The conductance of ballistic rings, and the absorption of radiation by small metallic grains

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$ISF, $GIF, $DIP

References

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Google "Doron Cohen"

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cond-mat archive

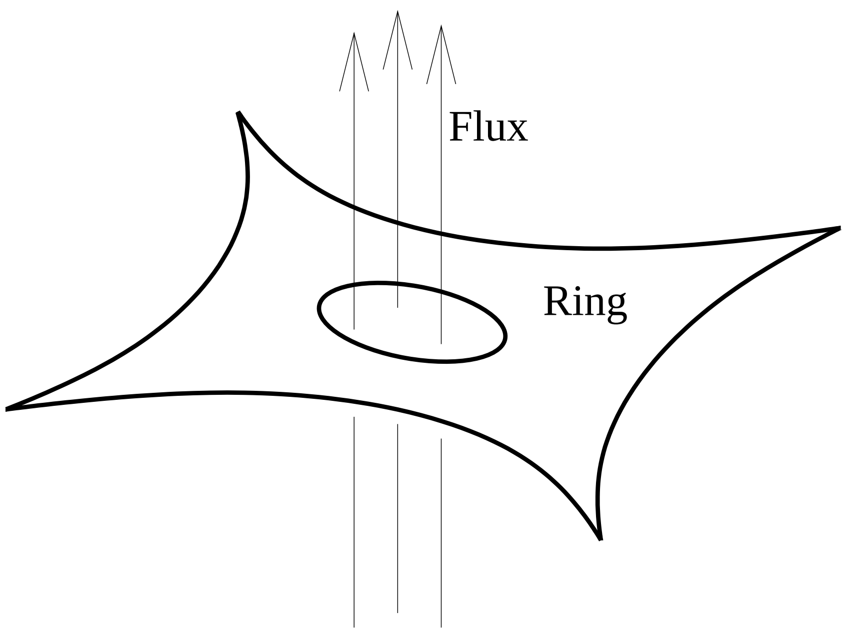
Driven Systems

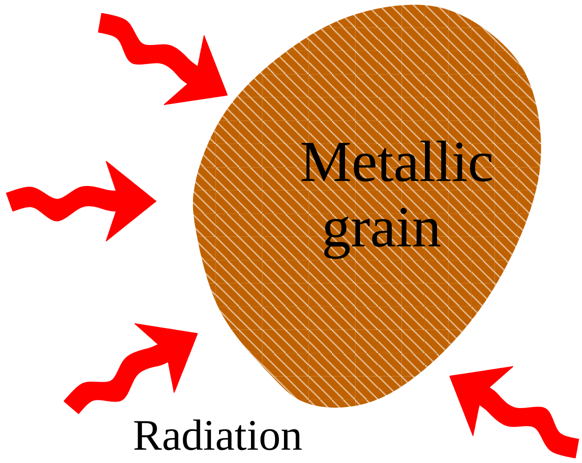
Non interacting "spinless" electrons.

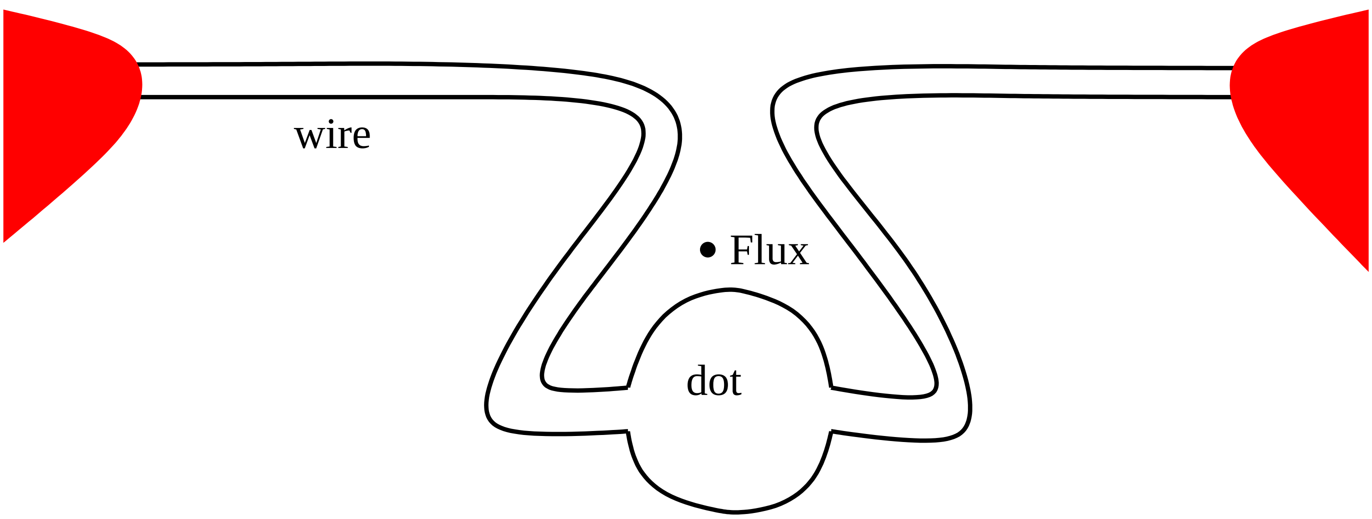
Held by a potential (e.g. AB ring geometry).

$\mathcal{H}(Q, P; X(t))$ = quantized chaotic system

X = some parameter in the Hamiltonian







“Quantum Chaos”

$\mathcal{H}(Q, P; X(t))$ = quantized chaotic system

X = some parameter in the Hamiltonian

Universality on small energy scales ($\propto \hbar^d$)

Fingerprints on larger energy scales ($\propto \hbar$)

Questions:

How is the $\hbar^d$ scale reflected in the response???

How is the $\hbar$ scale reflected in the response???

The message:

The Kubo formalism should be revised

$\Longrightarrow$ Semi-linear response theory

The main idea

There are circumstance in which the rate of energy absorption depends on the possibility to make connected sequences of transitions.

The energy landscape of a quantized chaotic system is not uniform: The perturbation matrix may have structures and sparsity.

