

Counting Statistics in closed mesoscopic devices

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Counting statistics for a coherent transition

MC and D. Cohen, Phys. Rev. A (2008, in press)

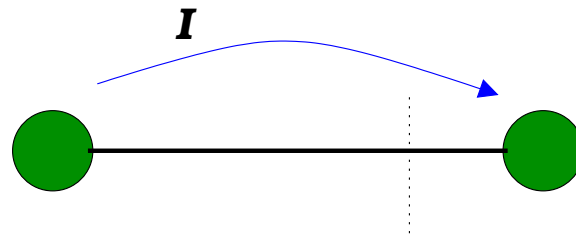
Counting statistics in multiple path geometries

MC and D. Cohen, J. Phys. A (2008, in press)

Outline

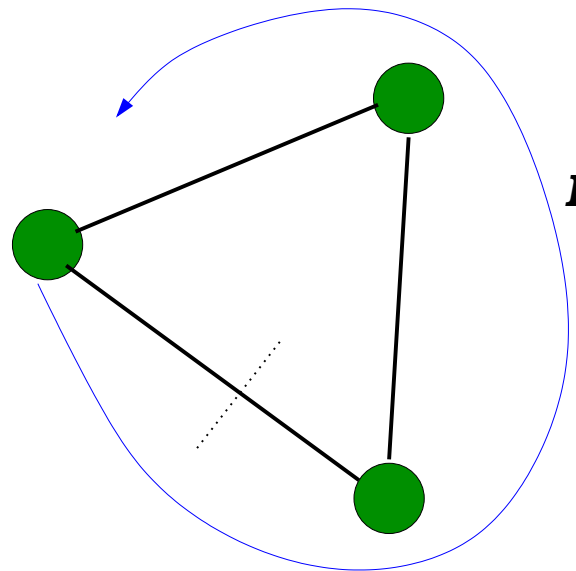
Counting

$$Q = \int_0^t \mathcal{I}(t') dt'$$



2 site system

FCS for a coherent transition

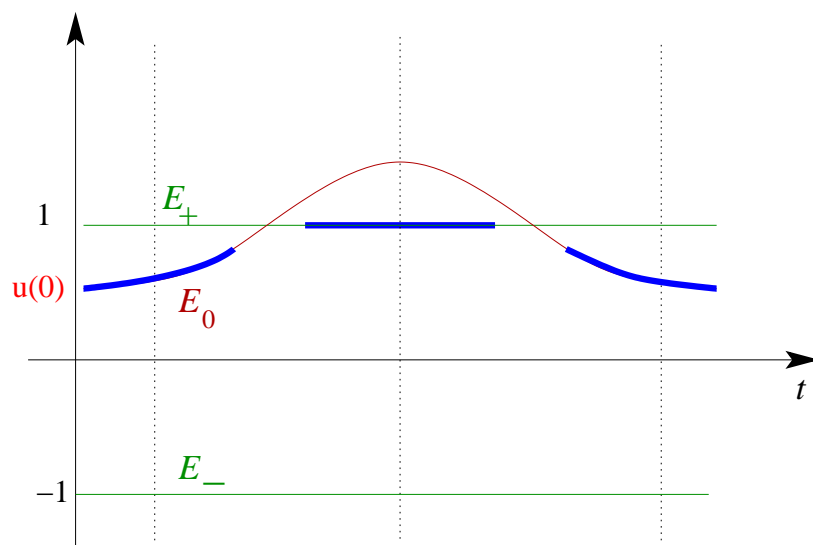
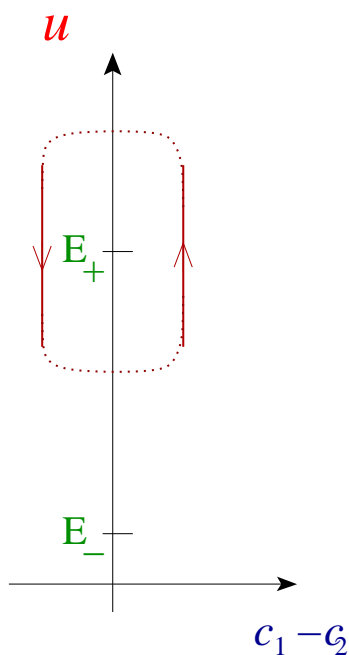
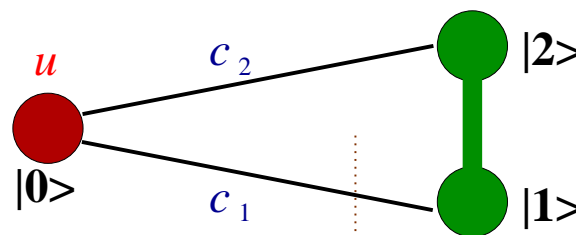


