The conductance of small mesoscopic disordered rings

Doron Cohen, Ben-Gurion University

Collaborations:

Tsampikos Kottos (Wesleyan)
Holger Schanz (Gottingen)
Swarnali Bandopadhyay (BGU)
Yoav Etzioni (BGU)
Tal Peer (BGU)
Rangga Budoyo (Wesleyan)
Alex Stotland (BGU)

Discussions:

Michael Wilkinson (UK)
Bernhard Mehlig (Goteborg)
Yuval Gefen (Weizmann)
Shmuel Fishman (Technion)

$ISF, $GIF, $DIP, $BSF

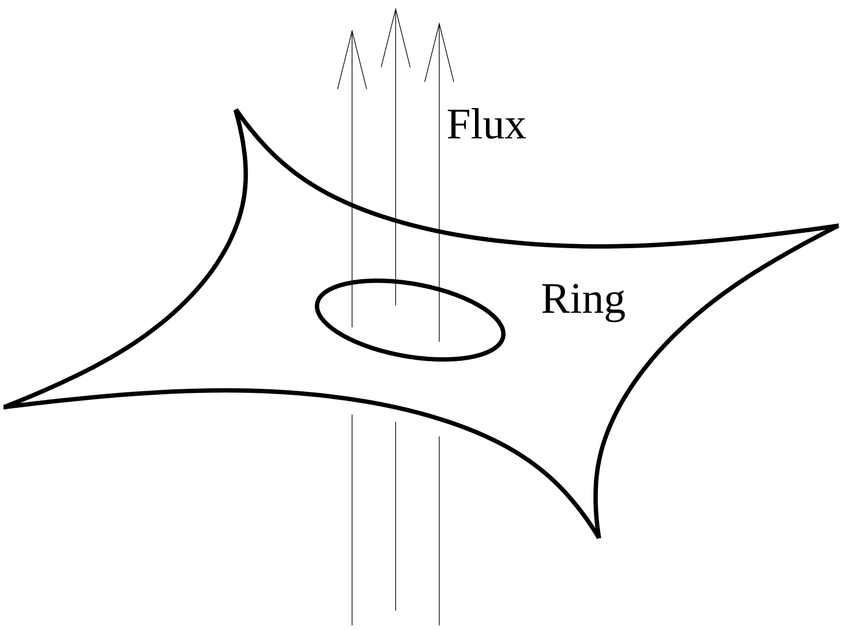
Driven Systems

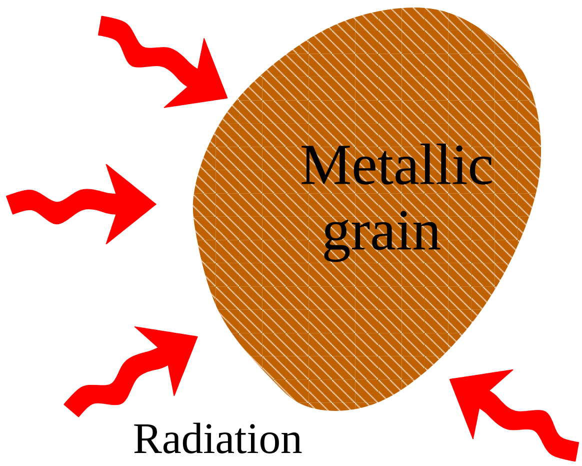
Non interacting “spinless” electrons in a ring.

