Disorder in Quantum spin Hall systems

Dedicated to Baruch Horovitz on his 60^{th} birthday

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Collaboration with

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RAPID COMMUNICATIONS

PHYSICAL REVIEW B VOLUME 35, NUMBER 1 1 JANUARY 1987

Phase transition of the Aharonov-Bohm periodicity in metallic cylinders

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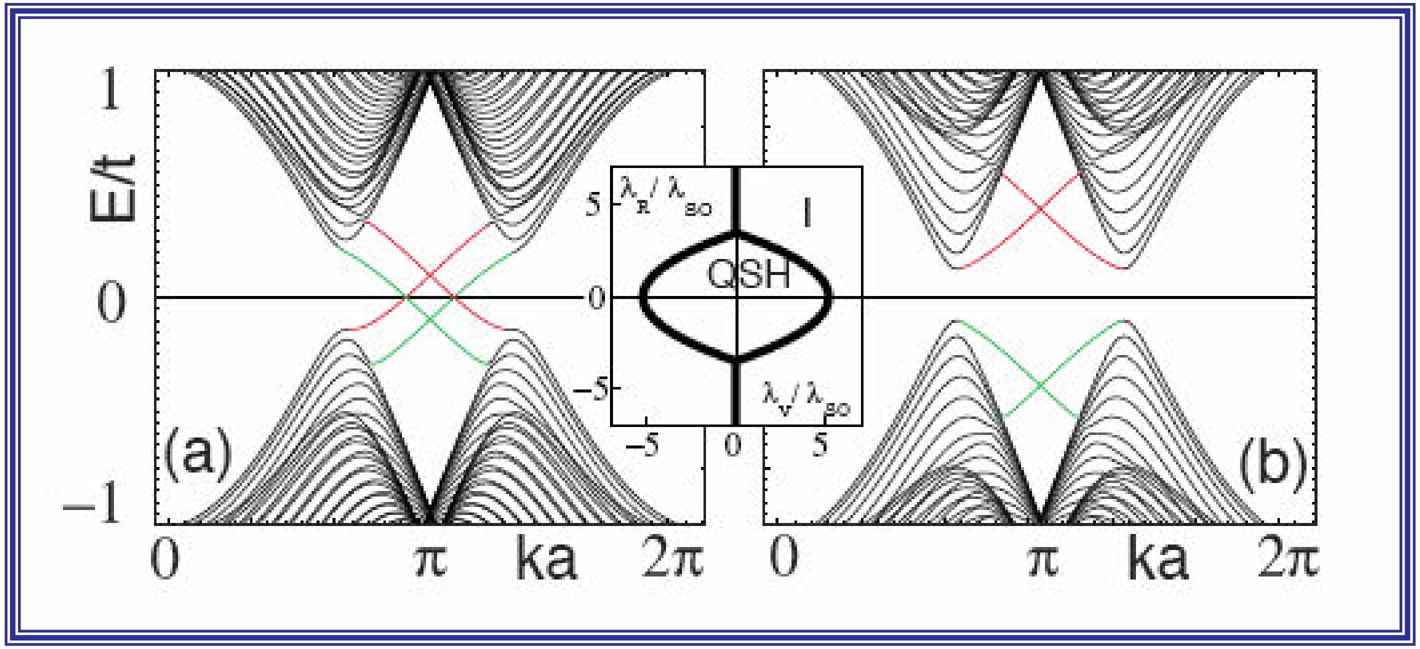
(Received 16 July 1986; revised manuscript received 20 October 1986)

A scattering formalism is developed for a random potential on a cylinder with flux ϕ along its axis. Simulations show the possibility of a phase transition in the ϕ dependence of an ensemble-averaged conductance: The periodicity in ϕ changes from $h/2e$ to h/e as the potential is weakened. The critical potential decreases as the sample size increases such that the zero-flux conductivity is $\sim 150e^2/h$.