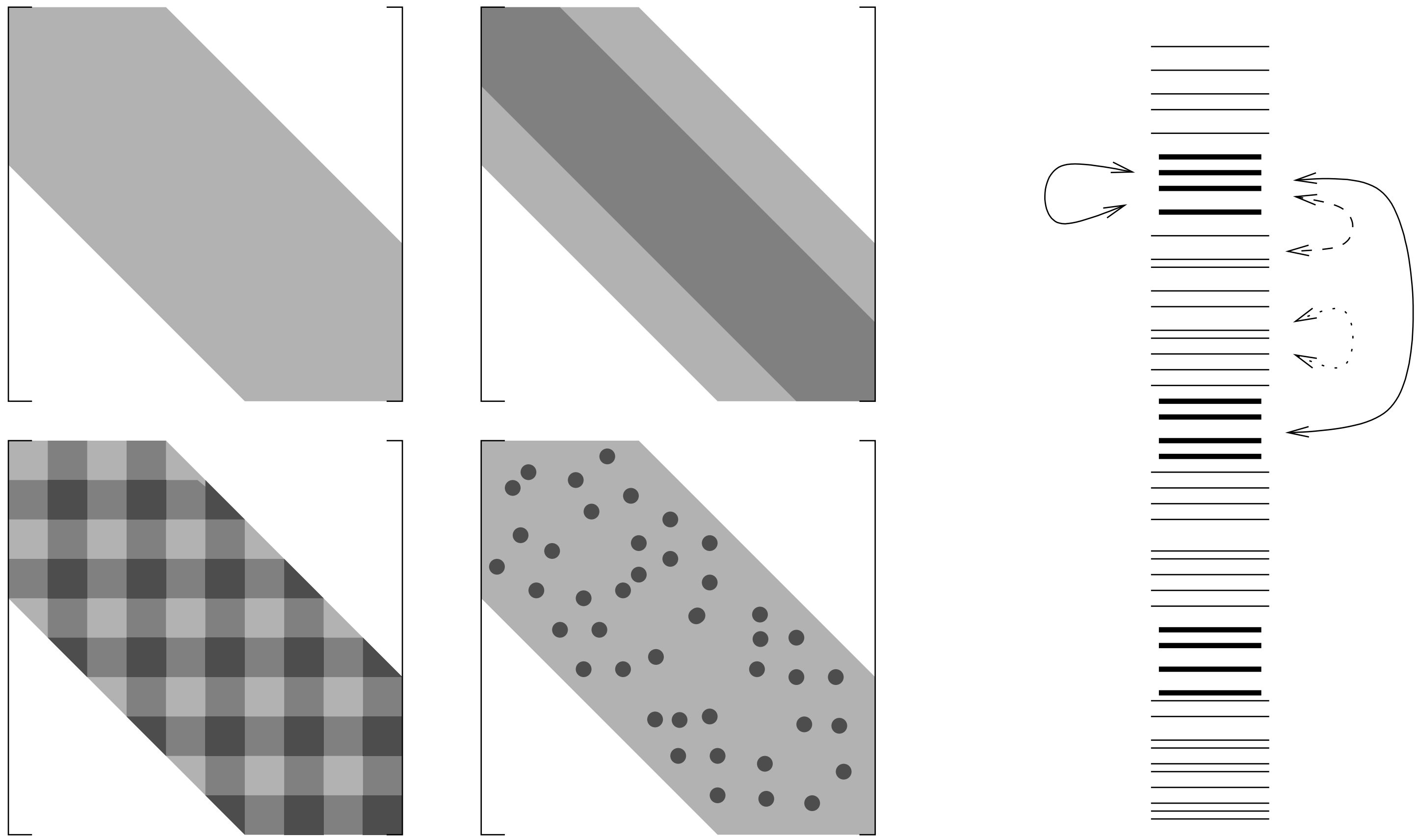
Even if the matrix elements are very large (on the average), still bottlenecks may lead to the suppression of the absorption rate.

The Kubo formalism should be revised! (*)

(*) **However, the Kubo formula becomes valid if one assumes strong environmentally-induced relaxation.**

Type of structures

$$\mathcal{H} \longmapsto \text{diag}\{E_n\} + X(t)\,\mathcal{I}_{nm}$$



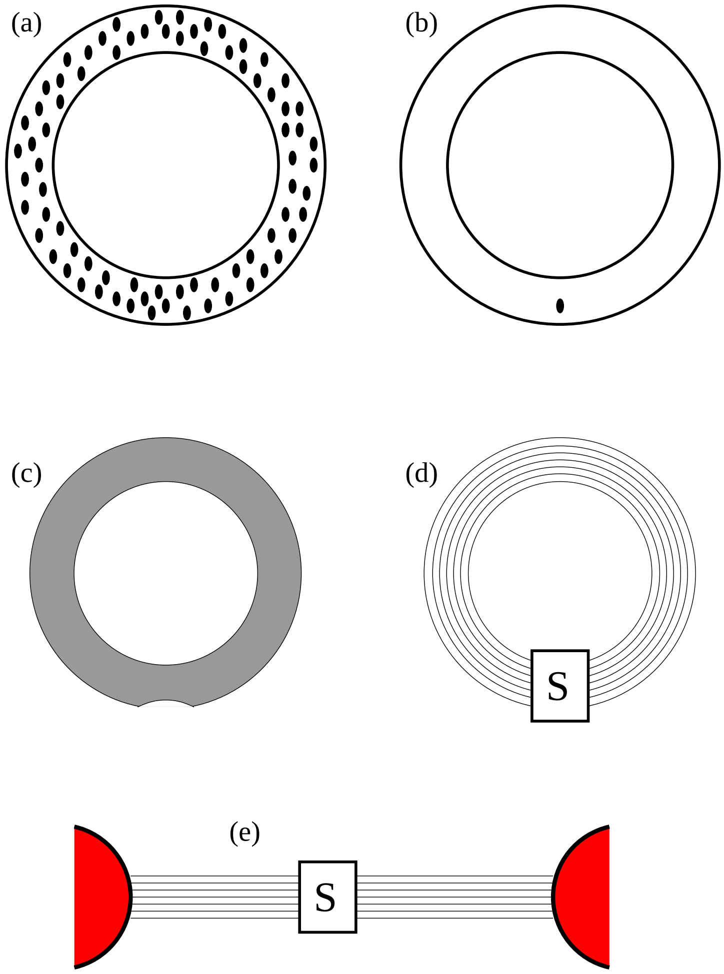
We are going to assume that FGR is valid!

Other issues:

Basko, Skvortsov, Kravtsov [PRL 2003] - weak localization

D.C. , Kottos [PRL 2000] - non-perturbative response

Main application - conductance of rings



$$G_{\text{Drude}} = \frac{e^2}{2\pi\hbar}\mathcal{M}\frac{\ell}{L} + \text{weak localization corrections}$$

L = perimeter of the ring

ℓ = mean free path

(!) Diffusive ring: $\ell \ll L$

??? Ballistic ring: $\ell \gg L$

The definition of G

$$\mathcal{H}(Q,P;X(t)) \;=\; \text{quantized chaotic system}$$

$$X \;=\; \text{some parameter in the Hamiltonian}$$
$$\dot{X} \;=\; \text{rate of the driving}$$

$$E \;=\; \langle \mathcal{H} \rangle \;=\; \text{the energy of the system}$$
$$\dot{E} \;=\; \text{rate of energy absorption}$$

$$\dot{E} \;=\; \boldsymbol{G}\,\dot{X}^2$$

Importent to remember:

The dissipation coefficient $\boldsymbol{G}$ reflects the stochastic-like diffusion D_E in energy space.