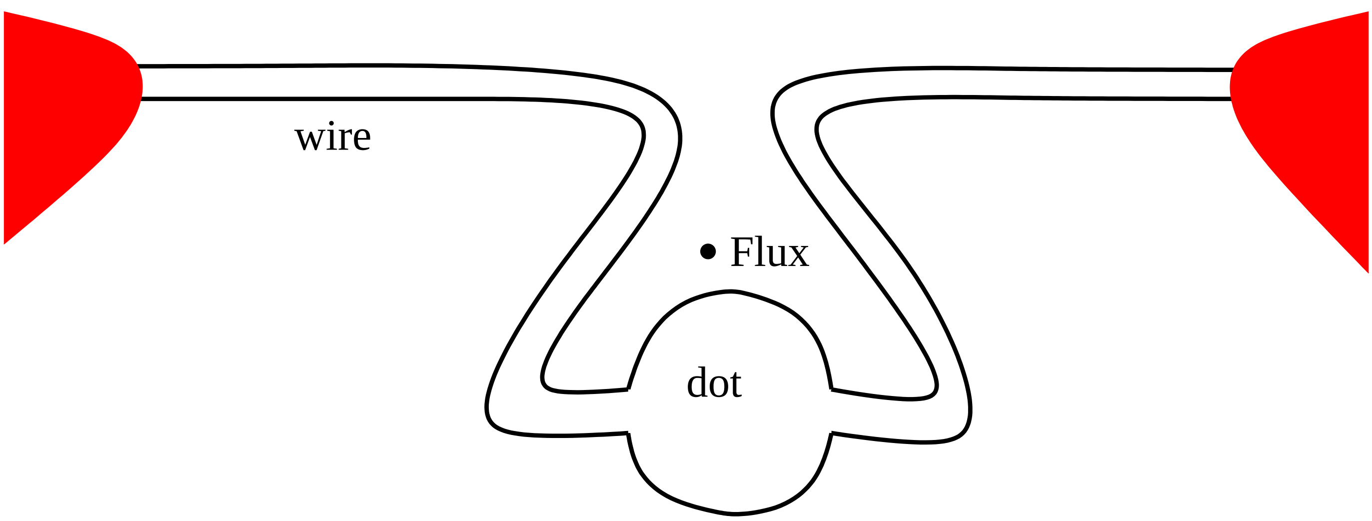
$$\mathcal{H}(Q, P; \Phi(t))$$

$$-\dot{\Phi} \quad = \quad \text{electro motive force (RMS)}$$

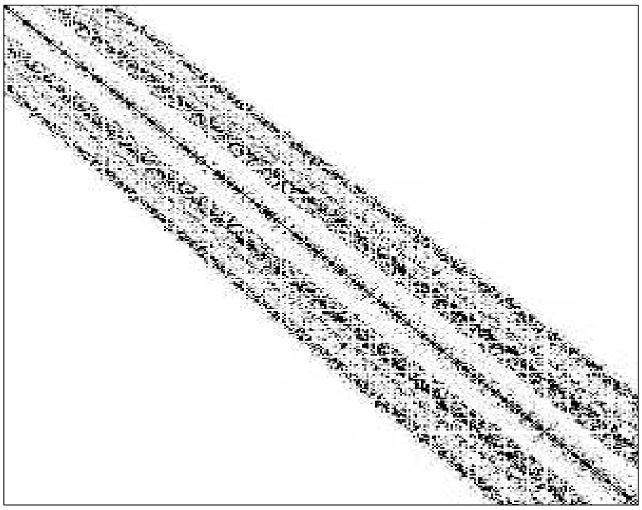
$$G\,\dot{\Phi}^2 \quad = \quad \text{rate of energy absorption}$$

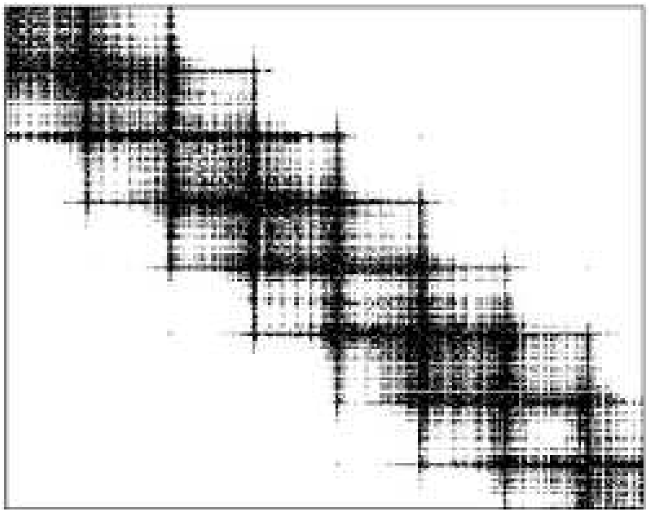






Linear Response Theory (LRT)





$$H = \{E_n\} - \Phi(t)\{\mathcal{I}_{nm}\}$$

$$\boldsymbol{G} = \pi\hbar \sum_{n,m} |\mathcal{I}_{mn}|^2 \ \delta_T(E_n - E_F)\ \delta_\Gamma(E_m - E_n)$$

$$\boldsymbol{G} \ = \ \pi\hbar(\varrho(E_F))^2 \ \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle$$

applies if

EMF driven transitions $\ll$ relaxation

otherwise

connected sequences of transitions are essential.

leading to

Semi Linear Response Theory (SLRT)

