Counting Statistics in closed mesoscopic devices

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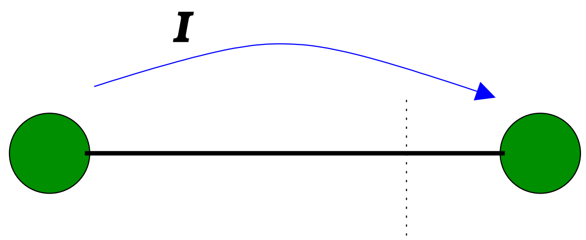
Counting statistics for a coherent transition
MC and D. Cohen, Phys. Rev. A (2008, in press)
Counting statistics in multiple path geometries
MC and D. Cohen, J. Phys. A (2008, in press)

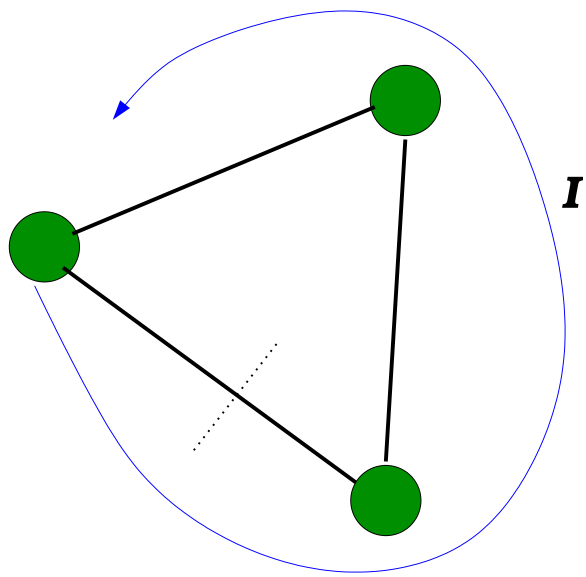
Outline

Counting $$\mathcal{Q} = \int_0^t \mathcal{I}(t')dt'$$



2 site system

FCS for a coherent transition

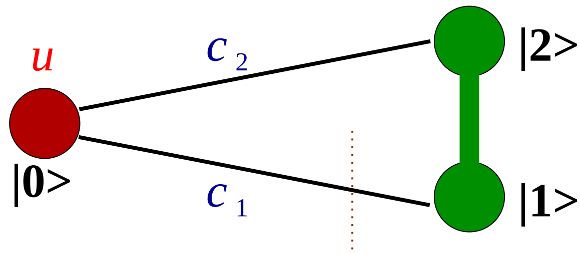


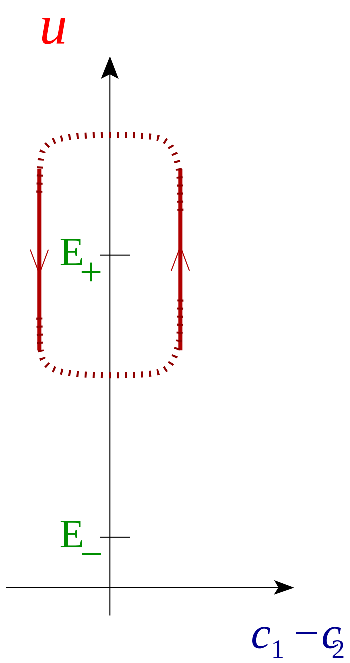
3 site system

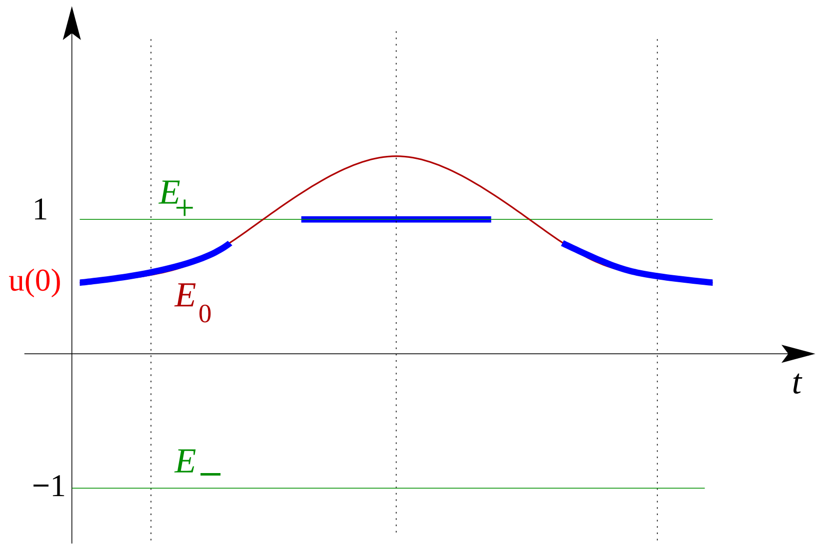
Var($\mathcal{Q}$) for quantum stirring

Counting Statistics, the model

$$\mathcal{H} = \begin{pmatrix} u & c_1 & c_2 \\ c_1 & 0 & 1 \\ c_2 & 1 & 0 \end{pmatrix}$$







