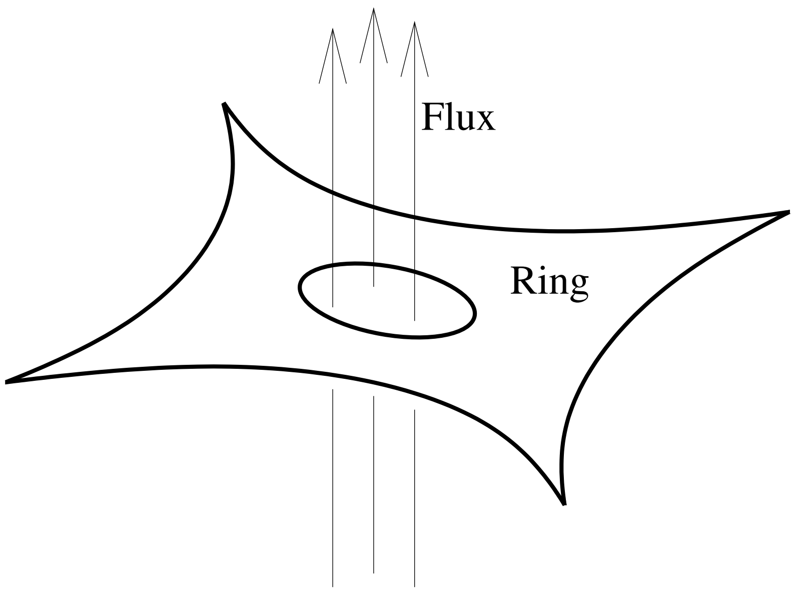
Driven Systems

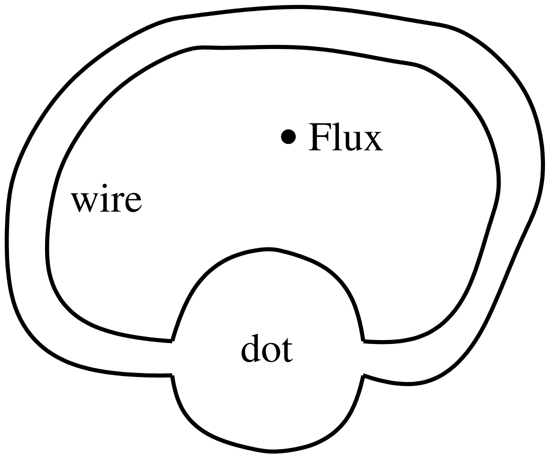
Non interacting “spinless” electrons.

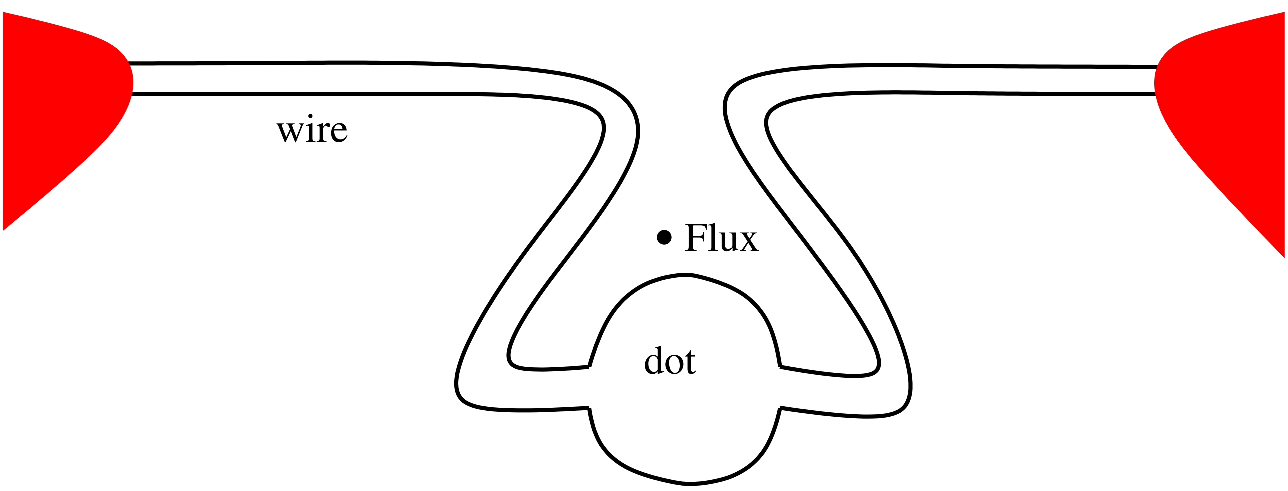
Held by a potential (e.g. AB ring geometry).