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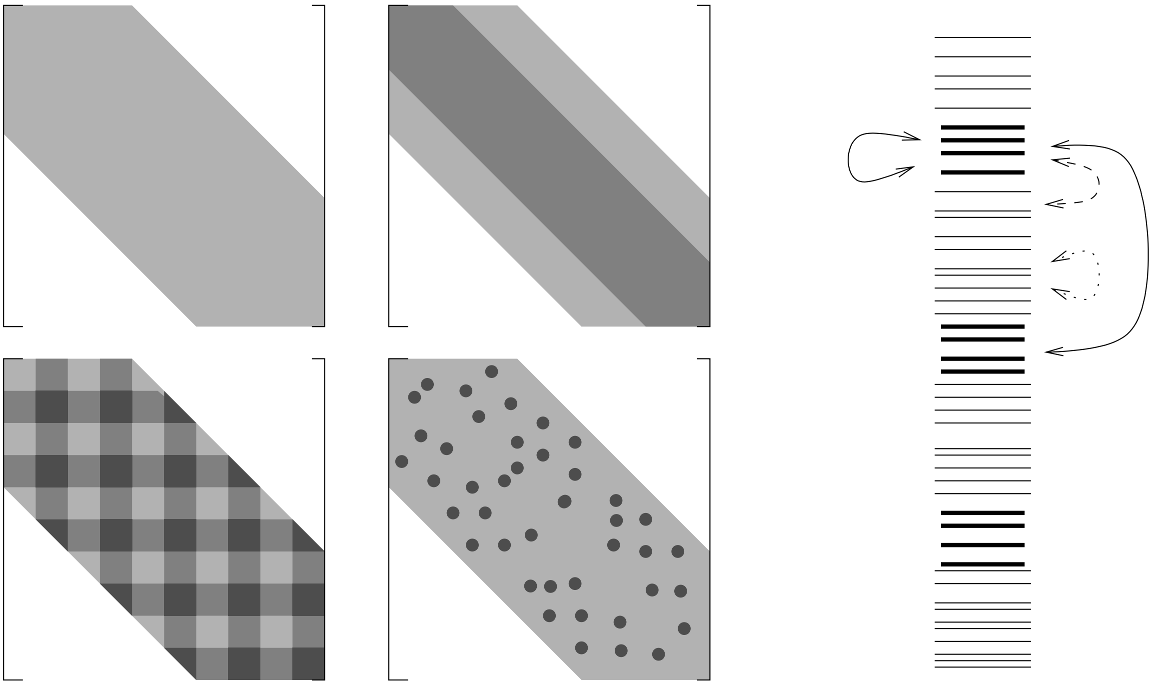


$$H = \{E_n\} - \Phi(t)\{\mathcal{I}_{nm}\}$$

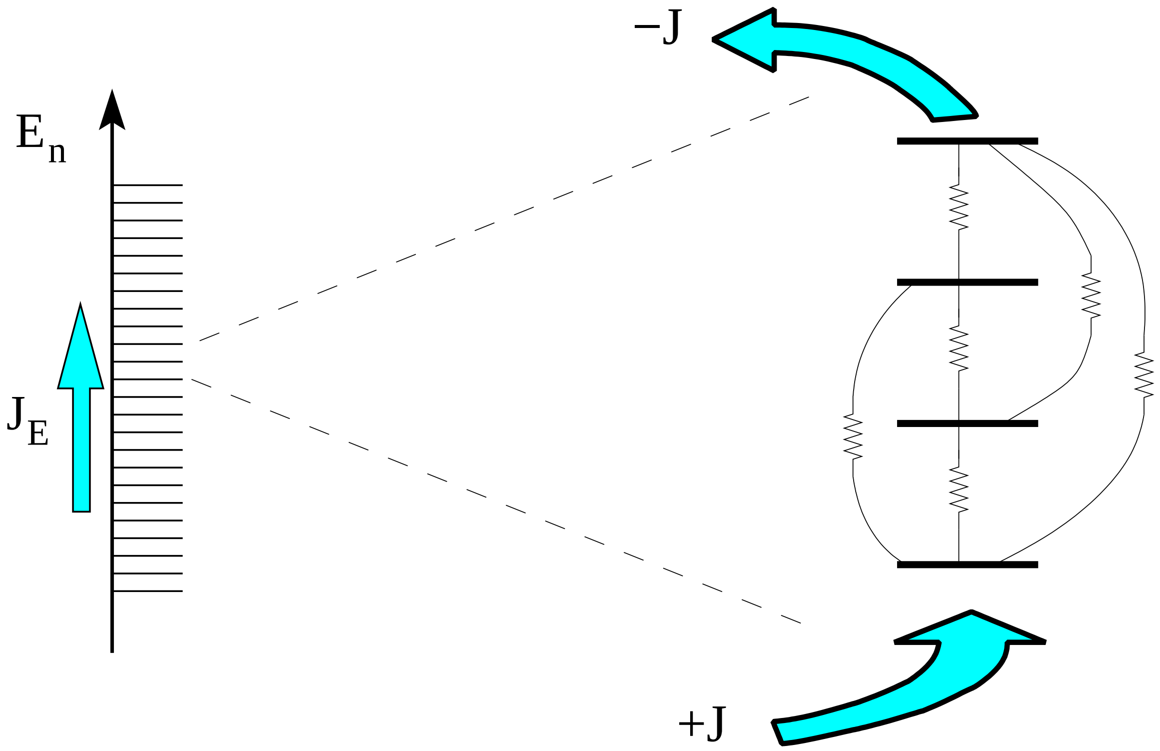
$$\frac{dp_n}{dt} = -\sum_m w_{nm}(p_n - p_m)$$

$$w_{nm} \quad = \quad \text{const} \times \mathrm{g}_{nm} \times \text{EMF}^2$$

Scaled transition rates:

$$\mathrm{g}_{nm} = 2\varrho_{\mathrm{F}}^{-3} \; \frac{|\mathcal{I}_{nm}|^2}{(E_n - E_m)^2} \; \delta_\Gamma(E_n - E_m)$$