The current through one bond operator:

$$\mathcal{I} = \begin{pmatrix} 0 & ic_1 & 0 \\ -ic_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The counting operator: $\mathcal{Q} \;=\; \int_0^t \mathcal{I}(t')dt'$

$\langle \mathcal{Q} \rangle =???$ $\quad$ $\mathrm{Var}(\mathcal{Q}) =???$ $\quad$ $\mathrm{P}(Q) =???$

Main results

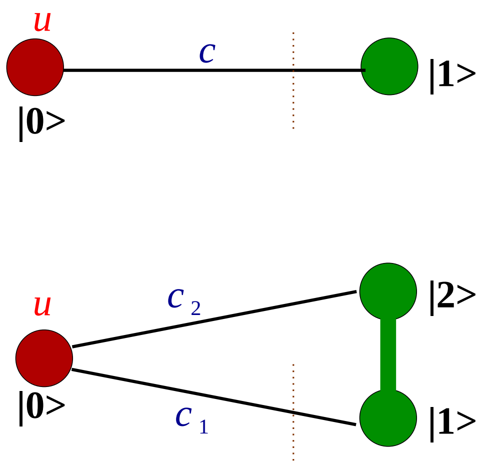
For a half a cycle:

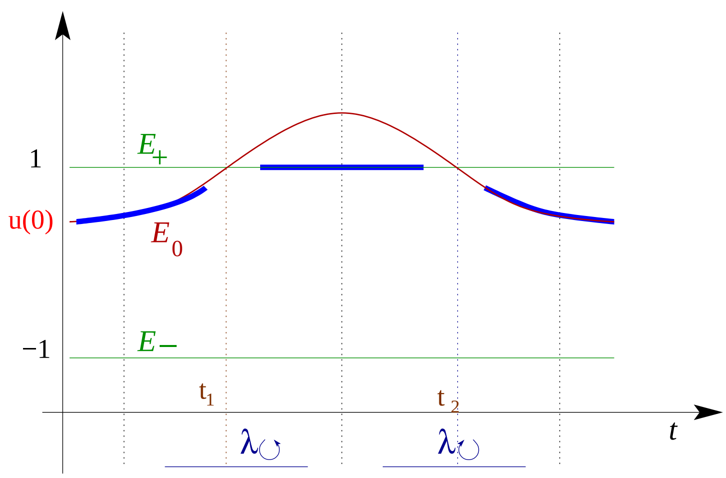
$$
\begin{aligned}
p &= 1 - P_{\mathrm{LZ}} \\
\langle \mathcal{Q} \rangle &= \lambda p \\
\mathrm{Var}(\mathcal{Q}) &= \lambda^2 (1-p)p \quad \neq \quad (1-\lambda p)\lambda p
\end{aligned}
$$

$$
P_{\mathrm{LZ}} = \mathrm{e}^{-\frac{\pi (c_1+c_2)^2}{\dot{u}}} \quad , \quad \lambda = \frac{c_1}{c_1+c_2} \; = \; \text{splitting ratio}
$$

For a full stirring cycle:

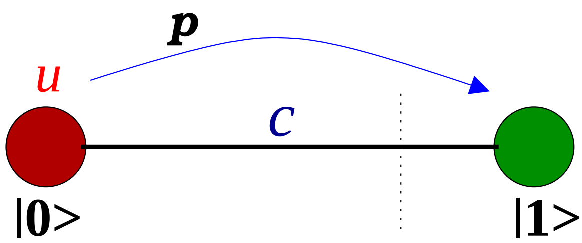
$$
\begin{aligned}
p &\approx \left| \mathrm{e}^{i\varphi_1} - \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}} \\
\langle \mathcal{Q} \rangle &\approx \lambda_{\circlearrowleft} - \lambda_{\circlearrowright} \\
\mathrm{Var}(\mathcal{Q}) &\approx \left| \lambda_{\circlearrowleft} \mathrm{e}^{i\varphi_1} + \lambda_{\circlearrowright} \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}
\end{aligned}
$$





Counting statistics for a coherent transition

$$\mathcal{H} = \begin{pmatrix} u(t) & c \\ c & E_1 \end{pmatrix}$$



Naive expectation:

Given the probability p to make the transition

$$\mathrm{P}(Q) = \begin{cases} 1-p & \text{for } Q = 0 \\ p & \text{for } Q = 1 \end{cases}$$

$$\langle \mathcal{Q}^k \rangle \quad = \quad \mathrm{P}(1) \cdot 1^k + \mathrm{P}(0) \cdot 0^k \quad = \quad p$$

$$\mathrm{Var}(\mathcal{Q}) \quad = \quad (1-p)p$$

Quantum result:

$$\mathrm{P}(Q) = \begin{cases} p_- & \text{for } Q = Q_- \\ p_+ & \text{for } Q = Q_+ \end{cases}$$

where

$$Q_\pm = \pm\sqrt{p}\ , \qquad\qquad p_\pm = \frac{1}{2}(1 \pm \sqrt{p})$$

hence

$$\langle \mathcal{Q}^k \rangle = p_+ Q_+^k + p_- Q_-^k = p^{\left\lfloor \frac{k+1}{2} \right\rfloor}$$

The measurement of Q, FCS

The distribution $\mathrm{P}(Q)$ can be determined using a continuous measurement scheme.

In such setup the current induces (so to say) a "translation" of a Von-Neumann pointer.

$$H_{\text{total}} = H_{\text{system}} - Ix + H_{\text{pointer}}(x, q)$$

One can measure the quasi distribution $\mathrm{P}(Q; x)$:

$$\mathrm{P}(Q; x=0) = \frac{1}{2\pi}\int \left\langle \left[\mathcal{T}\mathrm{e}^{-i\frac{r}{2}\mathcal{Q}}\right]^{\dagger}\left[\mathcal{T}\mathrm{e}^{+i\frac{r}{2}\mathcal{Q}}\right]\right\rangle \mathrm{e}^{-iQr}dr$$

If we ignore time ordering we get:

$$\mathrm{P}(Q) \;=\; \frac{1}{2\pi}\int \left\langle \mathrm{e}^{+ir\mathcal{Q}}\right\rangle \mathrm{e}^{-iQr}dr \;=\; \langle\delta(Q-\mathcal{Q})\rangle$$

H. Everett, Rev. Mod. Phys. 29, 454 (1957).
L.S. Levitov and G.B. Lesovik, JETP Letters (1992).
L.S. Levitov and G.B. Lesovik, JETP Letters (1993).
Y.V. Nazarov and M. Kindermann, EPJ B (2003).