3 site system

$\text{Var}(Q)$ for quantum stirring

Counting Statistics, the model

$$\mathcal{H} = \begin{pmatrix} u & c_1 & c_2 \\ c_1 & 0 & 1 \\ c_2 & 1 & 0 \end{pmatrix}$$



The current through one bond operator:

$$\mathcal{I} = \begin{pmatrix} 0 & ic_1 & 0 \\ -ic_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The counting operator: $Q = \int_0^t \mathcal{I}(t') dt'$

$$\langle Q \rangle = ???$$

$$\text{Var}(Q) = ???$$

$$P(Q) = ???$$

Main results

For a **half a cycle**:

$$p = 1 - P_{\text{LZ}}$$

$$\langle Q \rangle = \lambda p$$

$$\text{Var}(Q) = \lambda^2 \underbrace{(1 - p)p} \neq (1 - \lambda p)\lambda p$$

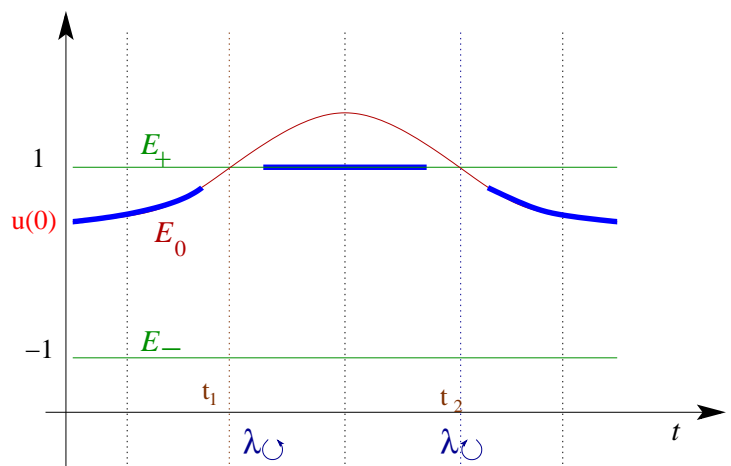
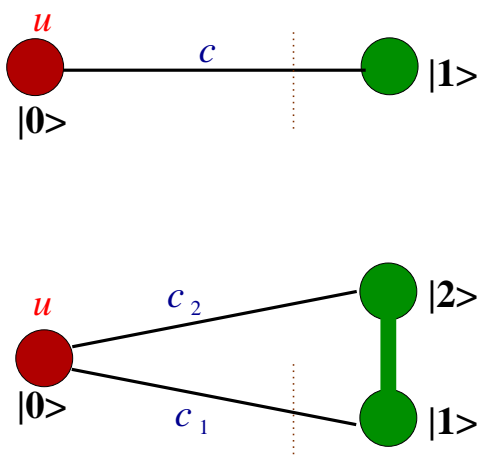
$$P_{\text{LZ}} = e^{-\frac{\pi(c_1+c_2)^2}{i}} , \quad \lambda = \frac{c_1}{c_1+c_2} = \text{splitting ratio}$$

For a **full stirring cycle**:

$$p \approx \left| e^{i\varphi_1} - e^{i\varphi_2} \right|^2 P_{\text{LZ}}$$

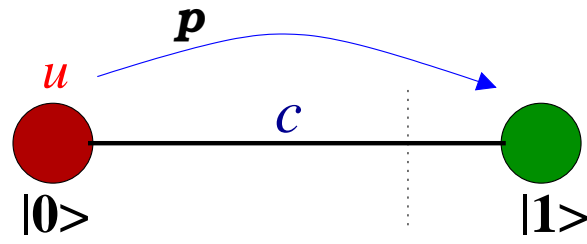
$$\langle Q \rangle \approx \lambda_{\odot} - \lambda_{\ominus}$$

$$\text{Var}(Q) \approx \left| \lambda_{\odot} e^{i\varphi_1} + \lambda_{\ominus} e^{i\varphi_2} \right|^2 P_{\text{LZ}}$$