Semi Linear Response Theory (cont.)



$$g_{nm} = 2\varrho_F^{-3} \; \frac{|\mathcal{I}_{nm}|^2}{(E_n - E_m)^2} \; \delta_\Gamma(E_n - E_m)$$

The SLRT analog of the Kubo formula:

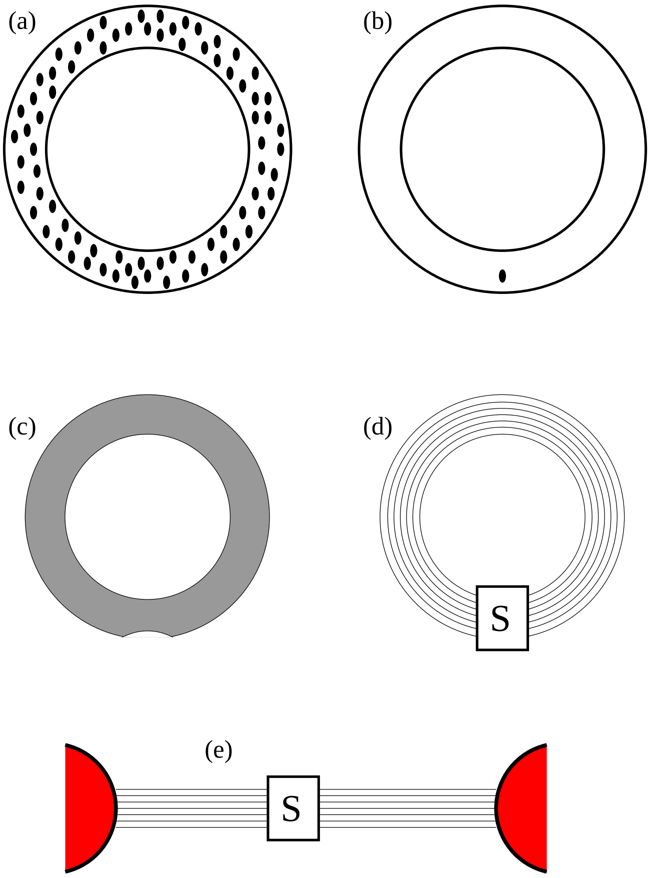
$$\boldsymbol{G} \;\; = \;\; \pi\hbar(\varrho(E_F))^2 \; \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle$$

where

$\langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle \equiv$ inverse resistivity of the network

$$\langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle_{\text{harmonic}} \ll \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle \ll \langle\langle|\mathcal{I}_{mn}|^2\rangle\rangle_{\text{algebraic}}$$

Conductance of mesoscopic rings



Naive expectation (assuming $\Gamma > \Delta$):

$$G = \frac{e^2}{2\pi\hbar}\mathcal{M}\frac{\ell}{L} + \mathcal{O}\left(\frac{\Delta}{\Gamma}\right)$$

L = perimeter of the ring

ℓ = mean free path $\propto W^2$

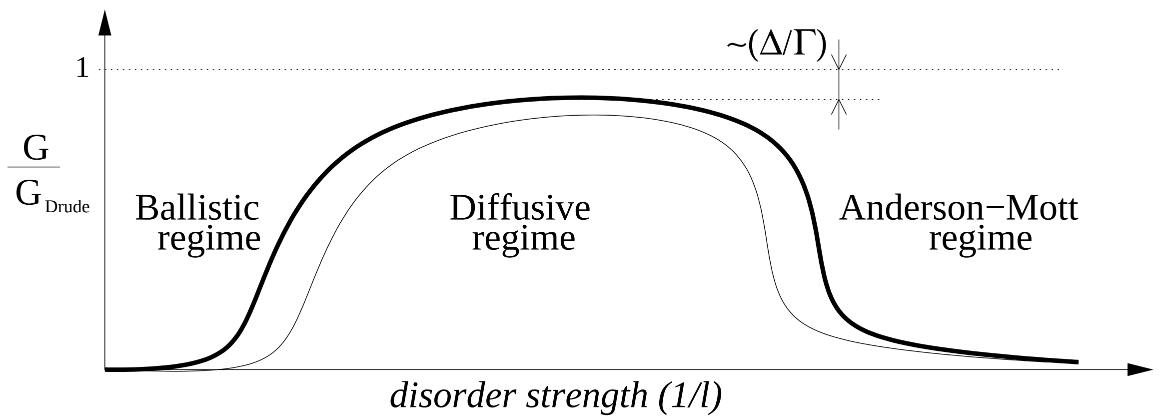
ℓ_∞ = localization length $\approx \mathcal{M}\ell$