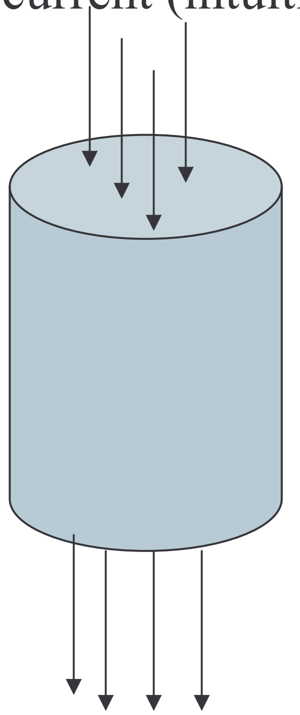
QSH phase: Time reversal invariant electronic state with a bulk band gap that supports charge and spin currents in gapless edge states (SO interaction leads to non-conservation of spin-G_s not quantized!).

Time reversal invariance: H=2Nx2N matrix with N modes and 2 spin states (space $1_N \, x \, 2$). Then: $H=\tau H^* \tau^{-1}$. $\tau = K \, e^{i s y}$

bulk band gap gapless edge states: (figure from Kane-Mele)



Charge and spin current (intuitive description)

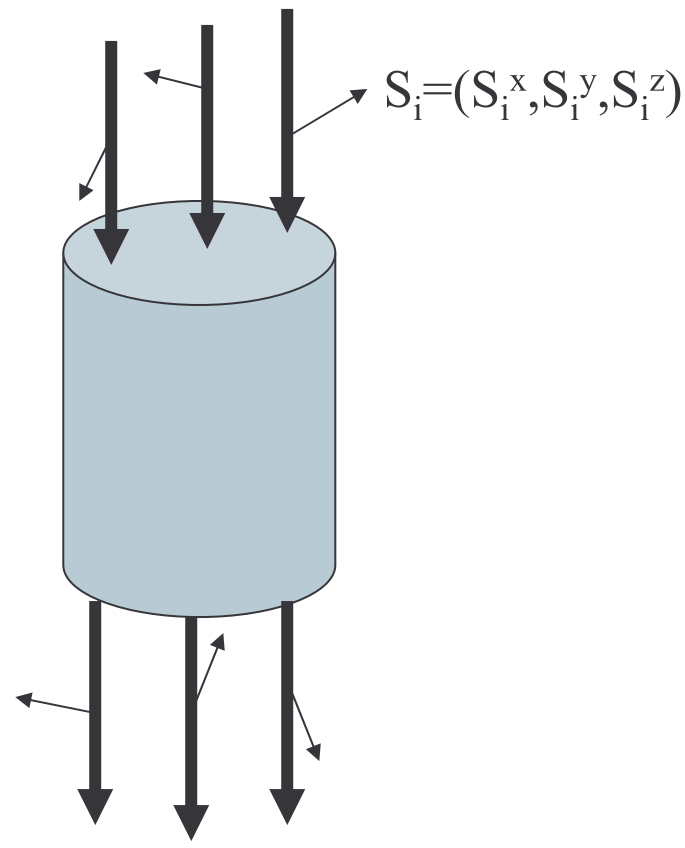


$$\mathbf{J}_c = \frac{Ne}{At} = \rho \mathbf{v}$$

Conserved!

$$\mathbf{J}_s^a = \frac{\sum_i S_i^a}{At} = \rho \mathbf{v} S^a$$

Conserved?
What is the measurable quantity?
Change in polarization?



charge and spin currents

Hamiltonian

$$\hat{H} = \frac{1}{2m}[\mathbf{p} - e\mathbf{A}(\mathbf{r},t) - \alpha\mathcal{A}(\mathbf{r},t)^a\sigma^a]^2 + \hat{V}(\mathbf{r}) = \frac{1}{2m}\hat{\mathbf{\Pi}}^2 + \hat{V}(\mathbf{r}).$$

$$\mathcal{A}(\mathbf{r},t)_i^a = \varepsilon_{iaj}E_j(\mathbf{r},t)$$

Vector

Charge current operator - U(1)

$$\hat{\mathbf{J}}_c(\mathbf{r}',t) = \frac{\delta\hat{H}}{\delta\mathbf{A}(\mathbf{r}',t)} = -\frac{e}{2m}[\hat{\mathbf{\Pi}}\delta(\mathbf{r}-\mathbf{r}') + \delta(\mathbf{r}-\mathbf{r}')\hat{\mathbf{\Pi}}]$$