$x_1, \; x_2 \;\; = \;\;$ shape parameters

$x_3 \; = \; \Phi \; = \; (\hbar/e)\phi \; = \;$ magnetic flux







"Ohm law"

For one parameter driving by EMF

$$I = \boldsymbol{G}^{33} \times (-\dot{x}_3)$$

$$dQ = -\boldsymbol{G}^{33}\ dx_3$$

For driving by changing another parameter

$$I = -\boldsymbol{G}^{31}\ \dot{x}_1$$

$$dQ = -\boldsymbol{G}^{31}\ dx_1$$

For two parameter driving

$$I = -\boldsymbol{G}^{31}\dot{x}_1\ - \boldsymbol{G}^{32}\dot{x}_2$$

$$dQ = -\boldsymbol{G}^{31}\ dx_1\ - \boldsymbol{G}^{32}\ dx_2$$

$$Q = -\oint \boldsymbol{G} \cdot dx$$

and in general

$$\langle \mathcal{F}^k \rangle = -\sum_j \boldsymbol{G}^{kj}\ \dot{x}_j$$

What is the problem?

From Kubo formula we get
a formal expression for $\boldsymbol{G}^{kj}$.

Can we trust this expression? Conditions?
Quantum chaos!

How to use this expression?
The bare Kubo formula gives no dissipation!

To define an energy scale Γ
Beyond first order perturbation theory!

Γ in case of isolated system is due to
non-adiabaticity.

Γ affects both the dissipative and the
non-dissipative (geometric) part of the response.

From Kubo to a "FD relation"

$$\mathcal{H} \;=\; \mathcal{H}(\boldsymbol{r},\boldsymbol{p};\; x_1(t), x_2(t), x_3(t))$$

$$\mathcal{F}^k \;=\; -\frac{\partial \mathcal{H}}{\partial x_k}$$

Generalized Ohm law:

$$\langle \mathcal{F}^k \rangle = -\sum_j \boldsymbol{G}^{kj}\; \dot{x}_j$$

$$K^{kj}(\tau) \;=\; \frac{i}{\hbar}\langle [\mathcal{F}^k(\tau), \mathcal{F}^j(0)] \rangle$$

$$C^{kj}(\tau) \;=\; \frac{1}{2}\Big(\langle \mathcal{F}^k(\tau)\mathcal{F}^j(0) \rangle + cc \Big)$$

$$\boldsymbol{G}^{kj} \;=\; \lim_{\omega\to 0} \frac{\mathrm{Im}[\chi^{kj}(\omega)]}{\omega} \;=\; \int_0^\infty K_F^{kj}(\tau)\tau d\tau$$

$$=\; \mathsf{g}(E_F)\; \int_0^\infty C_{E_F}^{kj}(\tau) d\tau$$

DC, PRB(R) 2003

BPT versus Geometric magnetism

$$\boldsymbol{G}^{kj} \;=\; \mathrm{g}(E_F)\int_0^{\infty} C^{kj}_{E_F}(\tau)d\tau$$

Due to non-adiabaticity

$$C^{kj}_{E}(\tau) \;\mapsto\; C^{kj}_{E}(\tau)\, \mathrm{e}^{-(\Gamma/\hbar)t}$$

$$\Gamma \;=\; \left(\frac{\hbar\sigma}{\Delta^2}|\dot{x}|\right)^{2/3} \times\ \Delta \quad \sim \quad \left(L\,|\dot{x}|\right)^{2/3}\frac{1}{L}$$

