BEC dynamics in few site systems

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[1] Occupation dynamics & fluctuations (PRL 2009)

[2] Dynamical phase locking of BECs (PRA 2009)

[3] Semiclassical analysis & quantum dynamics (arXiv 2010)

Thanks:

Isaac Israel

Alexander Stotland

Itamar Sela

Yoav Etzioni

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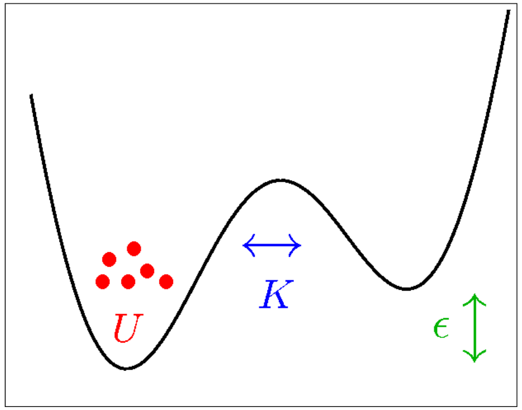


The model

The Bose-Hubbard Hamiltonian (BHH) for a dimer

$$\mathcal{H} \;=\; \sum_{i=1,2} \left[\mathcal{E}_i \hat{n}_i + \frac{U}{2}\hat{n}_i(\hat{n}_i - 1)\right] - \frac{K}{2}(\hat{a}_2^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2)$$

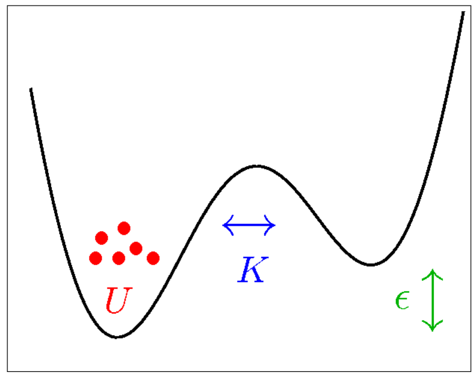
K = hopping

U = interaction

$\mathcal{E} = \mathcal{E}_2 - \mathcal{E}_1$ = bias

N particles in a double well is like spin $j = N/2$ system

$$\mathcal{H} \;=\; -\mathcal{E}\hat{J}_z + U\hat{J}_z^2 - K\hat{J}_x + \text{const}$$



$$\hat{J}_z \;=\; (N/2)\cos(\theta) \;=\; \hat{n} = \text{occupation difference}$$

$$\hat{J}_x \;\approx\; (N/2)\sin(\theta)\cos(\varphi), \qquad \varphi = \text{relative phase}$$

$$u \equiv \frac{NU}{K} \;, \quad \varepsilon \equiv \frac{\mathcal{E}}{K}$$

$$(J_x, J_y, J_z) \quad \Leftrightarrow \quad (\theta, \varphi) \quad \Leftrightarrow \quad (n, \varphi)$$

Phase space analysis

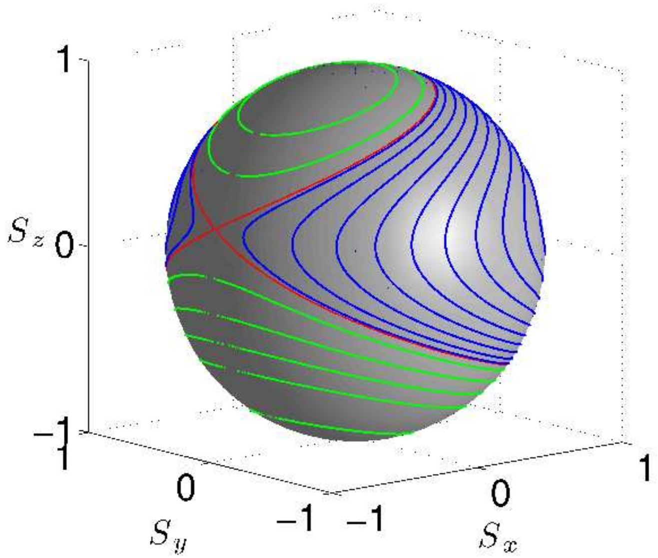
$$\mathcal{H} = -\mathcal{E}\hat{J}_z + U\hat{J}_z^2 - K\hat{J}_x \qquad u \equiv \frac{NU}{K}, \quad \varepsilon \equiv \frac{\mathcal{E}}{K}$$

$$\mathcal{H}(\theta,\varphi) = \frac{NK}{2}\left[\frac{1}{2}u(\cos\theta)^2 - \varepsilon\cos\theta - \sin\theta\cos\varphi\right]$$ spherical phase space

$\mathcal{H}(\hat{n},\varphi) =$ (similar to Josephson/pendulum Hamiltonian) cylindrical