Tensor

Spin current operator - SU(2)

$$\hat{\mathbf{J}}_s^a(\mathbf{r},t) = \frac{\delta\hat{H}}{\delta\mathcal{A}(\mathbf{r}',t)^a} = -\frac{\alpha}{2m}[\{\hat{\mathbf{\Pi}},\sigma^a\}\delta(\mathbf{r}-\mathbf{r}') + \delta(\mathbf{r}-\mathbf{r}')\{\hat{\mathbf{\Pi}},\sigma^a\}]$$

Charge current and density (numbers-first quantization)

$$\mathbf{J}_c(\mathbf{r},t) = -\frac{e}{m}[\Psi^\dagger(\mathbf{r},t)\hat{\mathbf{\Pi}}\Psi(\mathbf{r},t)] \quad \rho_c(\mathbf{r},t) = [\Psi^\dagger(\mathbf{r},t)\Psi(\mathbf{r},t)]$$

Spin current and density (numbers)

$$\mathbf{J}_s^a(\mathbf{r},t) = -\frac{\alpha}{m}[\Psi^\dagger(\mathbf{r},t)\{\hat{\mathbf{\Pi}},\sigma^a\}\Psi(\mathbf{r},t)] \quad \rho_s^a(\mathbf{r},t) = [\Psi^\dagger(\mathbf{r},t)\sigma^a\Psi(\mathbf{r},t)]$$

Charge conservation

$$\partial_i J_{ci}(\mathbf{r},t) + \partial_t \rho_c(\mathbf{r},t) = 0$$

Spin current conservation equation has a “source” on the RHS

$$\partial_i J_{si}^a(\mathbf{r},t) + \partial_t \rho_s^a(\mathbf{r},t) = \frac{i}{2\hbar}\{[H,\sigma^a],\rho_c(\mathbf{r},t)\}$$

Thus, Spin is not conserved since $[S^a, H]$ n.e 0.

To summarize the first part, we have defined:

1) Time reversal invariance
2) Bulk gap and gapless edge modes
3) (conserved) charge and (non-conserved) spin currents

4) QSH phase: Time reversal invariant electronic state with a bulk band gap that supports charge and spin currents in gapless edge states

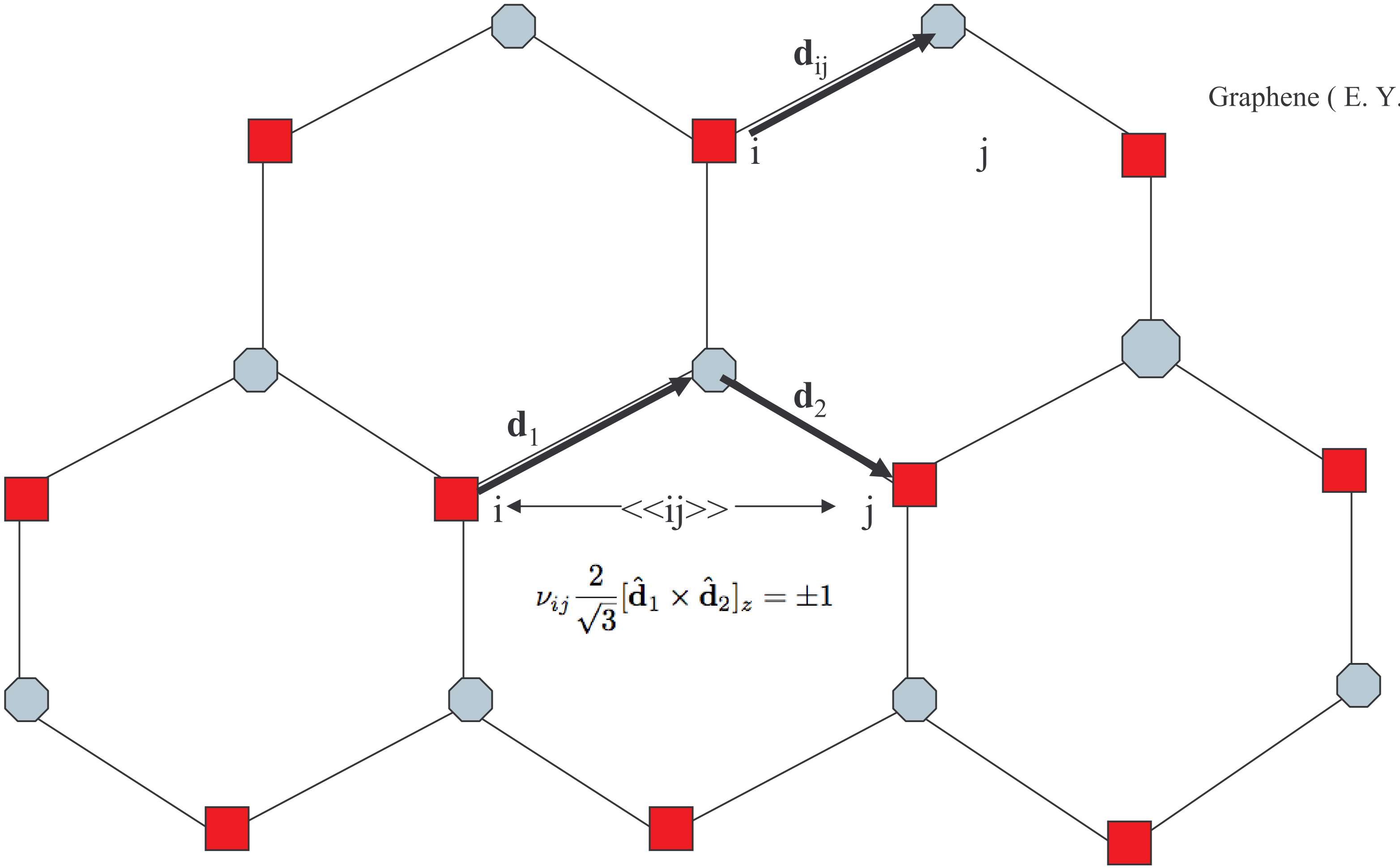
The central question at this point is:

How do we know if a given system is in a QSH phase?

If we solve the problem with periodic BC can we predict the existence of gapless edge modes in a cylinder BC?

[in the QHE the answer is positive, the Chern number calculated in a periodic BC counts the number of gapless edge states in an open system (TKNN, JEA et al). This relies on charge conservation (Laughlin argument). Here we cannot use it because Spin is not conserved].

For a tight-binding model on h.c lattice the answer is given by Kane and Mele. It is not trivial-Topological Z_2 order.



$$\nu_{ij}\frac{2}{\sqrt{3}}[\hat{\mathbf{d}}_1 \times \hat{\mathbf{d}}_2]_z = \pm 1$$

Graphene (E. Y. Andrei talk)

$$H_{KM} = t\sum_{<ij>} c_i^\dagger c_j + i\lambda_{SO}\sum_{<<ij>>} \nu_{ij} c_i^\dagger s^z c_j + i\lambda_R \sum_{<ij>} c_i^\dagger [\mathbf{s}\times\mathbf{d}_{ij}]_z c_i + \lambda_v \sum_i \xi_i c_i^\dagger c_i$$

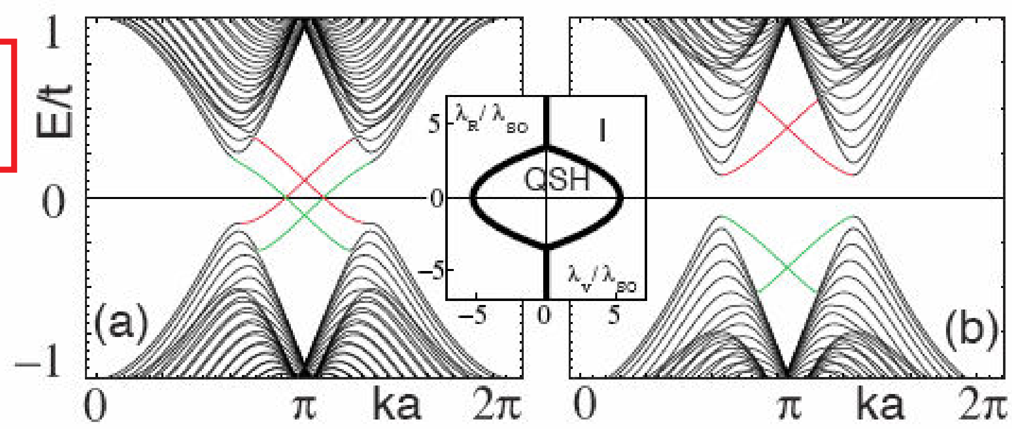
Kane Mele Hamiltonian (Modified Haldane)