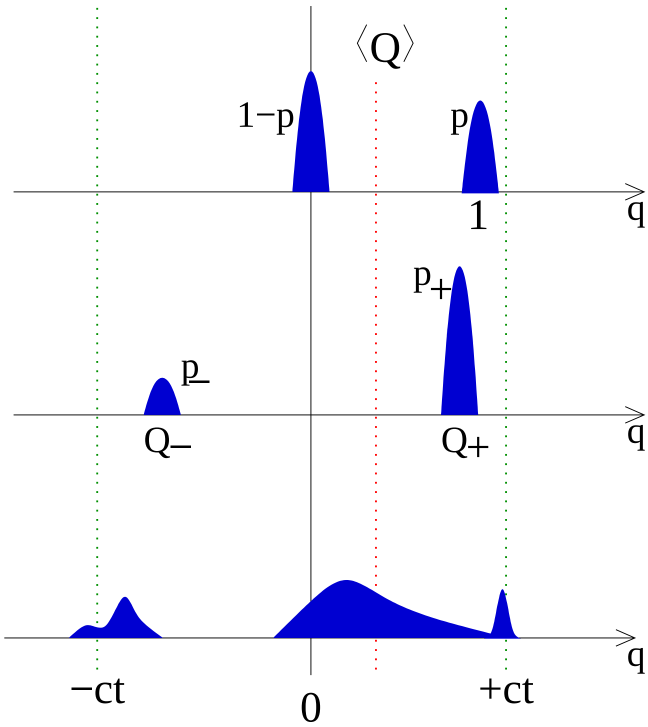
FCS for a coherent Bloch transition

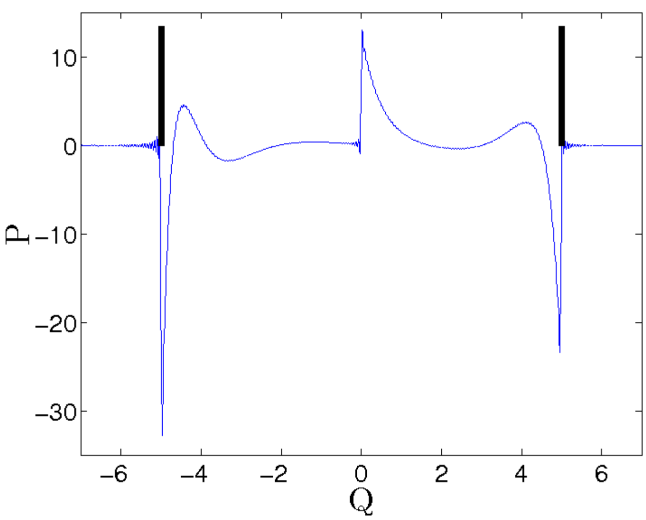
$$\rho_t(q,x) \;=\; \int \mathrm{P}(q-q';x)\rho_0(q',x)dq'$$

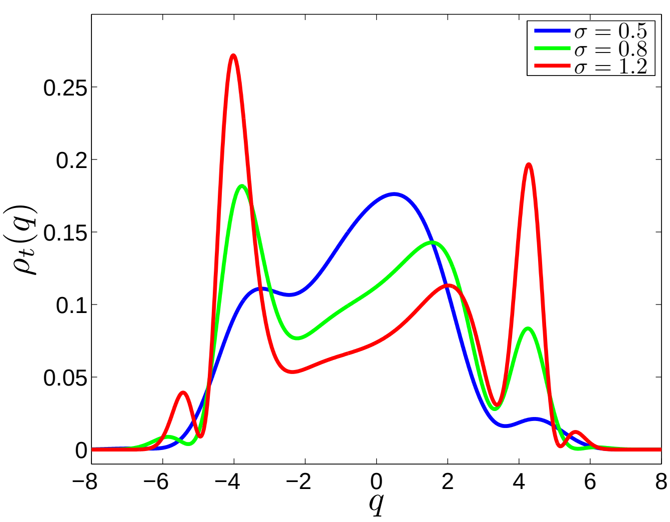
$$\mathrm{P}_{cl}(Q) \;=\; \begin{cases} 1-p & \text{for } Q=0 \\ p & \text{for } Q=1 \end{cases}$$

$$\mathrm{P}_{naive}(Q) \;=\; \begin{cases} p_- & \text{for } Q=Q_- \\ p_+ & \text{for } Q=Q_+ \end{cases}$$

