One body decoherence: fluctuations, recurrences, and the statistics of the quantum Zeno suppression due to erratic driving

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The Bose-Hubbard Hamiltonian (BHH) for a dimer

\[ H = \sum_{i=1,2} \left( \mathcal{E}_i \hat{n}_i + \frac{U}{2} \hat{n}_i(\hat{n}_i - 1) \right) - \frac{K}{2} (\hat{a}^\dagger_2 \hat{a}_1 + \hat{a}^\dagger_1 \hat{a}_2) \]

\( N \) particles in a double well is like spin \( j = N/2 \) system

\[ H = -\mathcal{E} \hat{J}_z + U \hat{J}_z^2 - K \hat{J}_x \]

Similar to the Josephson Hamiltonian

\[ \mathcal{H}(\hat{n}, \varphi) = U(\hat{n} - \epsilon)^2 - \frac{1}{2} K N \cos(\varphi) \]

\( \hat{n} = \hat{J}_z = \) occupation difference

\( \varphi = \) conjugate phase

Rabi regime: \( u < 1 \) (no islands)

Josephson regime: \( 1 < u < N^2 \) (sea, islands, separatrix)

Fock regime: \( u > N^2 \) (empty sea)

\( K = \) hopping
\( U = \) interaction
\( \mathcal{E} = \mathcal{E}_2 - \mathcal{E}_1 = \) bias

\( u \equiv \frac{N U}{K}, \quad \epsilon \equiv \frac{\mathcal{E}}{K} \)

Assuming \( u > 1 \) and \( |\epsilon| < \epsilon_c \)

Sea, Islands, Separatrix

\( \epsilon_c = \left( u^{2/3} - 1 \right)^{3/2} \)
WKB quantization (Josephson regime)

\[ h = \text{Planck cell area in steradians} = \frac{4\pi}{N+1} \]

\[ A(E_\nu) = \left( \frac{1}{2} + \nu \right) h \quad \nu = 0, 1, 2, 3, \ldots \]

\[ \omega(E) \equiv \frac{dE}{d\nu} = \left[ \frac{1}{h} A'(E) \right]^{-1} \]

\[ \omega_K \approx K = \text{Rabi Frequency} \]

\[ \omega_J \approx \sqrt{NUK} = \text{Josephson Frequency} \]

\[ \omega_+ \approx NU = \text{Island Frequency} \]

\[ \omega_x \approx \left[ \log \left( \frac{N^2}{u} \right) \right]^{-1} \omega_J \]

Eigenstates \( |E_\nu \rangle \) are like strips along contour lines of \( \mathcal{H} \).
Wavepacket dynamics

Mean Field theory (GPE) = classical evolution of a **point** in phase space
Semi Classical theory = classical evolution of a **distribution** in phase space
Quantum theory = recurrences, fluctuations (WKB is very good)

Any operator $\hat{A}$ can be presented by the phase-space function $A_W(\Omega)$

$$\langle \hat{A} \rangle = \text{trace}[\hat{\rho} \hat{A}] = \int \frac{d\Omega}{\hbar} \rho_W(\Omega) A_W(\Omega)$$
Recurrences and fluctuations

\[ \vec{S} = \langle \vec{J} \rangle / (N/2) = (S_x, S_y, S_z) \]

OccupationDifference = \( (N/2) \) \( S_z \)

OneBodyCoherence = \( S_x^2 + S_y^2 + S_z^2 \)

FringeVisibility = \( \left[ S_x^2 + S_y^2 \right]^{1/2} \)

Spectral analysis of the fluctuations: dependence on \( u \) and on \( N \), various preparations.
The preparations, and their LDOS $P(E)$

\[ \sim \left[ 1 - \left( \frac{2E}{NK} \right)^2 \right]^{-1/2} \]

\[ \sim \text{BesselI} \left[ \frac{E-E_x}{NU} \right] \]

\[ \sim \text{BesselK} \left[ \frac{E-E_x}{NU} \right] \]

\[ \sim \exp \left[ -\frac{1}{N} \left( \frac{E-E_x}{\omega_J} \right)^2 \right] \]
The participation number $M$

$$M \equiv \left[ \sum_{\nu} P(E_{\nu})^2 \right]^{-1} = \text{number of participating levels in the LDOS}$$