For $x_3 = 0$ one obtains $\boldsymbol{G}^{kj} = \boldsymbol{B}^{kj}$ where

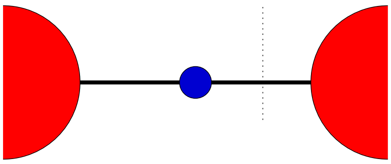
$$\boldsymbol{B}^{kj} \;=\; 2\hbar\sum_n f(E_n)\sum_{m(\neq n)}\frac{\mathrm{Im}\left[\mathcal{F}^k_{nm}\mathcal{F}^j_{mn}\right]}{(E_m-E_n)^2+(\Gamma/2)^2}$$

For dot-wire system in the $L\to\infty$ limit

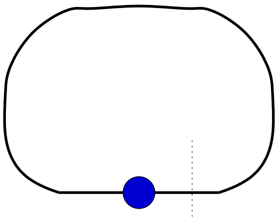
we have $\Delta \ll \Gamma \to 0$ leading to

$$\boldsymbol{G}^{3j} \;=\; \frac{e}{2\pi i}\mathrm{trace}\left(P_{\mathrm{A}}\frac{\partial S}{\partial x_j}S^{\dagger}\right) \qquad \text{[BPT]}$$

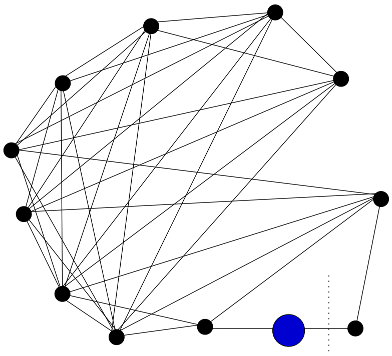
Simple model systems - networks



$$dQ = (1-g_0) \times \frac{e}{\pi} k_{\mathrm{F}} \times dX$$



$$dQ = 1 \times \frac{e}{\pi} k_{\mathrm{F}} \times dX$$

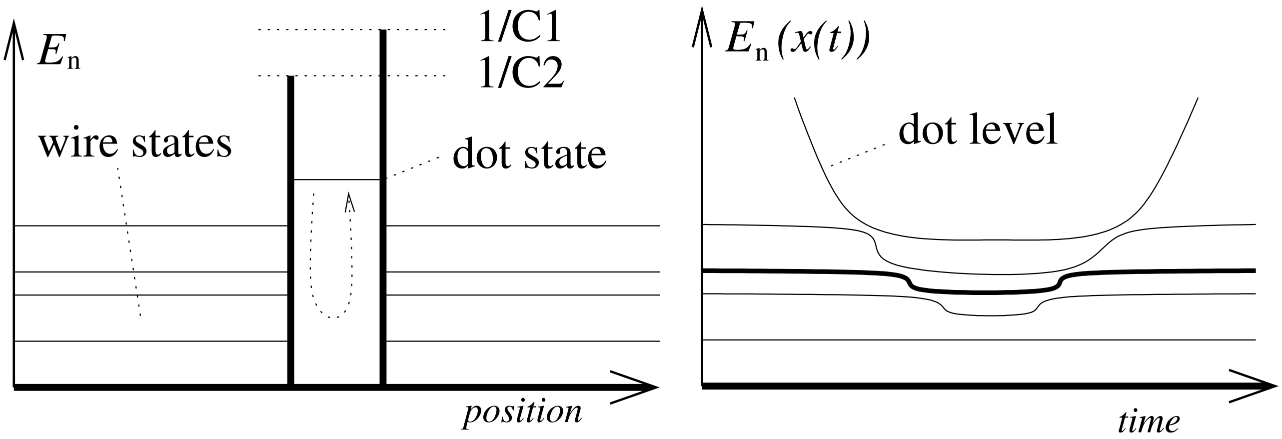


$$dQ = \left[\frac{g_T}{1-g_T}\right] \left[\frac{1-g_0}{g_0}\right] \times \frac{e}{\pi} k_{\mathrm{F}} \times dX$$

Adiabatic versus non-adiabatic result

DC, Kottos, Schanz, cond-mat 2004

Speculations...