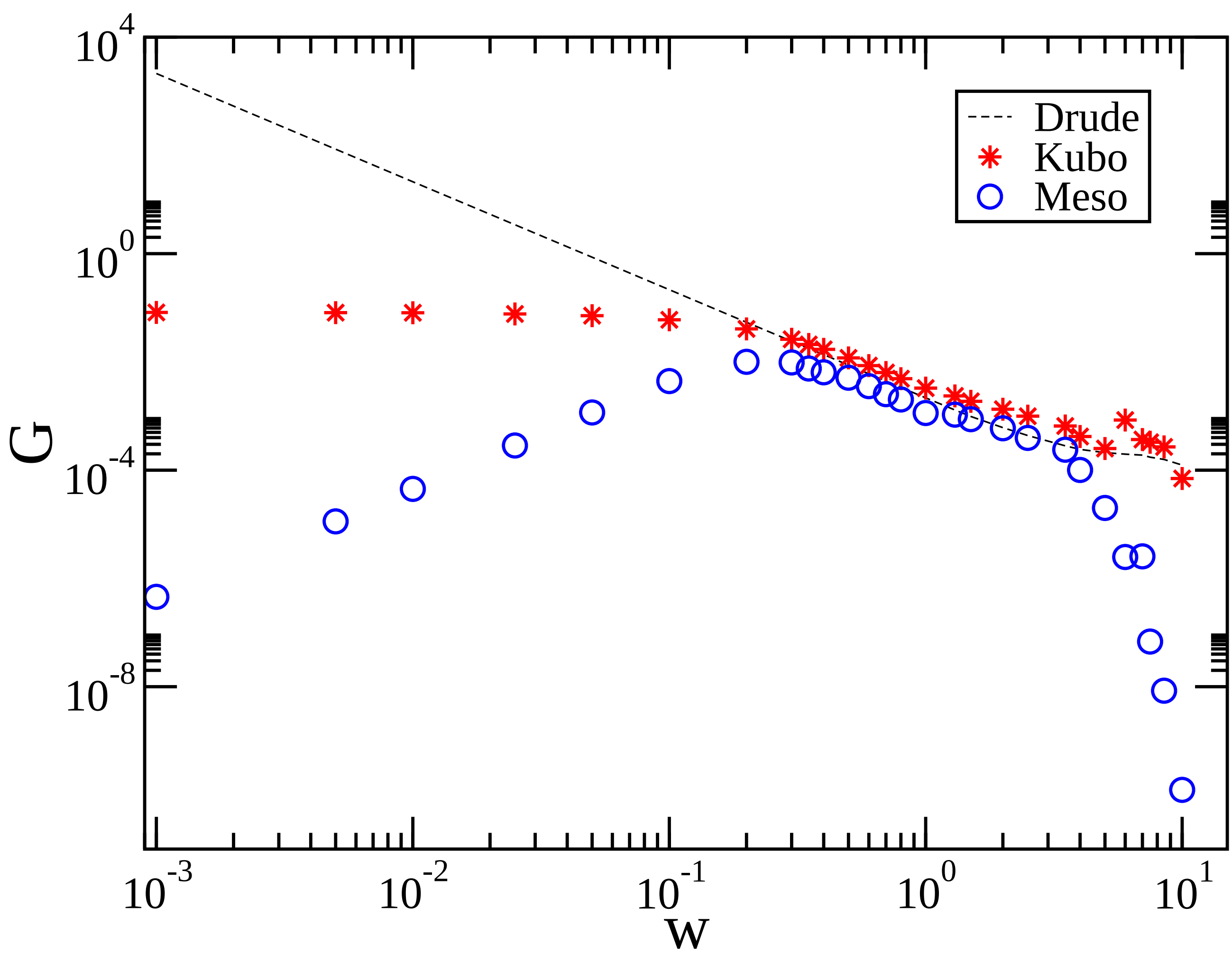
Ballistic regime: $L \ll \ell$

Diffusive regime: $\ell \ll L \ll \ell_\infty$

Anderson regime: $\ell_\infty \ll L$