In the semiclassical analysis there is scaling with respect to $(u/N)^{1/2}$ which is $\text{[the width of the wavepacket]} / \text{[the width of the separatix]}$

$$M[\text{generic}] \approx \sqrt{N}$$
$$M[\text{Edge}] \approx \left[ \log \left( \frac{N}{u} \right) \right] \sqrt{N}$$
$$M[\text{Pi}] \approx \left[ \log \left( \frac{N}{u} \right) \right] \sqrt{u}$$
$$M[\text{Zero}] \approx \sqrt{u}$$
Analysis

* Spectral content: $\omega \sim \omega_{\text{osc}}$

* Fluctuations: $\text{Var}[S(t)]$

$$\text{Var}[S] = \frac{1}{M} \int \tilde{C}_{\text{cl}}(\omega) d\omega$$

Zero prep  Pi prep  Edge prep
The spectral content of $S_x$:

\[ \omega_{osc} \approx 2\omega_J \quad \text{[Zero]} \]

\[ \omega_{osc} \approx 1 \times \left[ \log \left( \frac{N}{u} \right) \right]^{-1} 2\omega_J \quad \text{[Pi]} \]

\[ \omega_{osc} \approx 2 \times \left[ \log \left( \frac{N}{u} \right) \right]^{-1} 2\omega_J \quad \text{[Edge]} \]

\[ \omega_{osc} \approx \left( \frac{u}{N} \right)^{1/2} 2\omega_J \quad \text{[} u \gg N \text{]} \]
Fluctuations of $S_x$

Naive expectation: phase spreading diminishes coherence. In the Fock regime $\langle S_x \rangle_\infty \approx 0$ [Leggett’s “phase diffusion”]
In the Josephson regime $\langle S_x \rangle_\infty$ is determined by $u/N$.

\[
\overline{S_x} \approx \begin{cases} 1/3 & \text{[TwinFock]} \\
\exp[-(u/N)] & \text{[Zero]} \\
-1 - 4/\log\left(\frac{1}{32}(u/N)\right) & \text{[Pi]} \end{cases}
\]

RMS $\left[\langle A \rangle_t\right] = \left[\frac{1}{M} \int \tilde{C}_{cl}(\omega)d\omega\right]^{1/2}$
RMS $[S_x(t)] \sim N^{-1/4}$ [Edge]
RMS $[S_x(t)] \sim (\log(N))^{-1/2}$ [Pi]

**TwinFock:** Self induced coherence leading to $\overline{S_x} \approx 1/3$.

**Zero:** Coherence maintained if $u/N < 1$ (phase locking).

**Pi:** Fluctuations are suppressed by $u$.

**Edge:** Fluctuations are suppressed by $N$ (classical limit).
Periodic / Erratic / Noisy driving

\[ \mathcal{H} = U \hat{J}_z^2 - (K + f(t)) \hat{J}_x \]

**Periodic driving:** We distinguish between nearly resonant driving (∼Chaos), and high frequency driving (∼Kapitza).

\[ f(t) \propto \sin(\Omega t) \]

**Erratic driving:** deterministic but fluctuating \( f(t) \). An experimentalist can repeat the experiment with the same realization, or produce other realizations, as desired.

**Noisy driving:** arise due to the interaction with a stationary source (“high temperature bath”). The realizations of \( f(t) \) are not under experimental control. Nature is doing the averaging. (∼Zeno).

\[ \overline{f(t)f(t')} = 2D\delta(t - t') \]
Kapitza physics in spherical phase-space

\[
\frac{d\rho}{dt} = i[H + f(t)W, \rho]
\]

\[f(t) = \sin(\Omega t)\]

3 iterations and averaging over a cycle \(\sim\)

\[
\frac{d\rho}{dt} = i[H + V_{\text{eff}}, \rho]
\]

\[V_{\text{eff}} = -\frac{1}{4\Omega^2} [W, [W, H]]\]

Here:

\[W \propto J_x = (N/2) \sin \theta \cos \varphi\]

\[\sim V_{\text{eff}} \propto J_y^2 \sim \sin^2 \varphi\]
Erratic vs Noisy driving

$$\mathcal{H} = U \hat{J}_z^2 - (K + f(t))\hat{J}_x$$

$$f(t)f(t') = 2D\delta(t - t')$$

Initial state: \(S = (-1, 0, 0)\)

Master Equation:
\[
\frac{d\rho}{dt} = -i[H_0, \rho] - D[J_x, [J_x, \rho]]
\]