$$H_{KM} = t \sum_{<ij>} c_i^\dagger c_j + i\lambda_{SO} \sum_{<<ij>>} \nu_{ij} c_i^\dagger s^z c_j + i\lambda_R \sum_{<ij>} c_i^\dagger [\mathbf{s}\times\mathbf{d}_{ij}]_z c_i + \lambda_v \sum_i \xi_i c_i^\dagger c_i$$

$$\nu_{ij} \frac{2}{\sqrt{3}} [\hat{\mathbf{d}}_1 \times \hat{\mathbf{d}}_2]_z = \pm 1$$

Band insulator, each level is at least two-fold Kramer degenerate.

QSH - Odd # of Kramer Pairs on each edge



SHI - Even # of Kramer pairs annihilate each other

FIG. 1: Energy bands for a one dimensional "zigzag" strip in the (a) QSH phase $\lambda_v = .1t$ and (b) the insulating phase $\lambda_v = .4t$. In both cases $\lambda_{SO} = .06t$ and $\lambda_R = .05t$. The edge states on a given edge cross at $ka = \pi$. The inset shows the phase diagram as a function of λ_v and λ_R for $0 < \lambda_{SO} \ll t$.

Spin conductance in QSH phase: Generate E field at the edges using Laughlin gedanken experiment. After change of h/e flux a p-h excitation occurs at E_F (only for QSH not for SHI). This leads to spin imbalance at Left and Right edges.

$$\frac{d < S_z >}{dt} = G^s_{xy} E \quad G^s_{xy} = \frac{e}{h} [< S_z >_L - < S_z >_R]_{E_F}$$

$<O>_L$=expectation value with left edge state.
G^s is not quantized because edge states are not S_z eigenstates

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Volume 306(5703), 10 December 2004, pp 1910-1913

Observation of the Spin Hall Effect in Semiconductors
[Research Article: Applied Physics]
Kato, Y. K.; Myers, R. C.; Gossard, A. C.; Awschalom, D. D.*

Kane - Mele introduced "Z_2 topological order" to distinguishes QSH phase from SHI phase.

1) Use Trans. Inv. To define 4x4 Ham. H(k) (4x4 matrix - 2spin x 2bands)
2) H(k) defines Bloch states $u_n(\mathbf{k})$ (n=1,2,3,4).
3) Two bands n=1,2 are occupied and two n=3,4 are empty. Each band is Kramer degenerate.
4) $P(k)=\langle u_1(\mathbf{k})|T|\, u_2(\mathbf{k})\rangle$. (T=TR operator)
5) The # of pairs of 0 of P in BZ is a topological invariant.
6) This number Z_2 can be computed by sensitivity to BC.
7) In open system it gives # of gapless Kramers pairs on each edge.
8) QSH phase has odd Z_2.

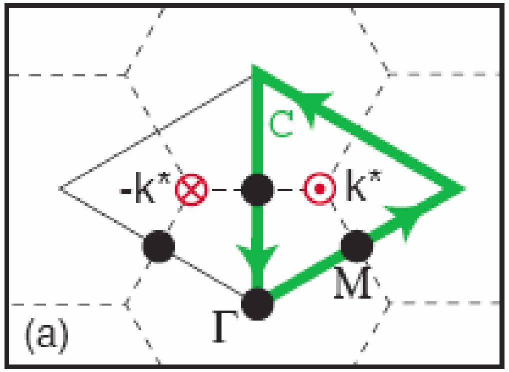


FIG. 2: The zeros of $P(\mathbf{k})$ in the QSH phase occur at points $\pm\mathbf{k}^*$ for (a) $\lambda_v \neq 0$

(Kane Mele PRL 95 226801 2005)

Alternatively, Z_2 can be defined in terms of Berry connection and curvature:

$$\mathcal{A} = \langle u_n(\mathbf{k})|\nabla_{\mathbf{k}}|u_n(\mathbf{k})\rangle, \quad \mathcal{F} = \nabla_{\mathbf{k}} \times \mathbf{A}$$