$$\mathrm{P}_{qm}(Q;x=0)$$

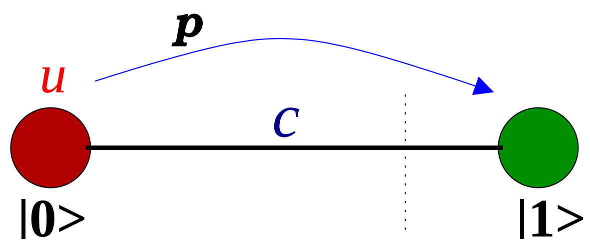






Calculation of the variance of a LZ crossing

$$\mathcal{H} = \begin{pmatrix} u & c \\ c & 0 \end{pmatrix},$$



leading order adiabatic approximation:

$$U(t) \approx \sum_n \Big| n(t) \Big\rangle \exp\left[-i \int_{t_0}^{t} E_n(t')dt' \right] \Big\langle n(t_0) \Big|$$

$$\mathcal{I}(t)_{nm} = \langle n | U(t)^{\dagger} \mathcal{I} U(t) | m \rangle$$

$$\approx \langle n(t) | \mathcal{I} | m(t) \rangle \exp\left[i \int_{t_0}^{t} E_{nm}(t')dt' \right]$$

$$\mathcal{Q} \equiv \begin{pmatrix} +Q_{\parallel} & iQ_{\perp} \\ -iQ_{\perp}{}^{*} & -Q_{\parallel} \end{pmatrix}$$

$$\mathrm{Var}(\mathcal{Q}) = |Q_{\perp}|^2 \approx \left| c \int_{-\infty}^{\infty} \mathrm{e}^{i\Phi(t)} dt \right|^2$$

$$\Phi(t) \equiv \int_0^t \sqrt{(\dot{u}t')^2 + (2c)^2} dt'$$

The LZ transition calculation

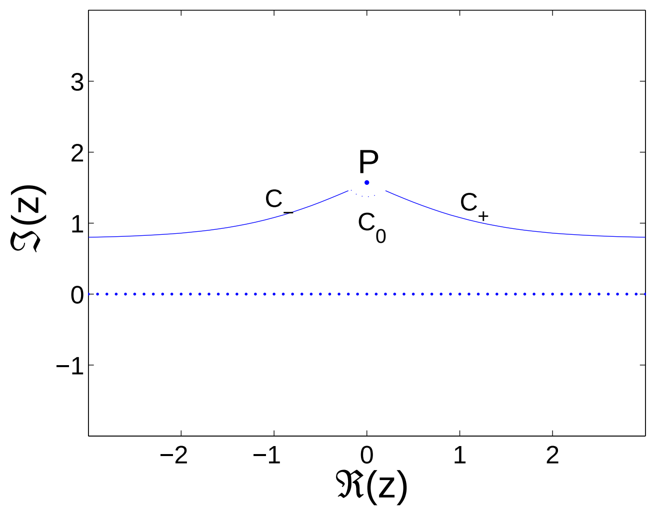
$$P_{\mathrm{LZ}} \approx \left| c \int_{-\infty}^{\infty} \frac{\dot{u}}{(\dot{u}t)^2 + (2c)^2} \, \mathrm{e}^{i\Phi(t)} \, dt \right|^2$$

$$= \left| \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{\cosh(z)} \, \mathrm{e}^{i\Phi(z)} dz \right|^2$$

$$\mathrm{Var}(\mathcal{Q}) \approx \left| c \int_{-\infty}^{\infty} \mathrm{e}^{i\Phi(t)} dt \right|^2$$

$$= \left| \frac{2c^2}{\dot{u}} \int_{-\infty}^{\infty} \cosh(z) \, \mathrm{e}^{i\Phi(z)} dz \right|^2$$

??? $\mathrm{Var}(\mathcal{Q}) = (1-P_{\mathrm{LZ}})P_{\mathrm{LZ}}$???



$$P_{\mathrm{LZ}} \sim \left(\frac{\pi}{3}\right)^2 \exp\left[-\frac{\pi c^2}{\dot{u}}\right]$$

$$\mathrm{Var}(\mathcal{Q}) \sim \left(\frac{2c^2}{\dot{u}}\right)^{2/3} \exp\left[-\frac{\pi c^2}{\dot{u}}\right]$$