Counting statistics for a coherent transition

$$\mathcal{H} = \begin{pmatrix} u(t) & c \\ c & E_1 \end{pmatrix}$$



Naive expectation:

Given the probability p to make the transition

$$P(Q) = \begin{cases} 1-p & \text{for } Q = 0 \\ p & \text{for } Q = 1 \end{cases}$$

$$\langle Q^k \rangle = P(1) \cdot 1^k + P(0) \cdot 0^k = p$$

$$\text{Var}(Q) = (1-p)p$$

Quantum result:

$$P(Q) = \begin{cases} p_- & \text{for } Q = Q_- \\ p_+ & \text{for } Q = Q_+ \end{cases}$$

where

$$Q_{\pm} = \pm\sqrt{p}, \quad p_{\pm} = \frac{1}{2} (1 \pm \sqrt{p})$$

hence

$$\langle Q^k \rangle = p_+ Q_+^k + p_- Q_-^k = p \lfloor \frac{k+1}{2} \rfloor$$

The measurement of Q , FCS

The distribution $P(Q)$ can be determined using a **continuous** measurement scheme.

In such setup the current induces (so to say) a “translation” of a **Von-Neumann pointer**.

$$H_{\text{total}} = H_{\text{system}} - Ix + H_{\text{pointer}}(x, q)$$

One can measure the quasi distribution $P(Q; x)$:

$$P(Q; x = 0) = \frac{1}{2\pi} \int \left\langle \left[\mathcal{T} e^{-i\frac{r}{2}\mathcal{Q}} \right]^\dagger \left[\mathcal{T} e^{+i\frac{r}{2}\mathcal{Q}} \right] \right\rangle e^{-iQr} dr$$

If we ignore time ordering we get:

$$P(Q) = \frac{1}{2\pi} \int \langle e^{+ir\mathcal{Q}} \rangle e^{-iQr} dr = \langle \delta(Q - \mathcal{Q}) \rangle$$

H. Everett, Rev. Mod. Phys. 29, 454 (1957).

L.S. Levitov and G.B. Lesovik, JETP Letters (1992).

L.S. Levitov and G.B. Lesovik, JETP Letters (1993).

Y.V. Nazarov and M. Kindermann, EPJ B (2003).

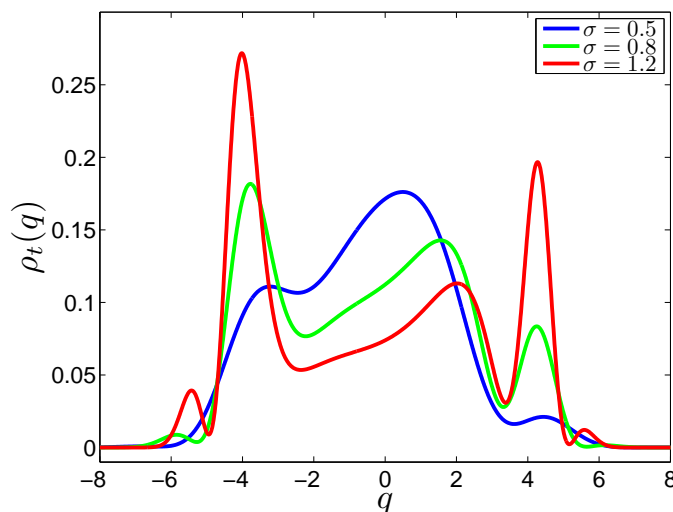
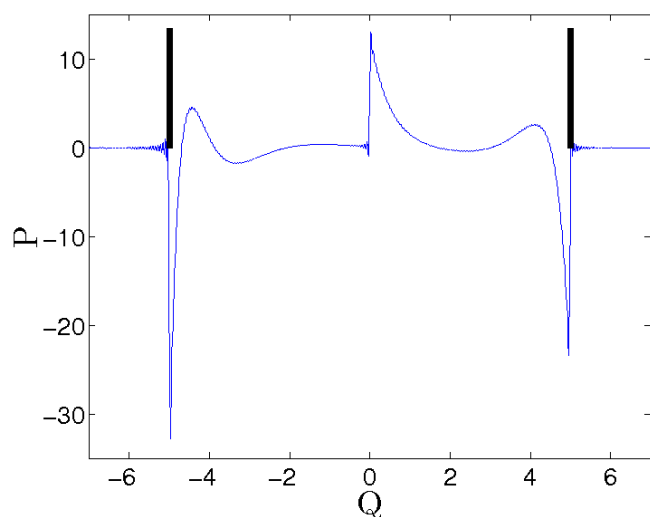
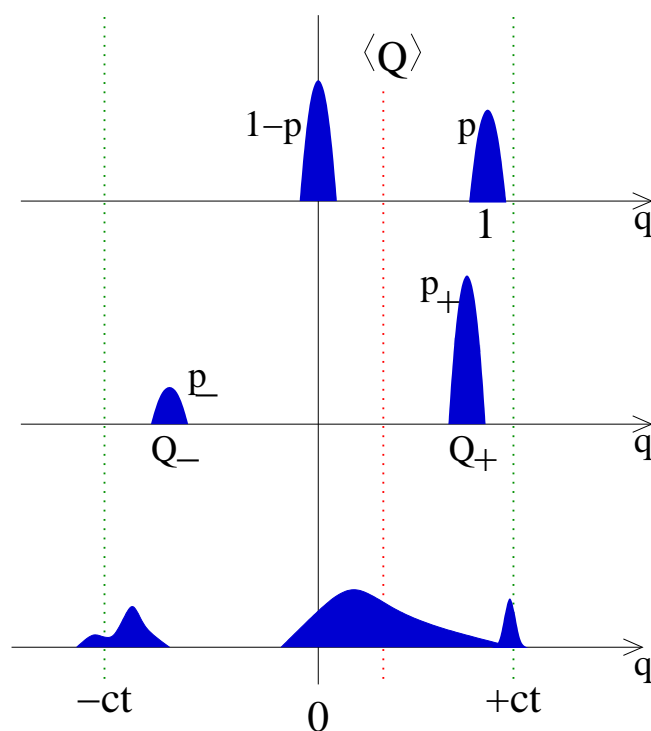
FCS for a coherent Bloch transition