For the open system, using BPT:

$$Q \approx 1 - g_0$$

- Is it due to non-adiabaticity?
- Is it a dissipative contribution?

What about strict adiabatic cycle in a closed system?

Shutenko, Aleiner, Altshuler (2000):

"If the system were closed [and strictly adiabatic], the charge distribution after each period of the perturbation would return to the original distribution, and therefore, the pumped charge would be *exactly quantized*."

Not correct!

The $\boldsymbol{B}$ field

$$\vec{\boldsymbol{B}} = (\boldsymbol{B}^{12}, \boldsymbol{B}^{23}, \boldsymbol{B}^{31})$$

$$\boldsymbol{B}^{kj} \;\; = \;\; \sum_n f(E_n)\boldsymbol{B}_n^{kj}$$

$$\boldsymbol{B}_n^{kj} \;\; = \;\; 2\hbar \sum_{m(\neq n)} \frac{\mathrm{Im}\left[\mathcal{F}_{nm}^k \mathcal{F}_{mn}^j\right]}{(E_m - E_n)^2 + (\Gamma/2)^2}$$

This field is divergenceless (for $\Gamma = 0$)

[geometric magnetism]

A chain of degeneracies:

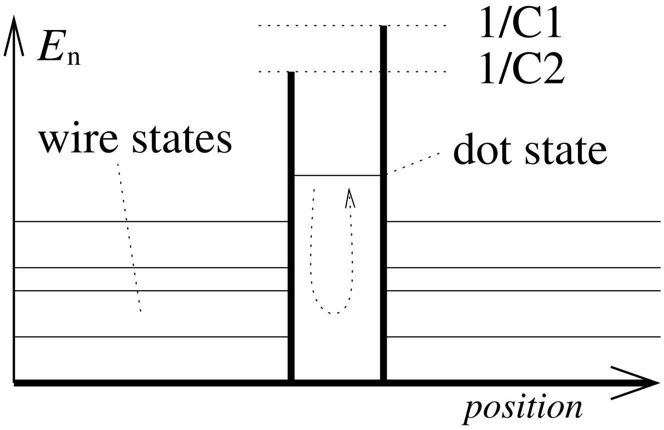
$$\left(x_1^{(0)}, \quad x_2^{(0)}, \quad \Phi^{(0)} + 2\pi\frac{e}{\hbar} \times \text{ integer}\right)$$

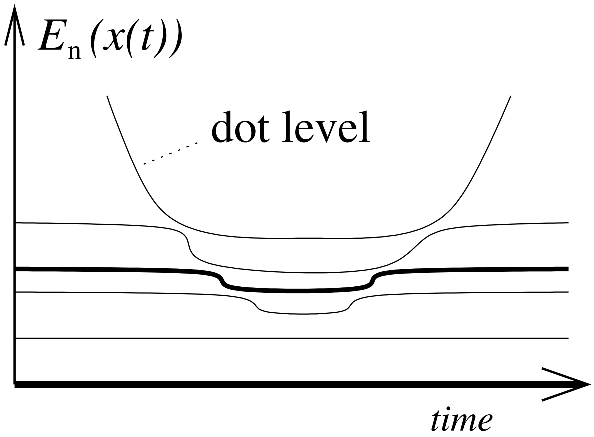
The degeneracies are like Dirac monopoles

- The issue of bandwidth

- The effect of screening

The two barrier model





$$X_1 = \frac{1}{2}(C_1 - C_2) = \text{left-right bias}$$

$$X_2 = -\frac{1}{2}(C_1 + C_2) = \text{dot potential floor}$$