Semi-linear Response

Joule law:

$$\dot{E} = \boldsymbol{G}\,\dot{X}^2$$

More generally:

$$\dot{E} = \int \frac{d\omega}{2\pi} G(\omega) |\dot{X}_\omega|^2 = \int \alpha(\omega) F(\omega) d\omega$$

leading to

$$F(\omega) \mapsto \lambda F(\omega) \quad \Longrightarrow \quad \dot{E} \mapsto \lambda \dot{E}$$

$$F(\omega) \mapsto \sum_i F_i(\omega) \quad \Longrightarrow \quad \dot{E} \mapsto \sum_i \dot{E}_i \ .$$

But we shall find circumstance such that

$$\dot{E} = \left[\int \mu(\omega) \left[F(\omega) \right]^{-1} d\omega \right]^{-1}$$

The Kubo formula and Drude

The Kubo formula

$$G = \varrho_F \times \frac{1}{2}\int_{-\infty}^{\infty} \langle\langle \mathcal{I}(\tau)\mathcal{I}(0)\rangle\rangle \, d\tau$$

The Drude assumption

$$\langle\langle \mathcal{I}(\tau)\mathcal{I}(0)\rangle\rangle = \left(\frac{e}{L}v_F\right)^2 \mathrm{e}^{-(v_F/\ell)\tau}$$

Hence

$$G_{\text{Drude}} = \frac{e^2}{2\pi\hbar}\, \mathcal{M}\frac{\ell}{L}$$

The quantum mechanical calculation:

$$\boldsymbol{G} \;\; = \;\; \pi\hbar(\varrho(E_F))^2 \, \langle\langle |\mathcal{I}_{mn}|^2 \rangle\rangle$$

From FGR to the Kubo formula

$$\mathcal{H} \mapsto E_n \delta_{nm} + W_{nm}$$

$$W_{nm} = i\dot{X} \frac{\hbar \mathcal{I}_{nm}}{E_n - E_m}$$

$$w_{nm} = \frac{2\pi}{\hbar} \delta_\Gamma(E_n - E_m) |W_{nm}|^2$$

$$D_E = \pi\hbar\varrho(E) \ \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle \times \dot{X}^2$$

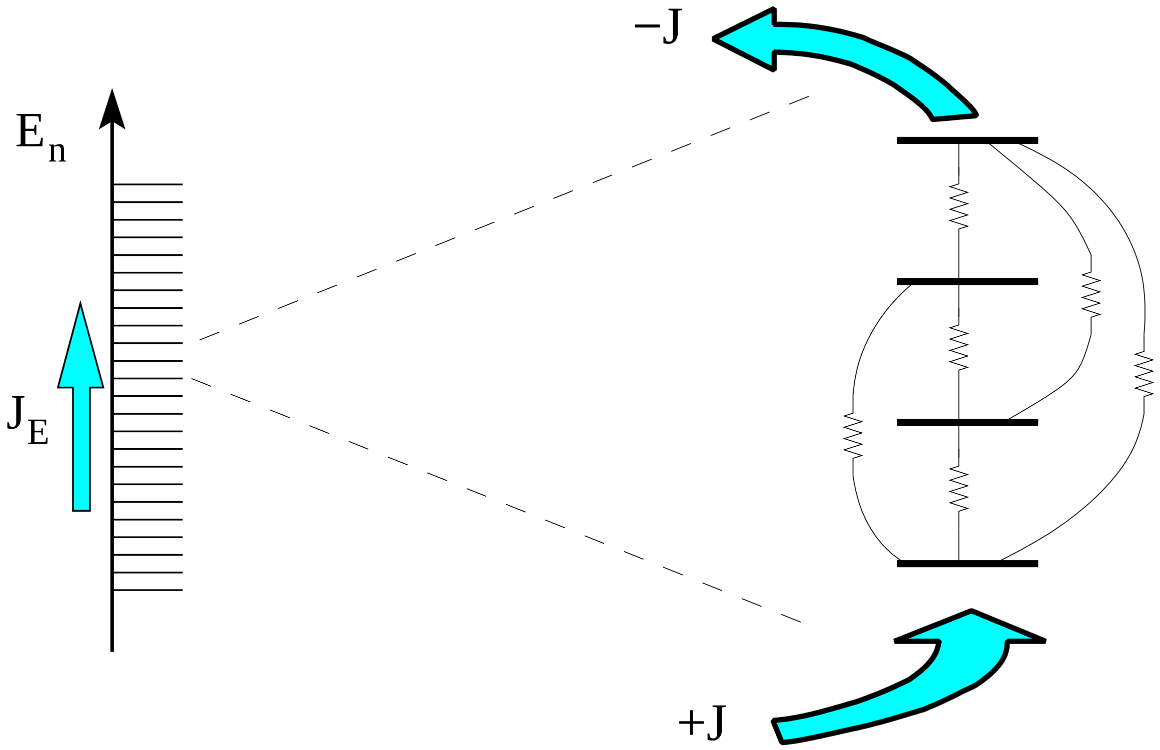
$$\dot{E} = \pi\hbar(\varrho(E_F))^2 \ \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle \times \dot{X}^2$$

$$\boldsymbol{G} = \pi\hbar(\varrho(E_F))^2 \ \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle$$

$$\boldsymbol{G} = \pi\hbar \sum_{n,m} |\mathcal{I}_{mn}|^2 \ \delta_T(E_n - E_F) \ \delta_\Gamma(E_m - E_n)$$

$$\delta_\Gamma(E_n - E_m) \ \rightarrow \ \frac{1}{\Gamma} F\left(\frac{E_n - E_m}{\Gamma}\right)$$