Inclusion of disorder (remember that Z_2 is defined for perfect lattice).
The question: Is it still possible to define a quantized "topological term" in terms of WF in a disordered system without edges?
If so how is it related to the number of gapless edge modes in a system with edges? In the absence of S_z conservation, what does it physically represent?
Compare with the IQHE: (TKNN+JEA et al first Chern # perfect system)

$$\sigma_{xy}^n = \frac{e^2}{\pi h}\int_{MBZ} Im < \frac{\partial u_{n\mathbf{k}}}{\partial k_x} | \frac{\partial u_{n\mathbf{k}}}{\partial k_y} > d^2\mathbf{k}$$

Thouless-Niu extended the definition For disordered systems (twisting BC)

$$\Psi(x+L_x,y) = e^{i\theta_x}\Psi(x,y) \quad \Psi(x,y+L_y) = e^{i\theta_y}\Psi(x,y)$$

$$\sigma_{xy}^n = \frac{e^2}{\pi h}\int_{[0,2\pi]^2} Im < \frac{\partial \Psi_n}{\partial \theta_x} | \frac{\partial \Psi_n}{\partial \theta_y} > d^2\theta$$

How to do it for TR conserving systems?

SU(2) (spin dependent) twisting (Sheng[2] , Weng & Haldane, Qui, Wu Zhang)

$$\Psi(x+L_x,y)=e^{iU_x\theta_x}\Psi(x,y) \quad \Psi(x,y+L_y)=e^{iU_y\theta_y}\Psi(x,y)$$

U_x, U_y are combinations of Pauli matrices. Then the results are:

1)

$$C_{sc}\equiv\frac{1}{4\pi}\int_{S^2[0,2\pi]} Im<\frac{\partial\Psi_{GS}}{\partial\theta_x}|\frac{\partial\Psi_{GS}}{\partial\theta_y}>d^2\theta$$

Is an even integer integer.

2) The parity of $C_{sc}/2$ is identical to the parity of Kramers pairs of gap-less edge modes when the toroidal system is "cyllindrized".

For disordered systems with bulk gap we now define two phases:

Quntum spin Hall, and Spin Hall insulator:

QSH-Odd $C_{sc}/2$ hence gapless edge modes on a cyllinder.

SHI- Even $C_{sc}/2$ hence no gapless edge modes on a cyllinder.

The main question addressed in this study is:How the presence of topological term C_{sc} affects localization of wave functions??

Localization problem in 2D systems:

GOE: all states are localized.
(H is real, TR conserved)

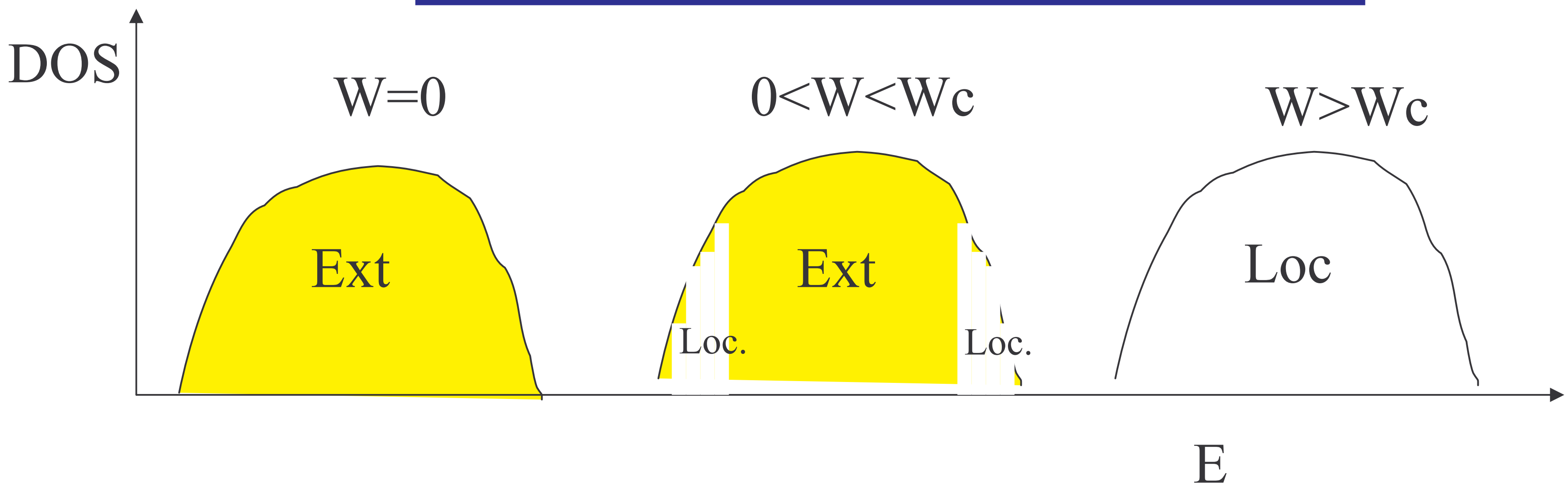
GUE without topological term: all states localized.
(H is complex, TR violated)