For $u>1$ and $|\varepsilon| < \varepsilon_c$

Sea, Islands, Separatrix



$$\varepsilon_c = \left(u^{2/3} - 1\right)^{3/2}$$

$$A_c \approx 4\pi\left(1 - u^{-2/3}\right)^{3/2}$$

Rabi regime: $u < 1$ (no islands)

Josephson regime: $1 < u < N^2$

Fock regime: $u > N^2$ (empty sea)

WKB quantization

$$h \;=\; \text{Planck cell area in steradians} \;=\; \frac{4\pi}{N+1}$$

$$A(E_n) \;=\; \left(\frac{1}{2}+n\right)h$$

$$\omega(E) \;\equiv\; \frac{dE}{dn} \;=\; \left[\frac{1}{h}A'(E)\right]^{-1}$$

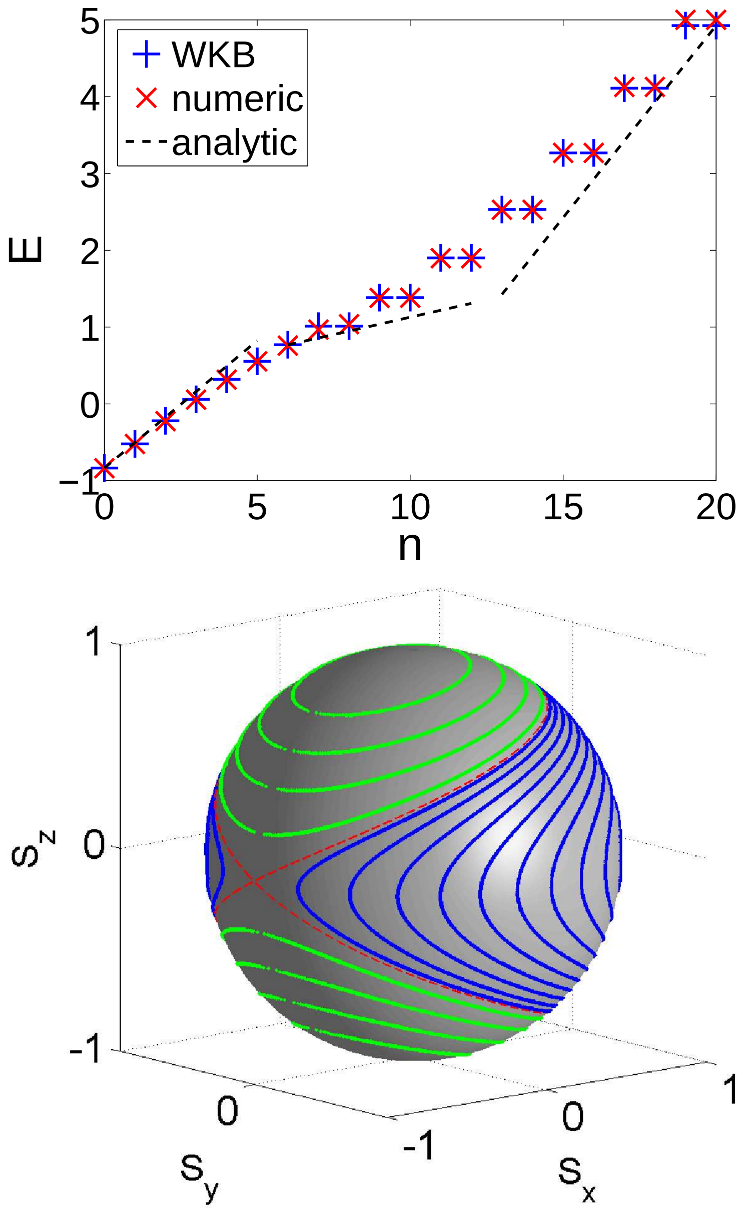
$$\omega_K \;\approx\; K \;=\; \text{Rabi Frequency}$$

$$\omega_J \;\approx\; \sqrt{NUK} \;=\; \sqrt{u}\,\omega_K$$

$$\omega_+ \;\approx\; NU \;=\; u\,\omega_K$$

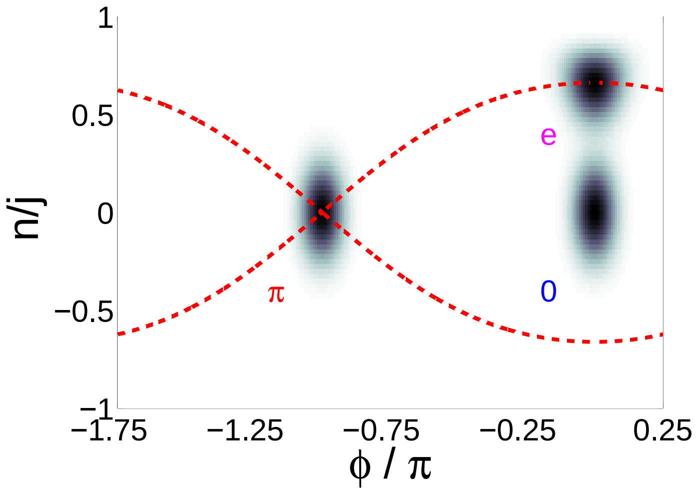
$$\omega_x \;\approx\; \left[\frac{1}{2}\log\left(\frac{N^2}{u}\right)\right]^{-1}\omega_J$$