Beyond the Kubo formula



w_{nm}^{-1} $\Longleftrightarrow$ resistor between node n and node m

D^{-1} $\Longleftrightarrow$ resistivity of the network

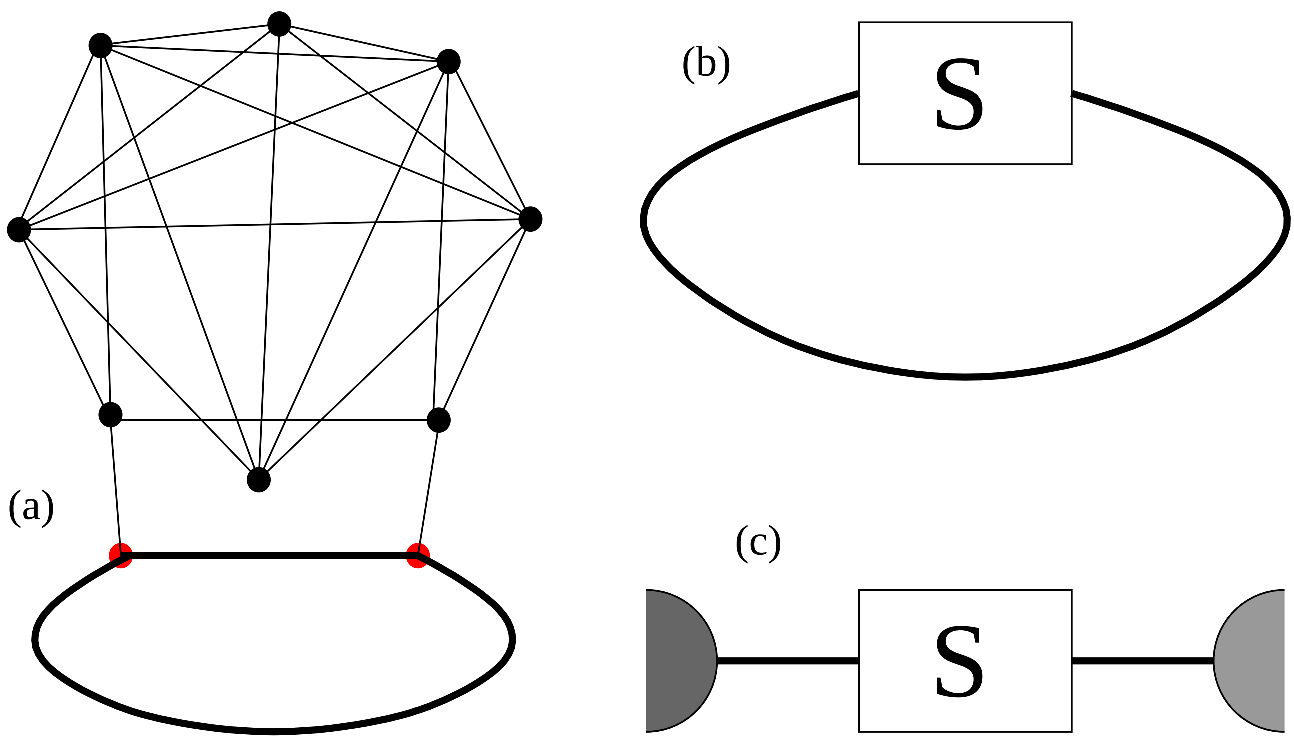
The dimensionless transition rates are

$$\mathrm{g}_{nm} = \frac{|I_{nm}|^2}{(n-m)^2}\,\frac{1}{\gamma}F\left(\frac{n-m}{\gamma}\right)$$

$$\mathrm{g}\Big|_{\mathrm{Meso}} \approx \left[\frac{1}{N}\sum_{n}^{N}\left[\frac{1}{2}\sum_{m}(m-n)^2\mathrm{g}_{nm}\right]^{-1}\right]^{-1}$$

$$\mathrm{g}\Big|_{\mathrm{Kubo}} = \left[\frac{1}{N}\sum_{n}^{N}\left[\frac{1}{2}\sum_{m}(m-n)^2\mathrm{g}_{nm}\right]\right]$$

Problem No.1 - single mode conductance



$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar}\, g_{cl}$$

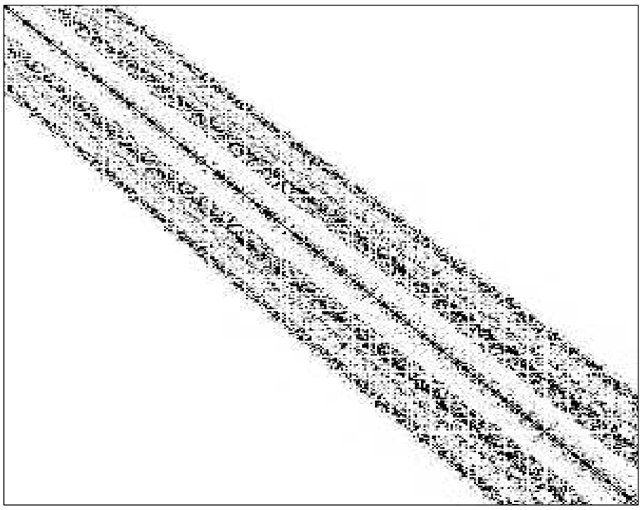
$$G_{\text{Drude}} = \frac{e^2}{2\pi\hbar}\left(\frac{g_{cl}}{1-g_{cl}}\right)$$

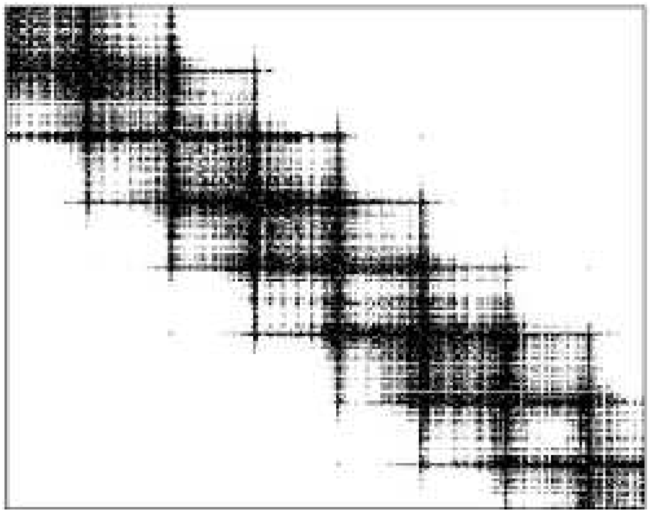
$$G_{\text{Mesoscopic}} \approx \frac{e^2}{2\pi\hbar}\,(1-g_{cl})^2\, g_{cl}$$