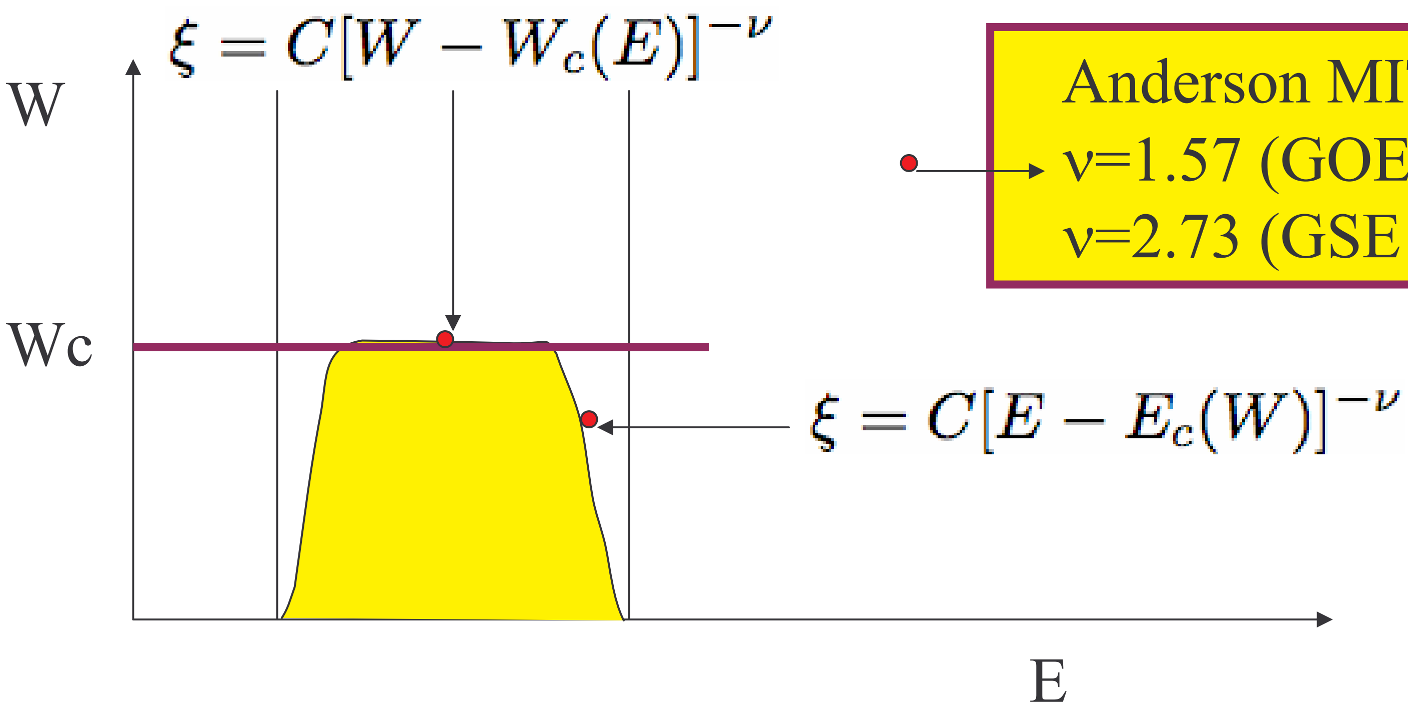
GSE without topological term: MIT (ν=2.7).
(H quaternion, TR conserved)

GUE with topological term: IQHE, (ν=2.4).
(Discrete energies of critical states, no metallic phase)

GSE with topological term:??
(expect metallic phase, what about criticality?).