Restricted quantum-classical correspondence

$\mathcal{N} =$ occupation operator (eigenvalues $= 0, 1$)

$\mathcal{I} =$ current operator

Heisenberg equation of motion:

$$\frac{d}{dt}\mathcal{N}(t) \;=\; \mathcal{I}(t)$$

leads to

$$\mathcal{N}(t) - \mathcal{N}(0) = \mathcal{Q}$$

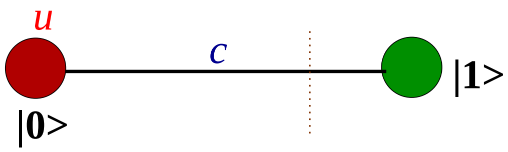
hence

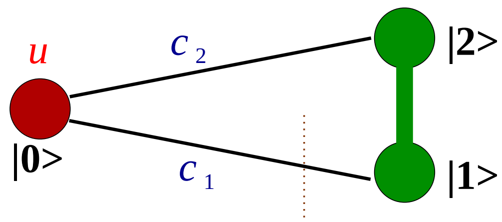
$$\langle \mathcal{Q}^k \rangle \;=\; \left\langle (\mathcal{N}(t) - \mathcal{N}(0))^k \right\rangle \;\stackrel{?}{=}\; \langle \mathcal{N}^k \rangle_t \;=\; p$$

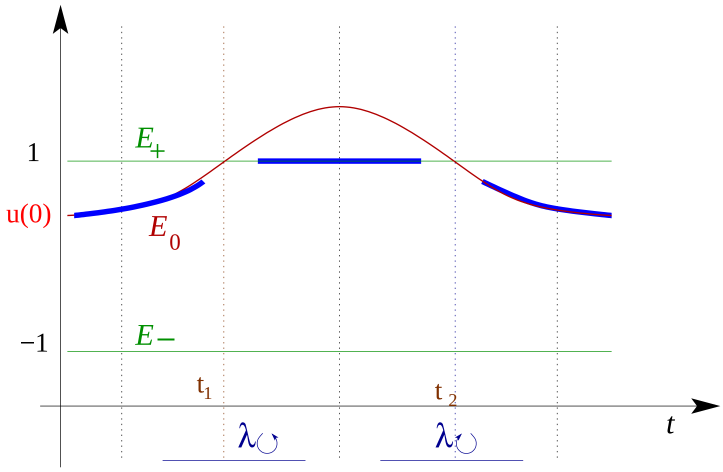
for $k = 1, 2$ only

Restricted QCC

Hamiltonians for 2 and 3 site systems







$$\mathcal{H} = \begin{pmatrix} u & c \\ c & 0 \end{pmatrix}, \qquad \mathcal{I} = \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u & c_1 & c_2 \\ c_1 & 0 & 1 \\ c_2 & 1 & 0 \end{pmatrix}, \qquad \mathcal{I} = \begin{pmatrix} 0 & ic_1 & 0 \\ -ic_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\mathcal{H} = \begin{pmatrix} u & \frac{(c_1+c_2)}{\sqrt{2}} \\ \frac{(c_1+c_2)}{\sqrt{2}} & 1 \end{pmatrix}, \qquad \mathcal{I} = \frac{c_1}{\sqrt{2}} \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

Double path adiabatic passage

$$\mathcal{H} = \begin{pmatrix} u(t) & c \\ c & 1 \end{pmatrix}, \qquad \mathcal{I} = \lambda \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix}$$

with effective coupling and splitting ratio

$$c \equiv \frac{(c_1+c_2)}{\sqrt{2}}, \qquad \lambda \equiv \frac{c_1}{c_1+c_2}$$