where

$$g_{cl} = \text{the average transmission}$$

The perturbation matrix



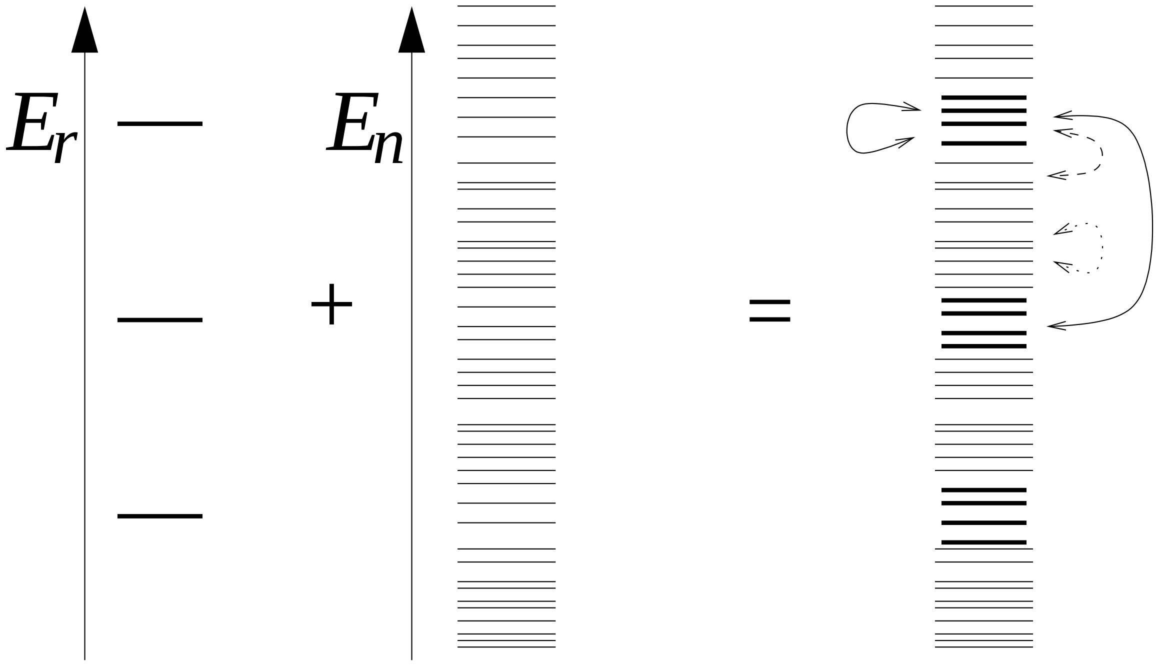


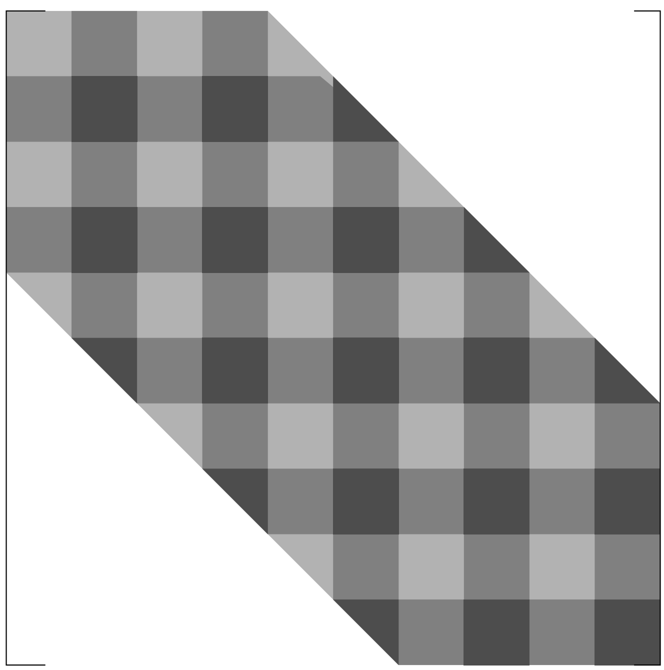
For structured matrix algebraic average is wrong!
Therefore the "classical" result is not obtained.

How is the coarse grained diffusion determined?

$$\langle\langle D_E \rangle\rangle \;=\; \left[\ \overline{1/D_E}\ \right]^{-1}$$

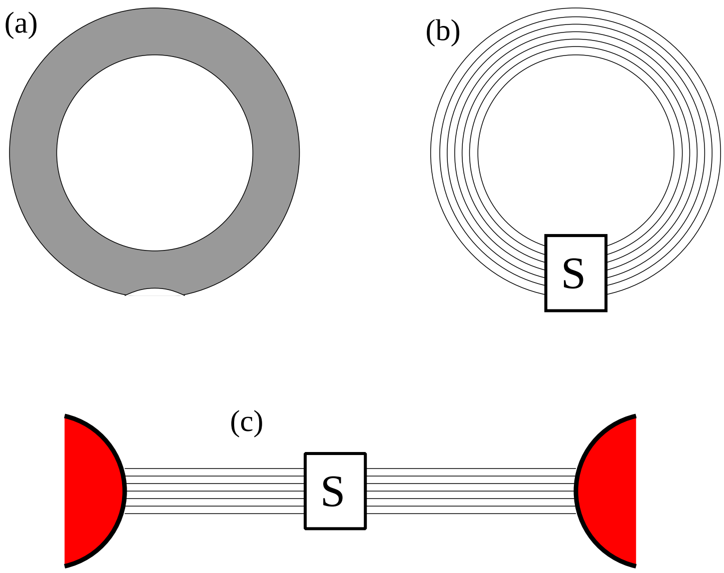
Why do we have structures?





Large scale $\mathcal{O}(\hbar)$ structures are the fingerprints of the "non-universal" classical limit.

Problem No.2 - multi mode conductance



The total transmission $g_T \sim 1$

$$L = \text{perimeter of the ring}$$

$$\ell = \text{mean free path}$$

Note: $\frac{\ell}{L} = \left(\frac{1}{1-g_T}\right)$

$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar}\,\mathcal{M}\,g_{cl}$$

$$G_{\text{Drude}} = \frac{e^2}{2\pi\hbar}\,\mathcal{M}\,\left(\frac{g_{cl}}{1-g_{cl}}\right)$$

Necessary condition for QCC: $\left(\frac{1}{1-g_{cl}}\right) \ll \mathcal{M}$

Digression: The "classical" result

Single mode versions:

$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar} g_{cl}$$

$$G = \frac{e^2}{2\pi\hbar} \left(\frac{g_{cl}}{1 - g_{cl}} \right)$$

Multimode versions:

$$\boldsymbol{g} = \begin{pmatrix} g^R & g^T \\ g^T & g^R \end{pmatrix}$$

$$G_{\text{Landauer}} = \frac{e^2}{2\pi\hbar} \sum_{n,m} g^T_{nm}$$

$$G = \frac{e^2}{2\pi\hbar} \sum_{nm} \left[2g^T/(1 - g^T + g^R) \right]_{nm}$$

[D.C. and Etzioni, JPA 2005]