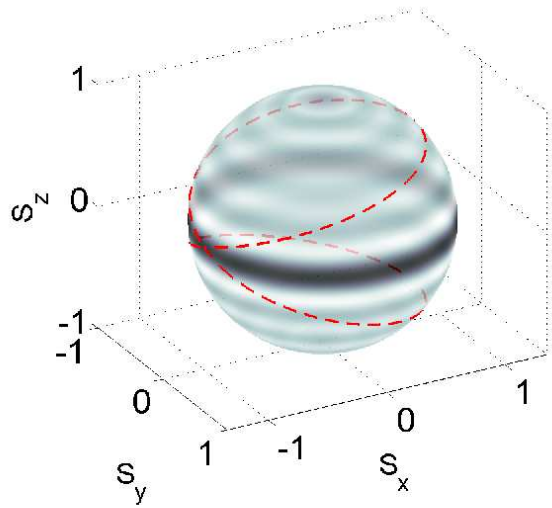
$$E_{-,x} = \mp(1/2)NK\;,\quad E_+ = (1/4)[u+(1/u)]NK$$



Typical preparations and their LDOS

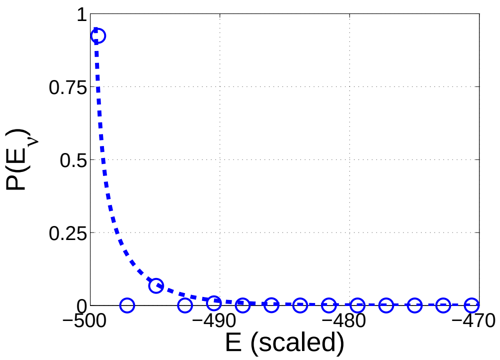
Coherent state
$|\theta\varphi\rangle$
is like a
minimal
Gaussian
wavepacket





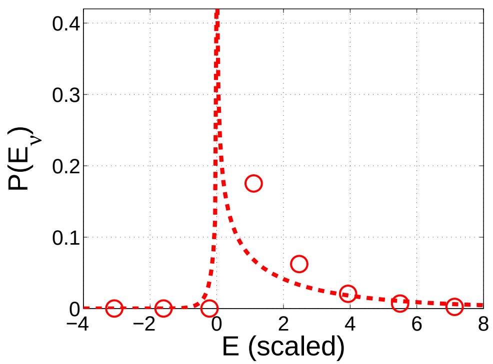
Fock state
$|n\rangle$
is like
equi-latitude
annulus

Zero preparation '0'



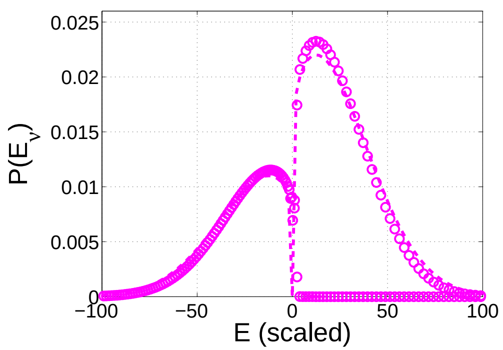
$$P(E) \sim \text{BesselI}\left[\frac{E - E_{-}}{NU}\right]$$

Pi preparation 'π'



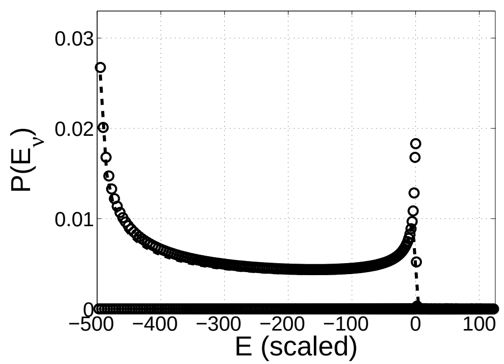
$$\sim \frac{\omega_x}{\omega_J}\,\text{BesselK}\left[\frac{E - E_x}{NU}\right]$$

Edge preparation 'e'



$$\sim \frac{\omega(E)}{\omega_J}\,\exp\left[-\frac{1}{N}\left(\frac{E - E_x}{\omega_J}\right)^2\right]$$