Conductance versus disorder



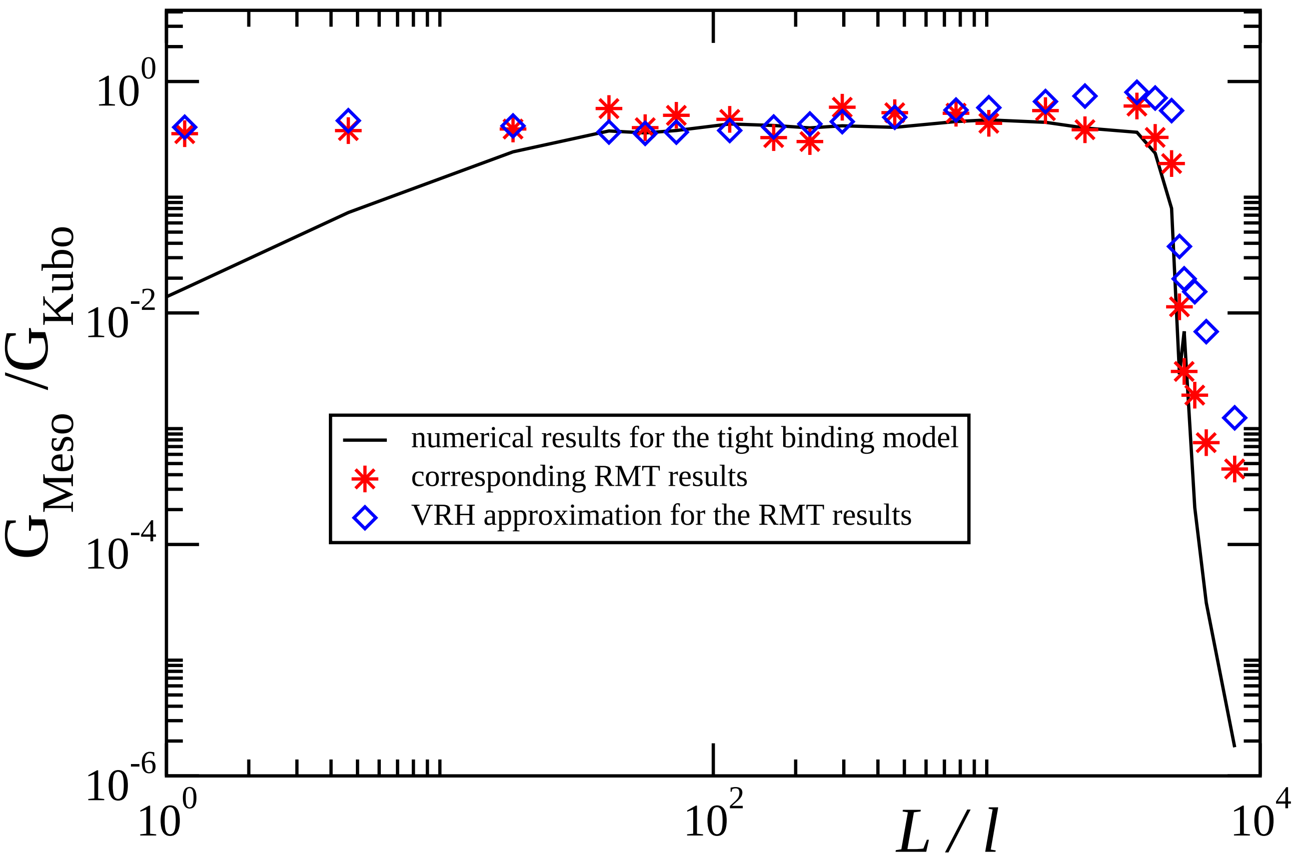


The RMT modeling

$$\{|v_{nm}|^2\} \;\equiv\; \{X\}$$

Characterization of the perturbation matrix:

- bandwidth (b)
- sparsity (p)
- texture

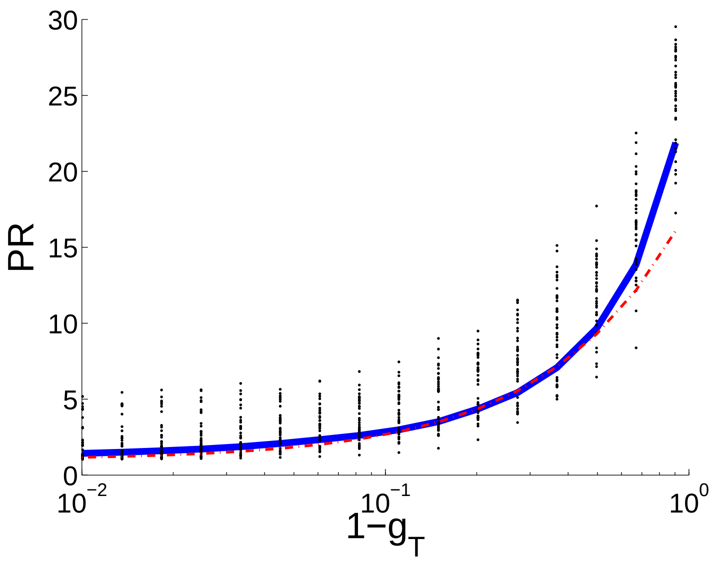


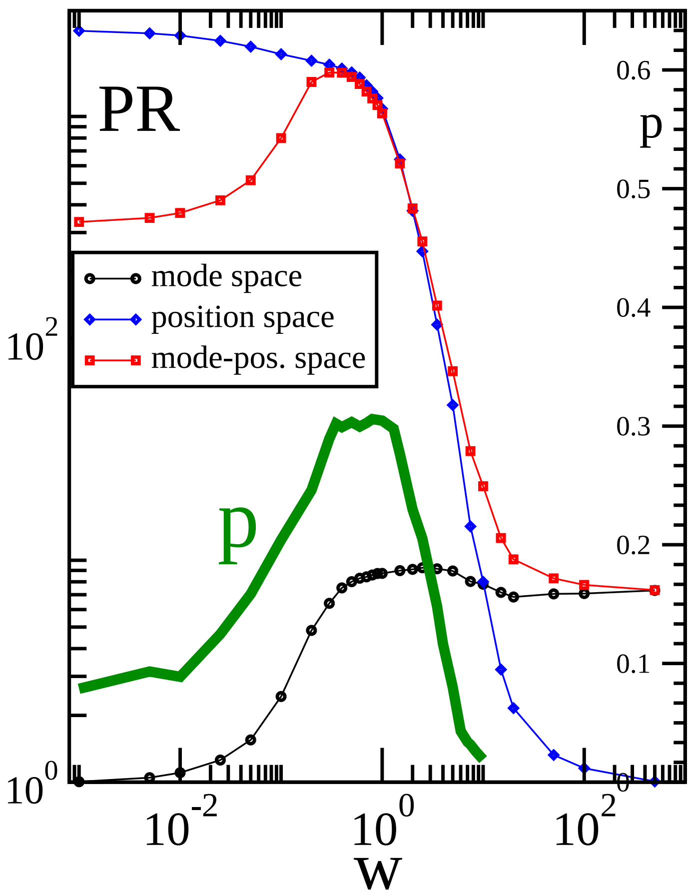
Comparison between:

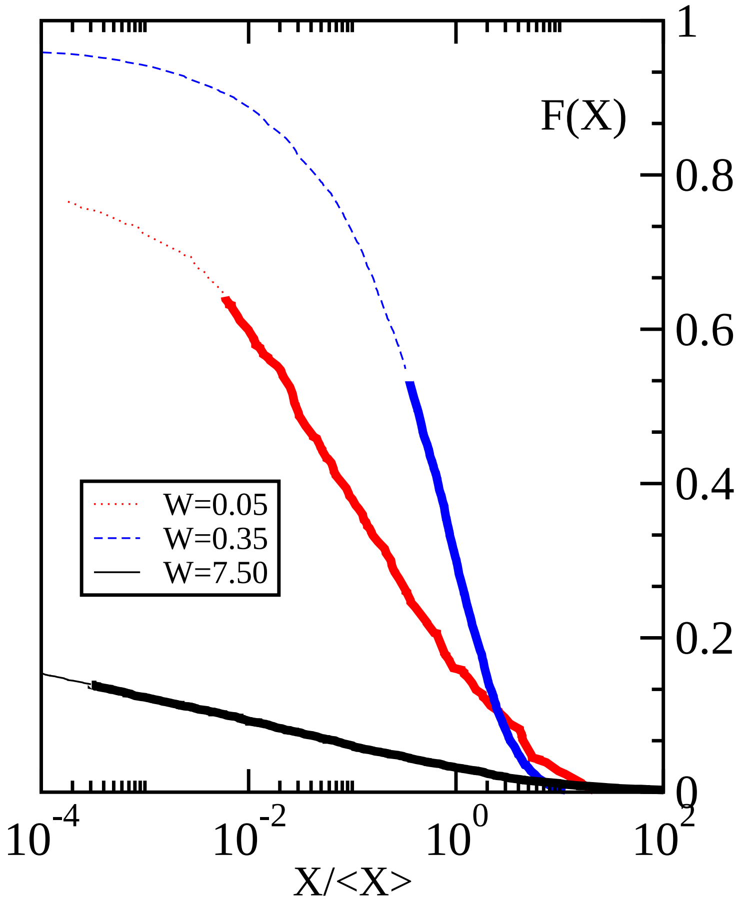
- Actual results based on “real” matrices
- RMT results based on “artificial” matrices
- Semi-analytical VRH estimate

Ergodicity of the eigenstates

- Weak disorder (ballistic rings): Wavefunctions are localized in mode space.
- Strong disorder (Anderson localization): Wavefunctions are localized in real space.