$$\rho_t(q, x) = \int P(q - q'; x) \rho_0(q', x) dq'$$

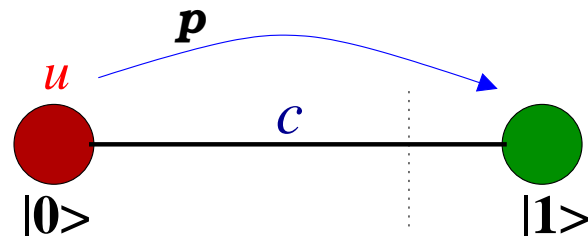
$$P_{cl}(Q) = \begin{cases} 1-p & \text{for } Q = 0 \\ p & \text{for } Q = 1 \end{cases}$$

$$P_{naive}(Q) = \begin{cases} p_- & \text{for } Q = Q_- \\ p_+ & \text{for } Q = Q_+ \end{cases}$$

$$P_{qm}(Q; x = 0)$$



Calculation of the variance of a LZ crossing

$$\mathcal{H} = \begin{pmatrix} u & c \\ c & 0 \end{pmatrix},$$


leading order adiabatic approximation:

$$U(t) \approx \sum_n |n(t)\rangle \exp \left[-i \int_{t_0}^t E_n(t') dt' \right] \langle n(t_0) |$$

$$\begin{aligned} \mathcal{I}(t)_{nm} &= \langle n | U(t)^\dagger \mathcal{I} U(t) | m \rangle \\ &\approx \langle n(t) | \mathcal{I} | m(t) \rangle \exp \left[i \int_{t_0}^t E_{nm}(t') dt' \right] \end{aligned}$$

$$\mathcal{Q} \equiv \begin{pmatrix} +Q_{\parallel} & iQ_{\perp} \\ -iQ_{\perp}^* & -Q_{\parallel} \end{pmatrix}$$

$$\text{Var}(\mathcal{Q}) = |Q_{\perp}|^2 \approx \left| c \int_{-\infty}^{\infty} e^{i\Phi(t)} dt \right|^2$$