Eigenstates

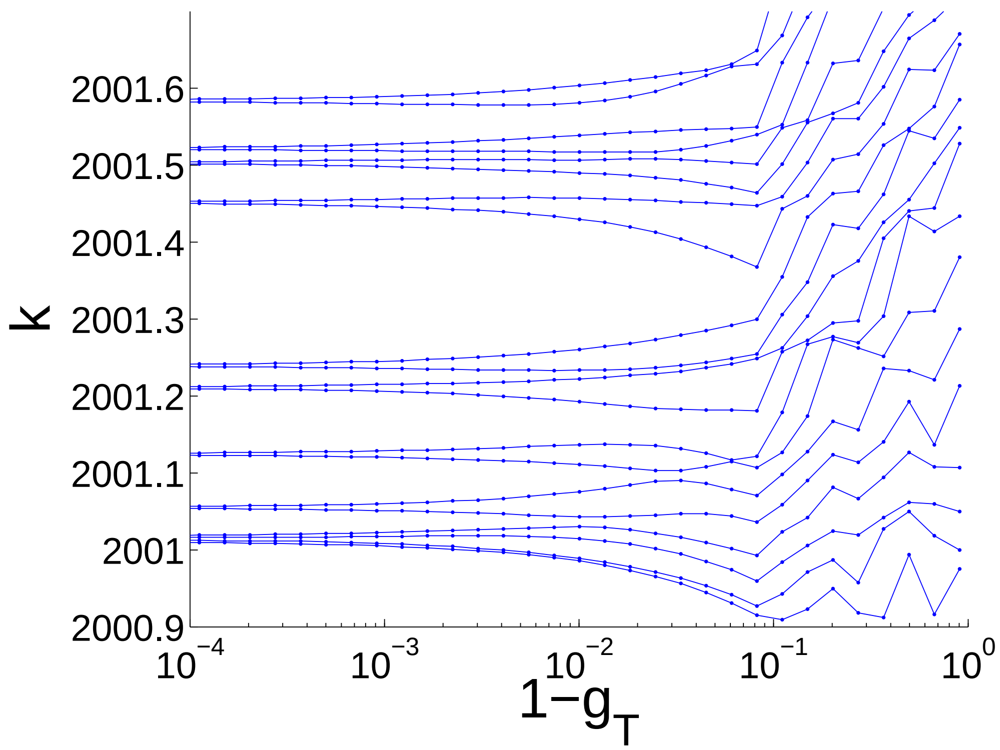
$a \equiv$ mode index $= 1, \cdots, \mathcal{M}$

The eigenfunctions of the ring

$$|\psi\rangle = \sum_a A_a \sin(kx + \varphi_a) \otimes |a\rangle$$

For a given g_T we find:

$(k_n, \varphi_a^{(n)}, A_a^{(n)})$ $\quad n =$ level index

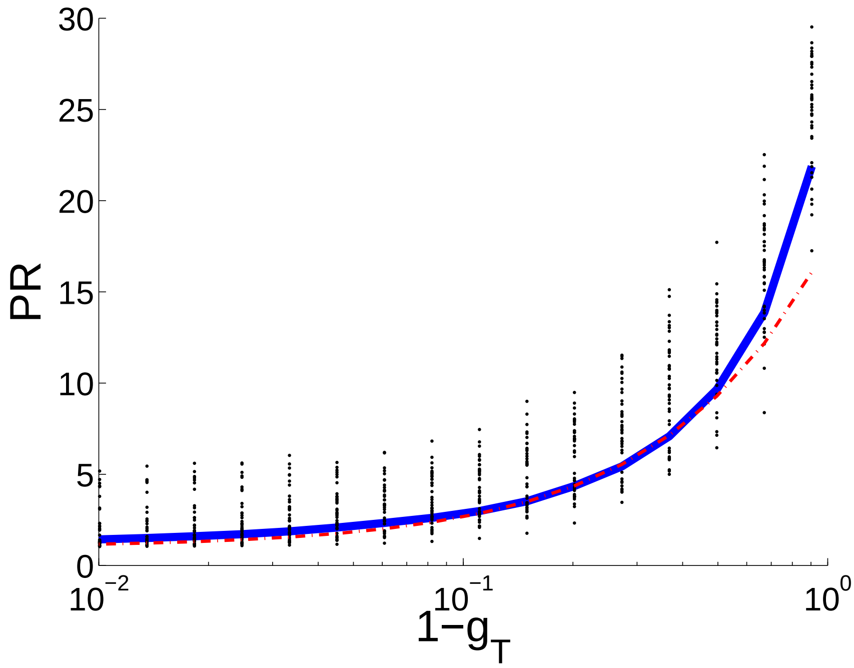


$\mathcal{M} = 50$ bonds of length $L_a \sim 1$ each

Lack of quantum ergodicity

Participation ratio

$$\mathrm{PR} \equiv \left[\sum_a \left(\frac{L_a}{2} A_a^2 \right)^2 \right]^{-1} = \begin{cases} 1 & \text{Localized} \\ \mathcal{M} & \text{Uniform} \end{cases}$$



$$\mathrm{PR} \approx 1 + \tfrac{1}{3}(1 - g_T)\mathcal{M}$$

The non-trivial ballistic regime:

$$1/\mathcal{M} \ll (1 - g_T) \ll 1$$

The perturbation matrix

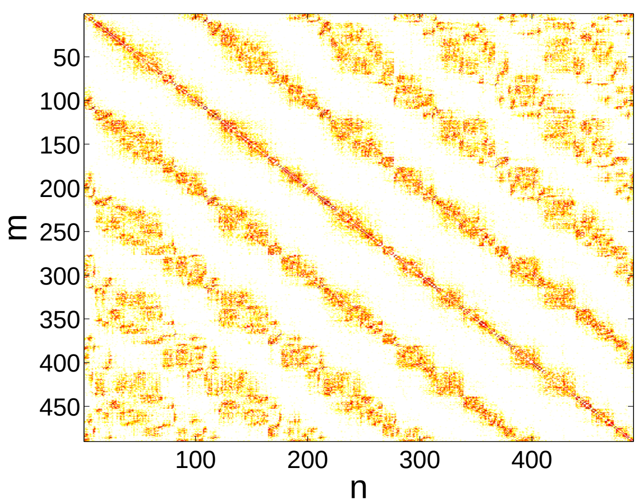
The current operator:

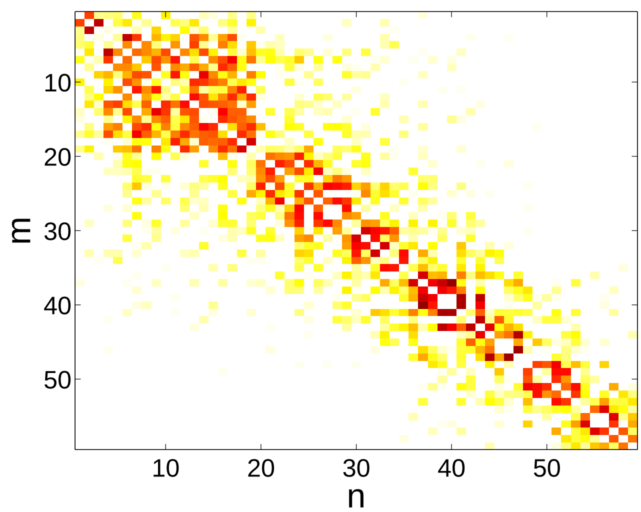
$$\hat{\mathcal{I}} \; = \; e\hat{v} \; \delta(\hat{x} - x_0) \qquad \text{(symmetrized)}$$