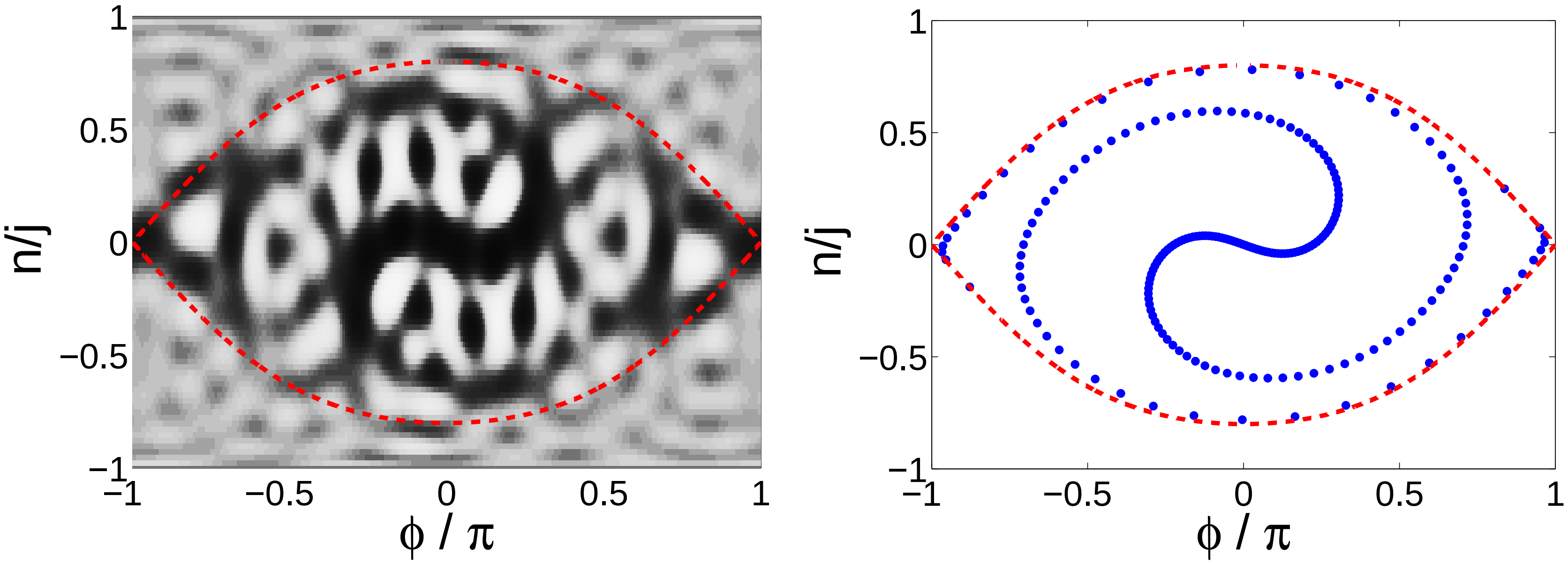
TwinFock preparation



$$\sim \frac{\omega(E)}{\omega_J}\left[1 - \left(\frac{2E}{NK}\right)^2\right]^{-1/2}$$

Numerical simulations - analysis of the evolution

MeanField theory (GPE) = classical evolution of a point in phase space
SemiClassical theory = classical evolution of a distribution in phase space
Quantum theory = recurrences, fluctuations (WKB is very good)



Any operator $\hat{A}$ can be presented by the phase-space function $A_{\mathrm{W}}(\Omega) \;=\; \mathrm{trace}[\hat{P}^{\Omega}\;\hat{\rho}]$

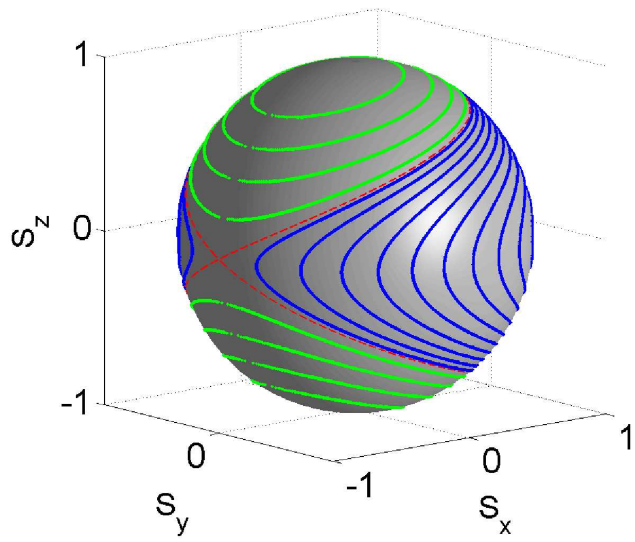
where the projector-like operators obey: $\int \frac{d\Omega}{h}\hat{P}^{\Omega} \;=\; \hat{\mathbf{1}}$ and for Hermitian operators $A_{\mathrm{W}} \in \mathcal{R}$