$$\Phi(t) \equiv \int_0^t \sqrt{(\dot{u}t')^2 + (2c)^2} dt'$$

The LZ transition calculation

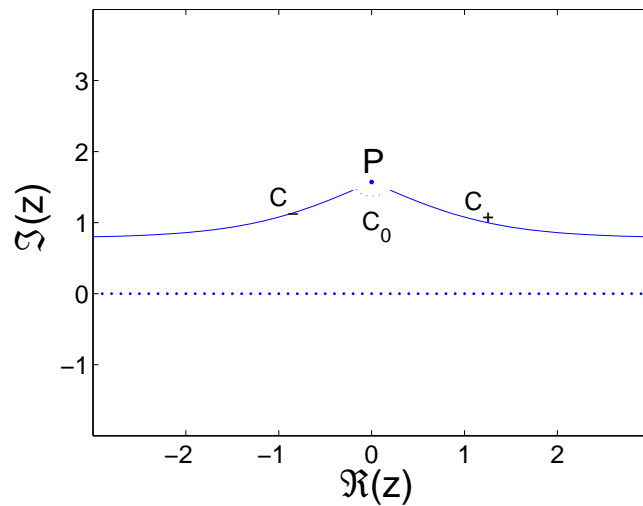
$$P_{\text{LZ}} \approx \left| c \int_{-\infty}^{\infty} \frac{\dot{u}}{(\dot{u}t)^2 + (2c)^2} e^{i\Phi(t)} dt \right|^2$$

$$= \left| \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{\cosh(z)} e^{i\Phi(z)} dz \right|^2$$

$$\text{Var}(\mathcal{Q}) \approx \left| c \int_{-\infty}^{\infty} e^{i\Phi(t)} dt \right|^2$$

$$= \left| \frac{2c^2}{\dot{u}} \int_{-\infty}^{\infty} \cosh(z) e^{i\Phi(z)} dz \right|^2$$

$$??? \quad \text{Var}(\mathcal{Q}) = (1 - P_{\text{LZ}}) P_{\text{LZ}} \quad ???$$



$$P_{\text{LZ}} \sim \left(\frac{\pi}{3} \right)^2 \exp \left[-\frac{\pi c^2}{\dot{u}} \right]$$

$$\text{Var}(\mathcal{Q}) \sim \left(\frac{2c^2}{\dot{u}} \right)^{2/3} \exp \left[-\frac{\pi c^2}{\dot{u}} \right]$$

Restricted quantum-classical correspondence

\mathcal{N} = occupation operator (eigenvalues = 0, 1)

\mathcal{I} = current operator

Heisenberg equation of motion:

$$\frac{d}{dt}\mathcal{N}(t) = \mathcal{I}(t)$$

leads to

$$\mathcal{N}(t) - \mathcal{N}(0) = \mathcal{Q}$$

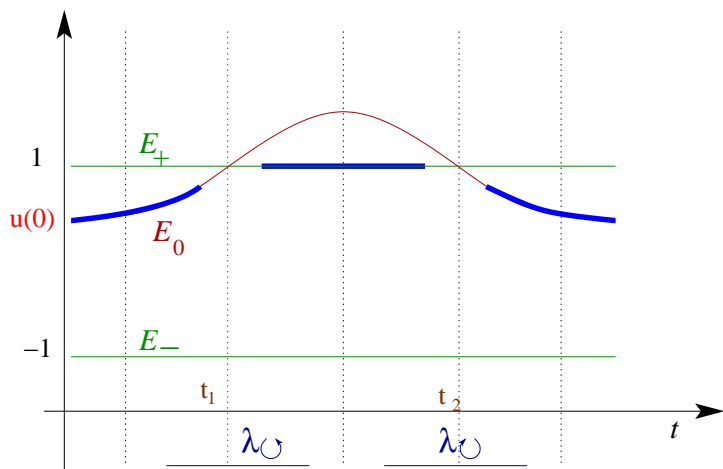
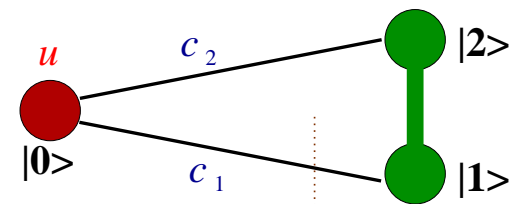
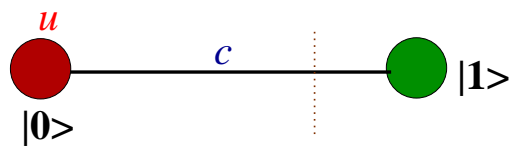
hence

$$\langle \mathcal{Q}^k \rangle = \langle (\mathcal{N}(t) - \mathcal{N}(0))^k \rangle \stackrel{?}{=} \langle \mathcal{N}^k \rangle_t = p$$