Its matrix elements:

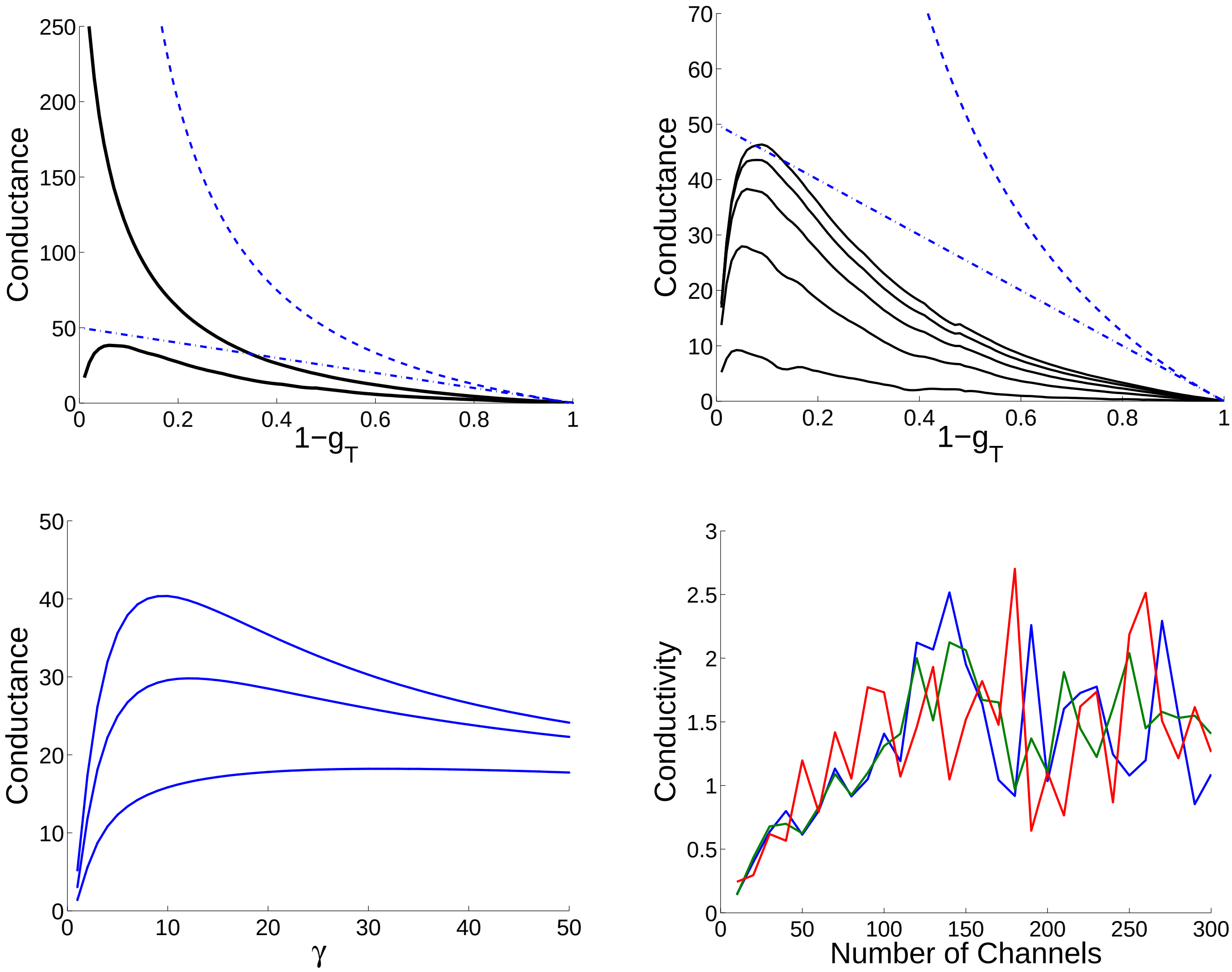
$$\mathcal{I}_{nm} \; \approx \; -iev_{\mathrm{F}} \sum_a \frac{1}{2} \mathsf{A}_a^{(n)} \mathsf{A}_a^{(m)} \sin(\varphi_a^{(n)} - \varphi_a^{(m)})$$

Small PR implies sparsity of $\mathcal{I}_{nm}$





Results for the conductance



$$\mathcal{M} \quad = \quad \text{number of open modes}$$

$$g_T \quad = \quad \text{total transmission}$$

$$\gamma \quad = \quad \Gamma/\Delta \quad = \quad \text{dimensionless level broadening}$$

Problem No.3 - metallic grains

The Hamiltonian:

$$\mathcal{H} = \mathcal{H}_0 - X(t)\mathcal{I}$$

$$\begin{aligned} \mathcal{H}_0 \quad &\mapsto \quad \{E_n\} \\ \mathcal{I} \quad &\mapsto \quad \{\mathcal{I}_{nm}\} \end{aligned}$$

The driving:

$$\langle X(t)X(t')\rangle = f(t-t')$$

$$F(\omega) = \int_{-\infty}^{\infty} f(\tau)\exp(\mathrm{i}\omega t)\mathrm{d}\tau$$

For example:

$$F(\omega) \;=\; \frac{\varepsilon^2}{\omega_0}\exp\left(-\frac{|\omega|}{\omega_0}\right)$$

Temperature $\omega_0 = k_{\mathrm{B}}T/\hbar$

We assume $k_{\mathrm{B}}T \ll \Delta$

The diffusion picture

Fermi Golden rule

$$w_{nm} \;=\; \frac{1}{\hbar^2}\, F\left(\frac{E_n - E_m}{\hbar}\right)\, |\mathcal{I}_{nm}|^2$$

Master equation

$$\frac{\mathrm{d}p_n}{\mathrm{d}t} = \sum_m w_{nm}(p_m - p_n)$$

Diffusion in energy space

$$\frac{\partial}{\partial t}p = \frac{\partial}{\partial n}\left[D\frac{\partial}{\partial n}p\right]$$

If the the transitions are only between neighboring levels:

$$D = \left\langle w_{\mathrm{n.n.}}^{-1} \right\rangle^{-1}$$

leading to semi-linear response:

$$D_E \;=\; \frac{\sigma^2}{(\varrho\hbar)^3}\left[\int \frac{\mathrm{d}\mathbf{x}\, \mathrm{e}^{-\mathbf{x}^2/2}}{(2\pi)^{N/2}\mathbf{x}^2}\right]^{-1}\left[\int_0^\infty \mathrm{d}\omega\, \frac{P_2(\varrho\hbar\omega)}{F(\omega)}\right]^{-1}$$