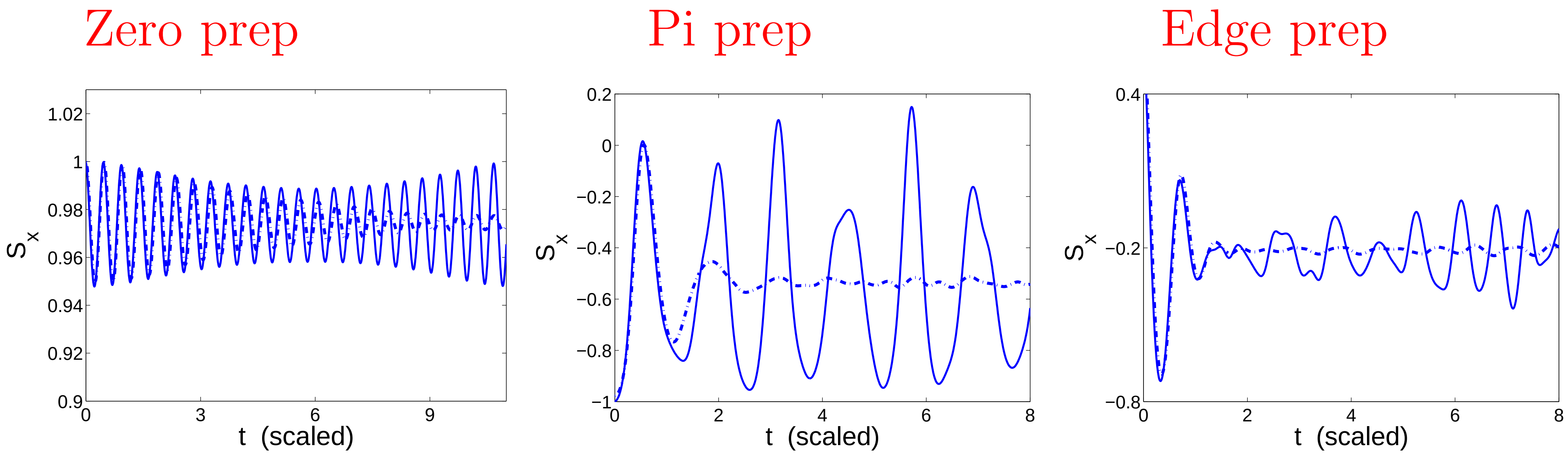
The expectation value: $\left\langle \hat{A} \right\rangle \;=\; \mathrm{trace}[\hat{\rho}\;\hat{A}] \;=\; \int \frac{d\Omega}{h}\rho_{\mathrm{W}}(\Omega)A_{\mathrm{W}}(\Omega)$

Recurrences and fluctuations

$$\vec{S} = \left\langle \vec{J} \right\rangle / j \quad , \quad j = \frac{N}{2}$$

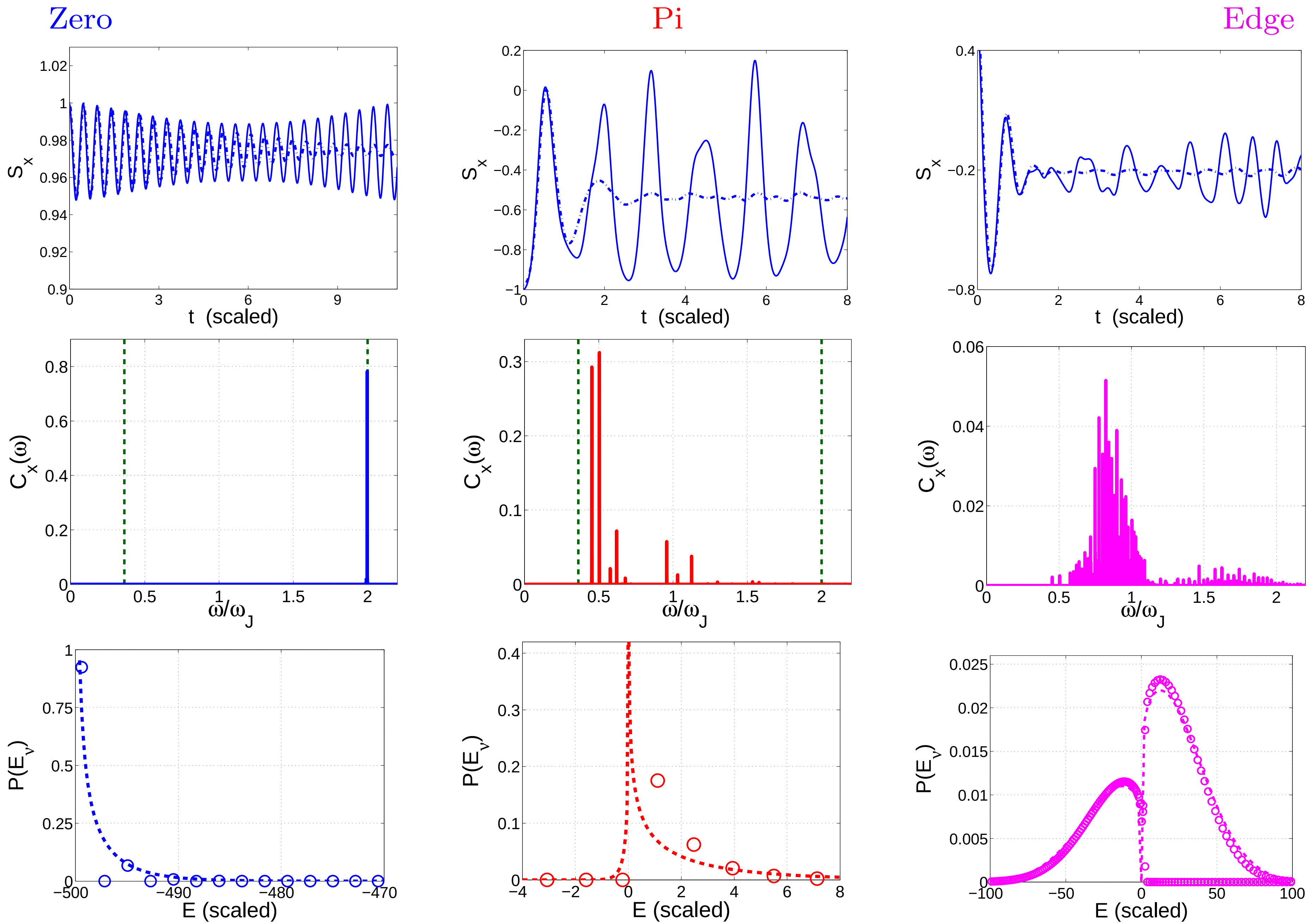
$$\begin{aligned} \text{OccupationDiff} &= (N/2)\,[1 + S_z] \\ \text{OneBodyPurity} &= (1/2)\,\left[1{+}S_x^2{+}S_y^2{+}S_z^2\right] \\ \text{FringeVisibility} &= \left[S_x^2 + S_y^2\right]^{1/2} \\ \text{RelativePhase} &= \arctan(S_x/S_y) \end{aligned}$$