for $k = 1, 2$ only

Restricted QCC

Hamiltonians for 2 and 3 site systems



$$\mathcal{H} = \begin{pmatrix} u & c \\ c & 0 \end{pmatrix},$$

$$\mathcal{I} = \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u & c_1 & c_2 \\ c_1 & 0 & 1 \\ c_2 & 1 & 0 \end{pmatrix},$$

$$\mathcal{I} = \begin{pmatrix} 0 & ic_1 & 0 \\ -ic_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u & \frac{(c_1+c_2)}{\sqrt{2}} \\ \frac{(c_1+c_2)}{\sqrt{2}} & 1 \end{pmatrix},$$

$$\mathcal{I} = \frac{c_1}{\sqrt{2}} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

Double path adiabatic passage

$$\mathcal{H} = \begin{pmatrix} u(t) & c \\ c & 1 \end{pmatrix}, \quad \mathcal{I} = \lambda \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

with effective coupling and splitting ratio

$$c \equiv \frac{(c_1 + c_2)}{\sqrt{2}}, \quad \lambda \equiv \frac{c_1}{c_1 + c_2}$$

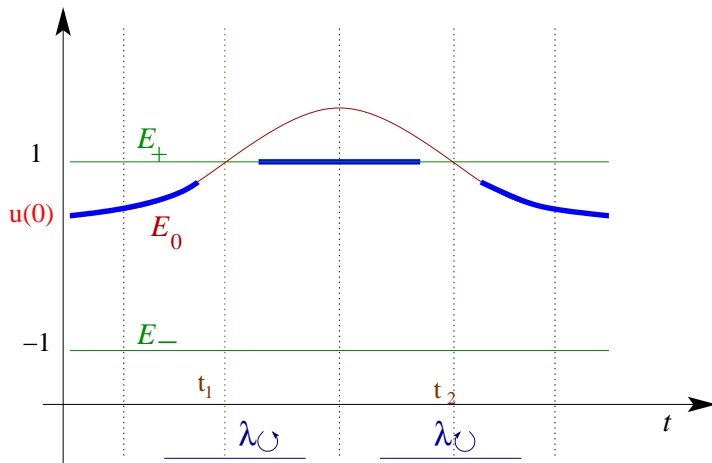
Accordingly:

$$\begin{aligned} \langle Q \rangle &= \lambda p \\ \text{Var}(Q) &= \lambda^2 \underbrace{(1-p)p} \neq (1-\lambda p)\lambda p \end{aligned}$$

Coherent splitting unlike probabilistic splitting of the wavepacket is “exact”.

$\lambda > 1 \Rightarrow$ The driving induces a **circulating current** within the ring, and illuminates the fallacy of the classical peristaltic point of view.

Full stirring cycle



a sequence of two
Landau Zener crossings

$$\begin{aligned} \langle Q \rangle &\approx \lambda_{\circlearrowleft} - \lambda_{\circlearrowright} \\ \text{Var}(Q) &= \left| \lambda c \int_{-\infty}^{\infty} e^{i\Phi(t)} dt \right|^2 \\ &\approx \left| \lambda_{\circlearrowleft} e^{i\varphi_1} + \lambda_{\circlearrowright} e^{i\varphi_2} \right|^2 P_{\text{LZ}} \end{aligned}$$

Due to interference the counting statistics becomes
in general unrelated to the occupation statistics:

$$\begin{aligned} p &= \left| \frac{1}{2} \int_{-\infty}^{\infty} \frac{(\dot{u}/2c) e^{i\Phi(t)}}{1 + (u/2c)^2} dt \right|^2 \\ &\approx \left| e^{i\varphi_1} - e^{i\varphi_2} \right|^2 P_{\text{LZ}} \end{aligned}$$

Conclusions

Quantum mechanics is a “deterministic” rather than a “probabilistic” theory. Coherent splitting unlike probabilistic splitting of a wavepacket is “exact”.

In a double path adiabatic passage one may find that (say) 170% of the particle goes via one path, while -70% goes via the second path, due to a circulating current induced by the driving.

There is at most restricted quantum-classical correspondence for the first and second moments.

In contrast to the single path crossing problem where the two types of statistics are a-priori related, for a full stirring cycle interference gets into the counting statistics calculation, so it is not generally related to the occupation statistics.