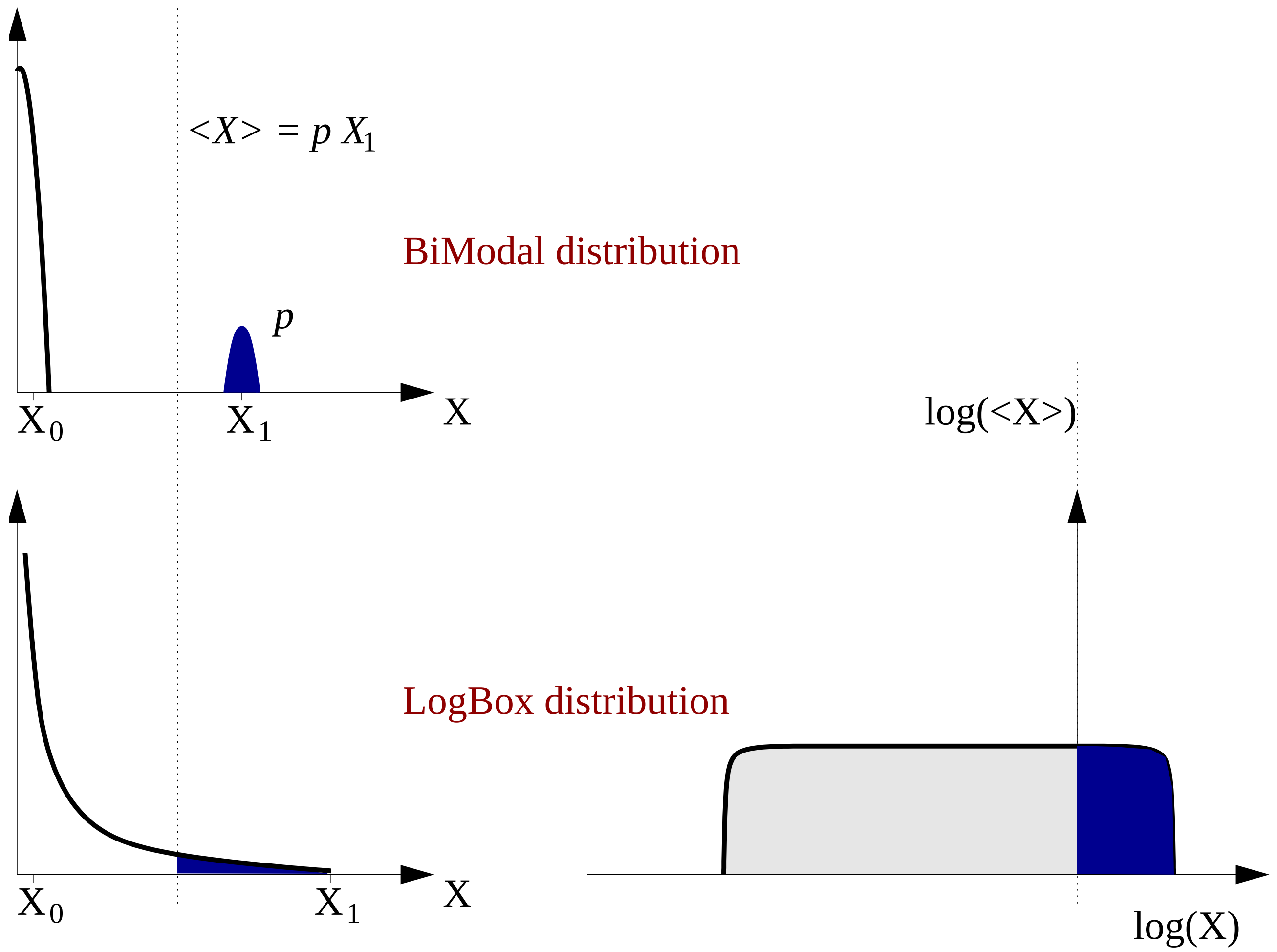






Modeling of sparsity

$$X \equiv |v_{nm}|^2 \sim \frac{1}{\mathcal{M}^2} v_{\mathrm{F}}^2 \exp\left(-\frac{x}{l_\infty}\right)$$



$$X \in \mathrm{LogBox}[X_0, X_1]$$

$$\tilde{p} \equiv (\ln(X_1/X_0))^{-1}$$

$$p \equiv \mathrm{Prob}\Big(X > \langle X \rangle\Big) \approx -\tilde{p}\ln\tilde{p}$$

$$\langle X \rangle \approx \tilde{p}X_1 \sim pX_1$$

The VRH estimate

$$\boldsymbol{G} \;=\; \pi\hbar \left(\frac{e}{L}\right)^2 \sum_{n,m} |v_{mn}|^2 \; \delta_T(E_n - E_F) \; \delta_\Gamma(E_m - E_n)$$

$$\boldsymbol{G} \;=\; \frac{1}{2} \left(\frac{e}{L}\right)^2 \varrho_{\mathrm{F}} \int \tilde{C}_{\mathrm{qm}}(\omega) \; \delta_\Gamma(\omega) \; d\omega$$

$$\tilde{C}_{\text{qm-LRT}}(\omega) \quad \equiv \quad 2\pi \varrho_{\mathrm{F}} \langle X \rangle$$

$$\tilde{C}_{\text{qm-SLRT}}(\omega) \quad \equiv \quad 2\pi \varrho_{\mathrm{F}} \overline{X}$$

where by definition: $\left(\frac{\omega}{\Delta}\right) \mathrm{Prob}\left(X > \overline{X}\right) \;\sim\; 1$

For strong disorder we get:

$$\overline{X} \;\approx\; v_{\mathrm{F}}^2 \exp\left(-\frac{\Delta_\ell}{\omega}\right)$$

$$\boldsymbol{G} \;\propto\; \int \exp\left(-\frac{\Delta_\ell}{|\omega|}\right) \exp\left(-\frac{|\omega|}{\omega_c}\right) d\omega$$

LRT, SLRT and beyond

$$-\dot{\Phi} \quad = \quad \text{electro motive force (RMS)}$$
$$G\,\dot{\Phi}^2 \quad = \quad \text{rate of energy absorption}$$

Semi linear response theory

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