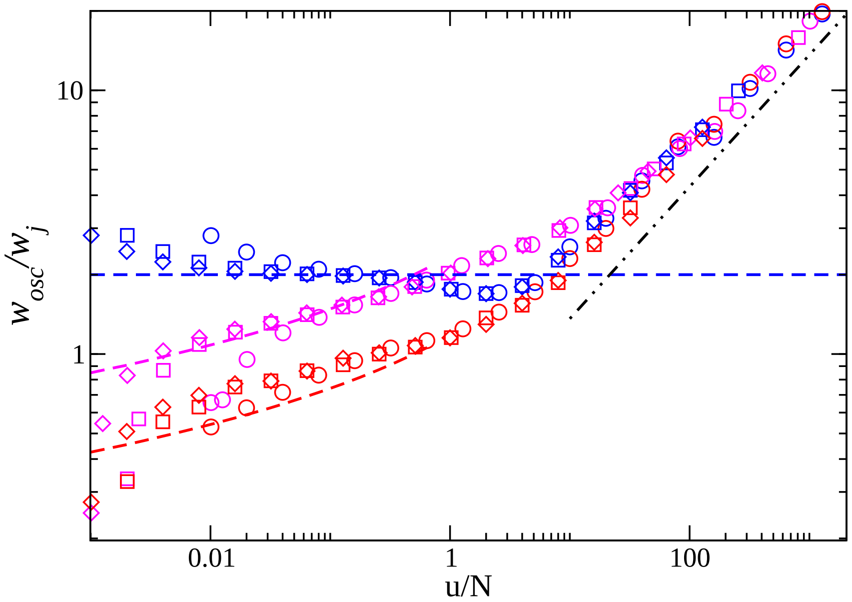


SC —·, QM —

Temporal behavior & the spectral content of S_x



The spectral content of S_x

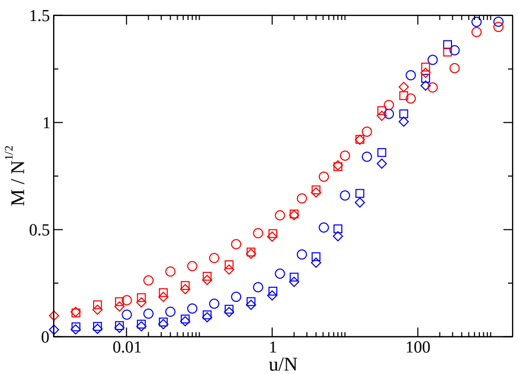


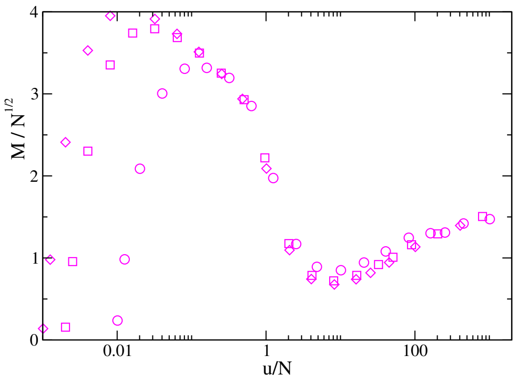
$$\omega_{\rm osc} \approx 2\left[\log\left(\frac{N}{u}\right)\right]^{-1}\omega_J \quad \text{[Edge]}$$