Quantum Zeno Effect:
[Khodorkovsky Kurizki Vardi 2008]
\[|S|_{\text{noise}} \approx \exp\left\{ -\frac{1}{N} \frac{\omega_\perp^2}{D} t \right\}\]

Statistics for erratic driving:
\[|S|_{f(t)} \approx \exp\left\{ -\frac{2}{N} \sinh^2(\Lambda) \right\}\]
\[|S|_{\text{median}} \approx \exp\left\{ -\frac{2}{N} \sinh^2(\mu(t)) \right\}\]
\[|S|_{\text{average}} \approx \exp\left\{ -\frac{1}{N} \left[ e^{2\sigma(t)^2} \cosh(2\mu(t)) - 1 \right] \right\}\]
Details of analysis (I)

Traditional approach to analyze the Quantum Zeno effect is based on an FGR picture. Instead we use a semi-classical picture.

\[ J_{\parallel} \mapsto [j(j+1)]^{1/2} \cos(r) \quad \text{[Wigner-Weyl]}, \quad r = \text{the radial coordinate} \quad r \equiv \theta \quad \text{if} \quad J_{\parallel} \equiv J_z \]

For squeezing along orthogonal directions with \( e^{\pm \Lambda} \)

\[
|S| = \left[ 1 + \frac{2}{N} \right]^{1/2} \langle \cos(r) \rangle = \exp \left\{ -\frac{1}{2} \left( \langle r^2 \rangle - \frac{2}{N} \right) \right\} = \exp \left\{ -\frac{2}{N} \sinh^2(\Lambda) \right\}
\]

(a) For a pure squeezing scenario

\[
|S| = \exp \left\{ -\frac{2}{N} \cot^2(2\Theta) \sinh^2(w_J t) \right\}
\]

\[ w_J = \sqrt{(NU-K)K}, \quad \Theta = \tan^{-1}(w_J/K) \]

(b) For an erratic squeezing scenario:
one has to figure out the \( \Lambda \) statistics.
\( \sim \) log-wide distribution.
Details of analysis (II)

\[ |S| = \left[ 1 + \frac{2}{N} \right]^{1/2} \langle \cos(r) \rangle = \exp \left\{ -\frac{1}{2} \left( \langle r^2 \rangle - \frac{2}{N} \right) \right\} = \exp \left\{ -\frac{2}{N} \sinh^2(\Lambda) \right\} \]

(c) For a noisy squeezing scenario there are two options for analysis.

Via \( \Lambda \) statistics:

\[ |S|_{\text{average}} \approx \exp \left\{ -\frac{1}{N} \left[ e^{2\sigma(t)^2} \cosh(2\mu(t)) - 1 \right] \right\} \]

Via convolution of small steps:

\[ D_w = \left[ \cot^2(2\Theta) \right] \frac{w_f^2}{8D} \quad \text{log-radial diffusion coefficient in multiplicative process} \]
\[ t_D = \frac{1}{2D} \quad \text{time to randomize direction due to transverse diffusion} \]

\[ |S|_{\text{average}} = \exp \left\{ -\frac{1}{N} \left[ \exp (8D_w t) - 1 \right] \right\} \quad \text{should be contrasted with} \quad \exp \left\{ -\frac{1}{N} 8D_w t \right\} \]
The many body Landau-Zener transition

Dynamical scenarios: adiabatic/diabatic/sudden
Occupation Statistics

Adiabatic-diabatic (quantum) crossover
Diabatic-sudden (semiclassical) crossover
Summary

• Semiclassical analysis (WKB and Wigner-Weyl are beyond MFT)
• The dependence of the participation number $M$ on $u$ and on $N$.
• Fluctuations and recurrences, study of $\omega_{osc}$ and $\overline{S(t)}$ and Var[$S(t)$]
• Noise driven dimer: Improved Quantum Zeno effect analysis for $\overline{S(t)}$
• Erratic driving: analysis of the statistics of $|S(t)|$ - challenging the system-bath paradigm
• Occupation statistics in a time dependent Landau-Zener scenario: identification of the adiabatic / diabatic / sudden crossovers.

\[ f = p \]
\[ f = \frac{4p}{5} \]
\[ f = \frac{3p}{5} \]
\[ f = \frac{2p}{5} \]
\[ f = \frac{p}{5} \]
\[ f = 0 \]