Results of LRT and SLR

Linear response theory (LRT):

$$D_E = \sigma^2 \hbar \varrho \int_0^{\infty} \mathrm{d}\omega\, \omega^2\, R_2(\hbar\omega) F(\omega)$$

Semi-linear response (SLR):

$$D_E = \frac{\sigma^2}{(\varrho\hbar)^3} \left[\int \frac{\mathrm{d}\mathbf{x}\, \mathrm{e}^{-\mathbf{x}^2/2}}{(2\pi)^{N/2} \mathbf{x}^2} \right]^{-1} \left[\int_0^{\infty} \mathrm{d}\omega \frac{P_2(\varrho\hbar\omega)}{F(\omega)} \right]^{-1}$$

Level spacing statistics:

$$P_2(S) \approx a_\beta S^\beta \exp(-c_\beta S^2) \qquad \text{with } \beta = 1, 2, 4$$

The LRT result of Gorkov and Eliashberg:

$$D_E = C_\beta \sigma^2 \varepsilon^2 (\hbar\varrho)^{\beta+1}\, \omega_0^{\beta+2}$$

Our SLR result (large S statistics!):

$$D_E = \frac{\varepsilon^2 \sigma^2}{2\hbar\varrho} \frac{1}{(\hbar\varrho\omega_0)^{\beta-1}} \exp\left[-\frac{1}{\pi (\hbar\varrho\omega_0)^2} \right]$$

The SLR result - details

For $\mathcal{H} = \mathcal{H}_0 - X(t)\mathcal{I}$ we would get $D_E = 0$ because RMT implies a non-zero probability to have a vanishingly small matrix element.

It is only in 3D that we get absorption:

$$\mathcal{H} = \mathcal{H}_0 - \sum_j X_j(t)\mathcal{I}^j$$

with

$$\langle X_i(t) X_j(t') \rangle = \delta_{ij} f(t-t')$$

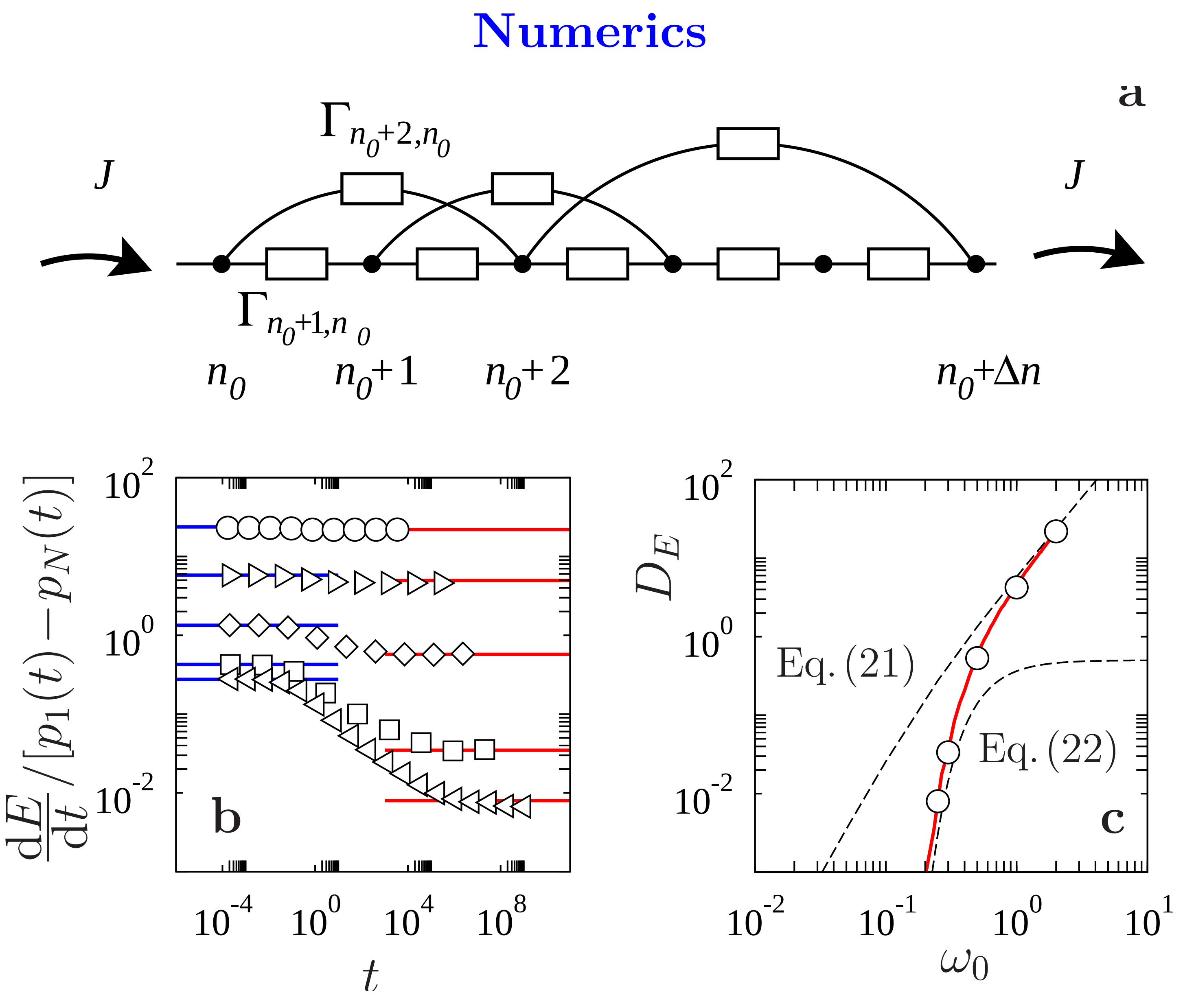
$$F(\omega) = \int_{-\infty}^{\infty} f(\tau) \exp(\mathrm{i}\omega t) \mathrm{d}\tau$$

We have used:

$$F(\omega) \ = \ \frac{\varepsilon^2}{\omega_0} \exp\left(-\frac{|\omega|}{\omega_0}\right)$$

Temperature $\omega_0 = k_{\mathrm{B}}T/\hbar$

We assume $k_{\mathrm{B}}T \ll \Delta$



Numerics
a
J
J
Γ_{n_0+2,n_0}
Γ_{n_0+1,n_0}
n_0
n_0+1
n_0+2
$n_0+\Delta n$
$\frac{dE}{dt}/[p_1(t)-p_N(t)]$
t
b
D_E
ω_0
Eq. (21)
Eq. (22)
c