$$\omega_{\rm osc} \approx \left[\log\left(\frac{N}{u}\right)\right]^{-1}\omega_J \quad \text{[Pi]}$$

$$\omega_{\rm osc} \approx \left(\frac{u}{N}\right)^{1/2}\omega_J \quad [u \gg N]$$

The semiclassical ratio $\sim (u/N)^{1/2}$





The participation number:

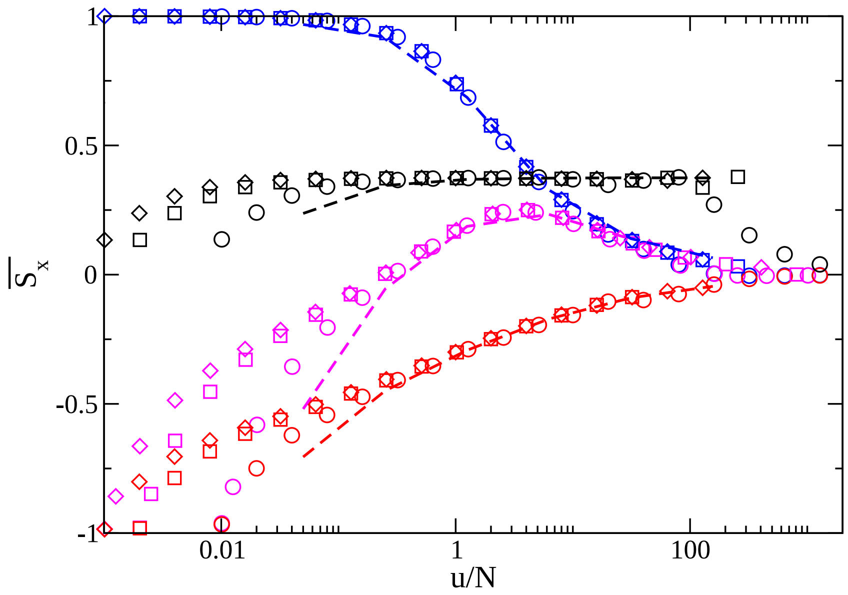
$$M = \left[\sum_\nu \mathrm{P}(E_\nu)^2\right]^{-1}$$

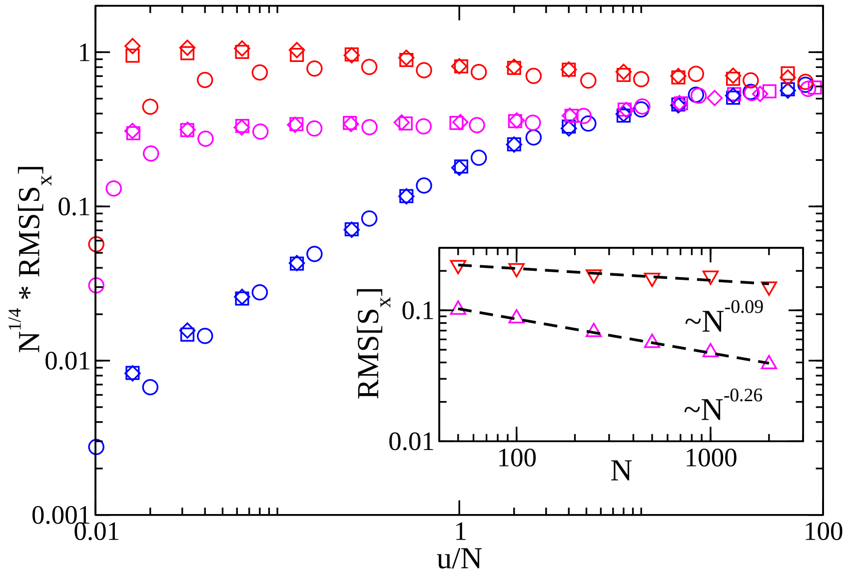
$$M \approx \left[\log\left(\frac{N}{u}\right)\right]\sqrt{N} \quad \text{[Edge]}$$

$$M \approx \left[\log\left(\frac{N}{u}\right)\right]\sqrt{u} \quad \text{[Pi]}$$

"Zero" , "Pi" , "Edge" . $N = 100$ ○, $N = 500$ □, $N = 1000$ ◇.

Fluctuations of S_x





"Zero" , "Pi" , "Edge" , "TwinFock" .