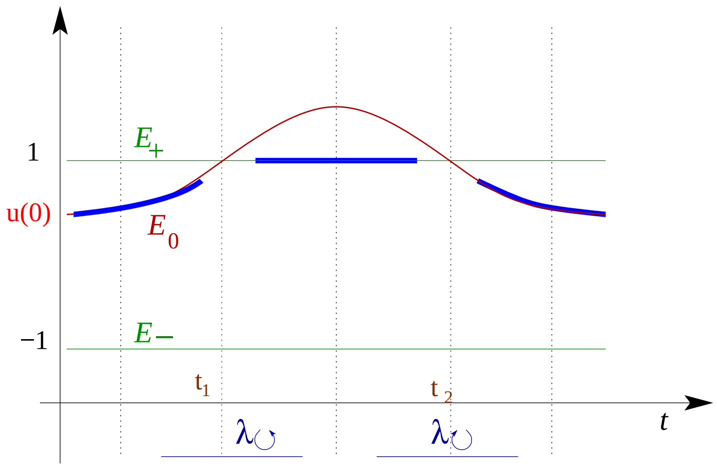
Accordingly:

$$\begin{aligned} \langle \mathcal{Q} \rangle &= \lambda p \\ \mathrm{Var}(\mathcal{Q}) &= \lambda^2 \underset{\sim\sim\sim\sim}{(1-p)p} \quad \neq \quad (1-\lambda p)\lambda p \end{aligned}$$

Coherent splitting unlike probabilistic splitting of the wavepacket is "exact".

$\lambda > 1 \quad \Rightarrow \quad$ The driving induces a circulating current within the ring, and illuminates the fallacy of the classical peristaltic point of view.

Full stirring cycle



a sequence of two
Landau Zener crossings

$$\begin{aligned}
\langle \mathcal{Q} \rangle &\approx \lambda_{\circlearrowleft} - \lambda_{\circlearrowright} \\
\mathrm{Var}(\mathcal{Q}) &= \left| \lambda \; c \int_{-\infty}^{\infty} \mathrm{e}^{i\Phi(t)} dt \right|^2 \\
&\approx \left| \lambda_{\circlearrowleft} \mathrm{e}^{i\varphi_1} + \lambda_{\circlearrowright} \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}
\end{aligned}$$

Due to interference the counting statistics becomes in general unrelated to the occupation statistics:

$$\begin{aligned}
p &= \left| \frac{1}{2} \int_{-\infty}^{\infty} \frac{(\dot{u}/2c) \; \mathrm{e}^{i\Phi(t)}}{1 + (u/2c)^2} dt \right|^2 \\
&\approx \left| \mathrm{e}^{i\varphi_1} - \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}}
\end{aligned}$$

Conclusions

Quantum mechanics is a “deterministic” rather than a “probabilistic” theory. Coherent splitting unlike probabilistic splitting of a wavepacket is “exact”.

In a double path adiabatic passage one may find that (say) 170% of the particle goes via one path, while −70% goes via the second path, due to a circulating current induced by the driving.

There is at most restricted quantum-classical correspondence for the first and second moments.

In contrast to the single path crossing problem where the two types of statistics are a-priori related, for a full stirring cycle interference gets into the counting statistics calculation, so it is not generally related to the occupation statistics.

Main results

For a half a cycle:

$$\begin{aligned} p &= 1 - P_{\mathrm{LZ}} \\ \langle \mathcal{Q} \rangle &= \lambda p \\ \mathrm{Var}(\mathcal{Q}) &= \lambda^2 (1-p)p \quad \neq \quad (1-\lambda p)\lambda p \end{aligned}$$

$$P_{\mathrm{LZ}} = \mathrm{e}^{-\frac{\pi (c_1+c_2)^2}{\dot{u}}} \quad , \quad \lambda = \frac{c_1}{c_1+c_2} \quad = \quad \text{splitting ratio}$$

For a full stirring cycle:

$$\begin{aligned} p &\approx \left| \mathrm{e}^{i\varphi_1} - \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}} \\ \langle \mathcal{Q} \rangle &\approx \lambda_{\circlearrowleft} - \lambda_{\circlearrowright} \\ \mathrm{Var}(\mathcal{Q}) &\approx \left| \lambda_{\circlearrowleft} \mathrm{e}^{i\varphi_1} + \lambda_{\circlearrowright} \mathrm{e}^{i\varphi_2} \right|^2 P_{\mathrm{LZ}} \end{aligned}$$