$N = 100$ ○, $N = 500$ □, $N = 1000$ ◇.

$$\overline{S_x} \approx \exp[-\tfrac{1}{4}(u/N)] \qquad \text{[Zero]}$$

$$\overline{S_x} \approx -1 - 4/\log\left[\tfrac{1}{32}(u/N)\right] \qquad \text{[Pi]}$$

$$\overline{S_x} \approx 1/3 \qquad \text{[TwinFock]}$$

$$\mathrm{RMS}\left[\langle A\rangle_t\right] = \left[\frac{1}{M}\int \tilde{C}_{\mathrm{cl}}(\omega)d\omega\right]^{1/2}, \quad \left[\frac{M}{\sqrt{N}} \sim f\left(\sqrt{\frac{u}{N}}\right)\right]$$

$$\mathrm{RMS}\left[S_x(t)\right] \sim N^{-1/4} \qquad \text{[Edge]}$$

$$\mathrm{RMS}\left[S_x(t)\right] \sim (\log(N))^{-1/2} \qquad \text{[Pi]}$$

"Zero" Coherence maintained if $u/N < 1$ (phase locking).

"Pi" Fluctuations are suppressed by u.

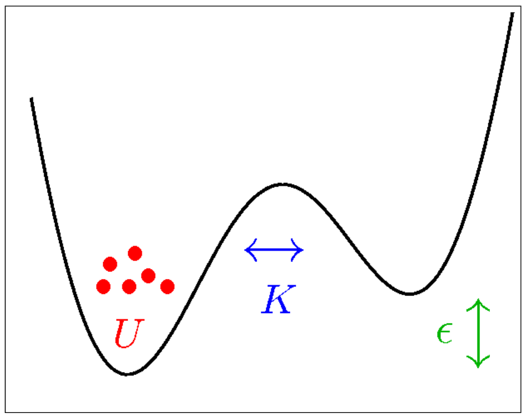
"Edge" Fluctuations are suppressed by N (classical limit).

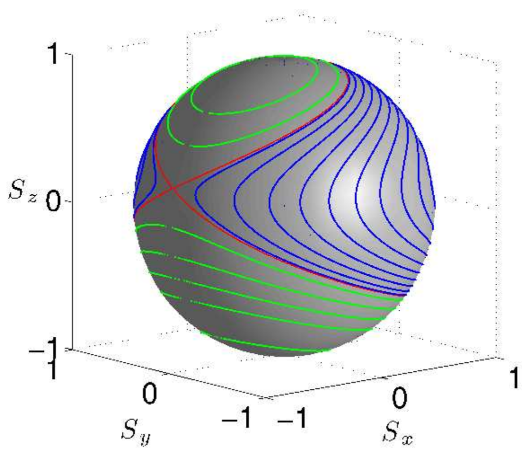
"TwinFock" Self induced coherence leading to $\overline{S_x} \approx 1/3$.

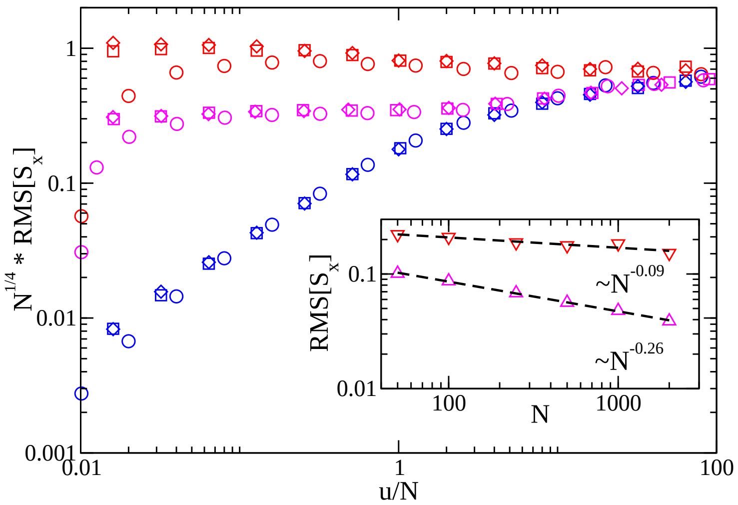
The semiclassical ratio $\sim (u/N)^{1/2}$

Summary

- One-particle coherence loss and buildup analysis using semiclassical phase-space picture.
- The simplicity of BHH allows semi-analytic WKB quantization.
- Closed semiclassical results for LDOS provide useful insights for the quantum evolution.
- The long-time fringe visibility of an initially coherent state in the Josephson interaction regime, has a u/N (semi-classical ratio) dependent value.
- The Zero prep remains roughly Gaussian throughout its motion, a Pi prep squeezes rapidly and its relative-phase information is lost.
- Two types of coherent preparations in the vicinity of the separatrix show significant differences in their M dependence on $(u; N)$.
- The Pi prep preparation exhibits u dependent fluctuations, whereas the Edge prep exhibits N dependent fluctuations.







Thank You