2D GSE or 3D GOE (Anderson model)

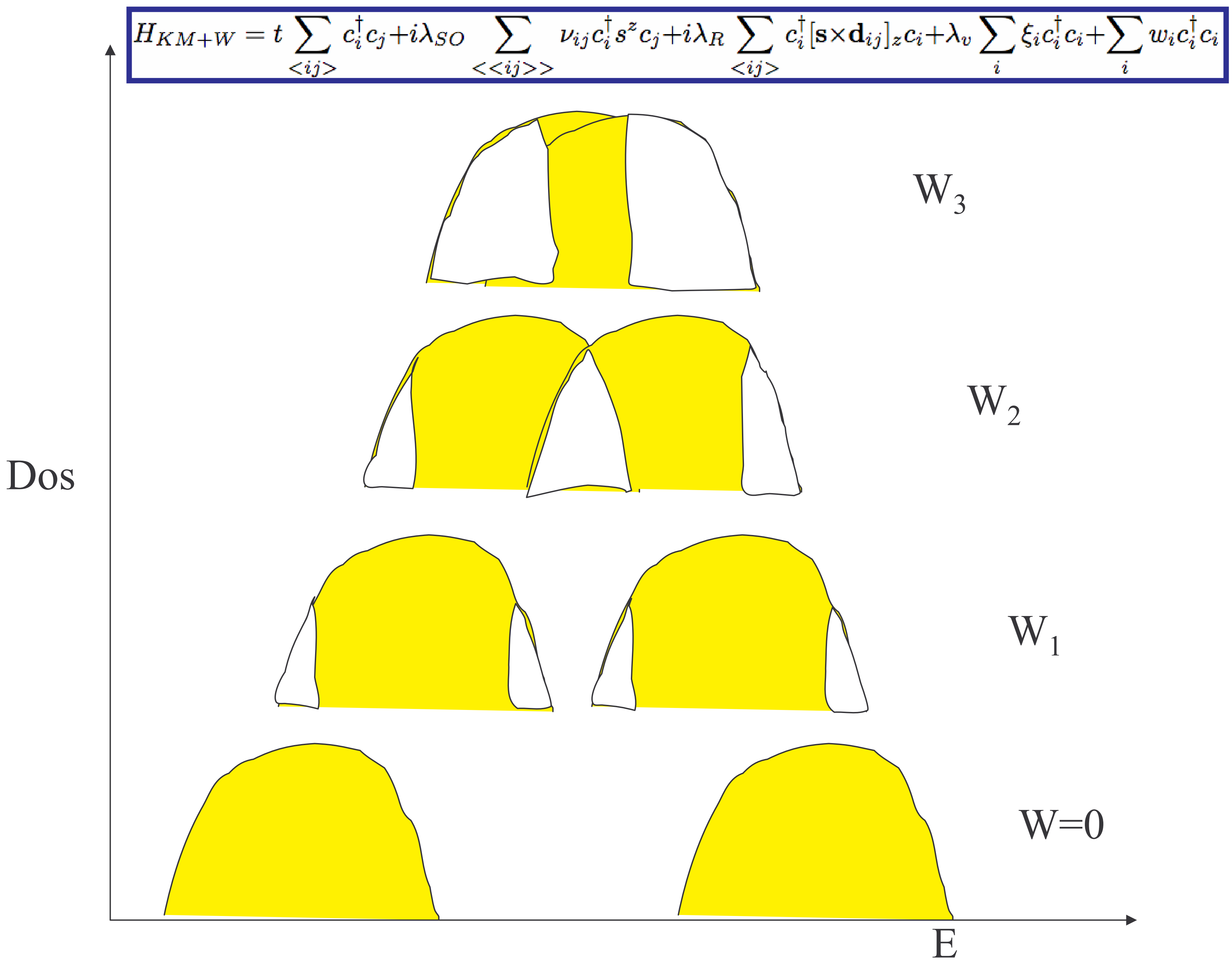


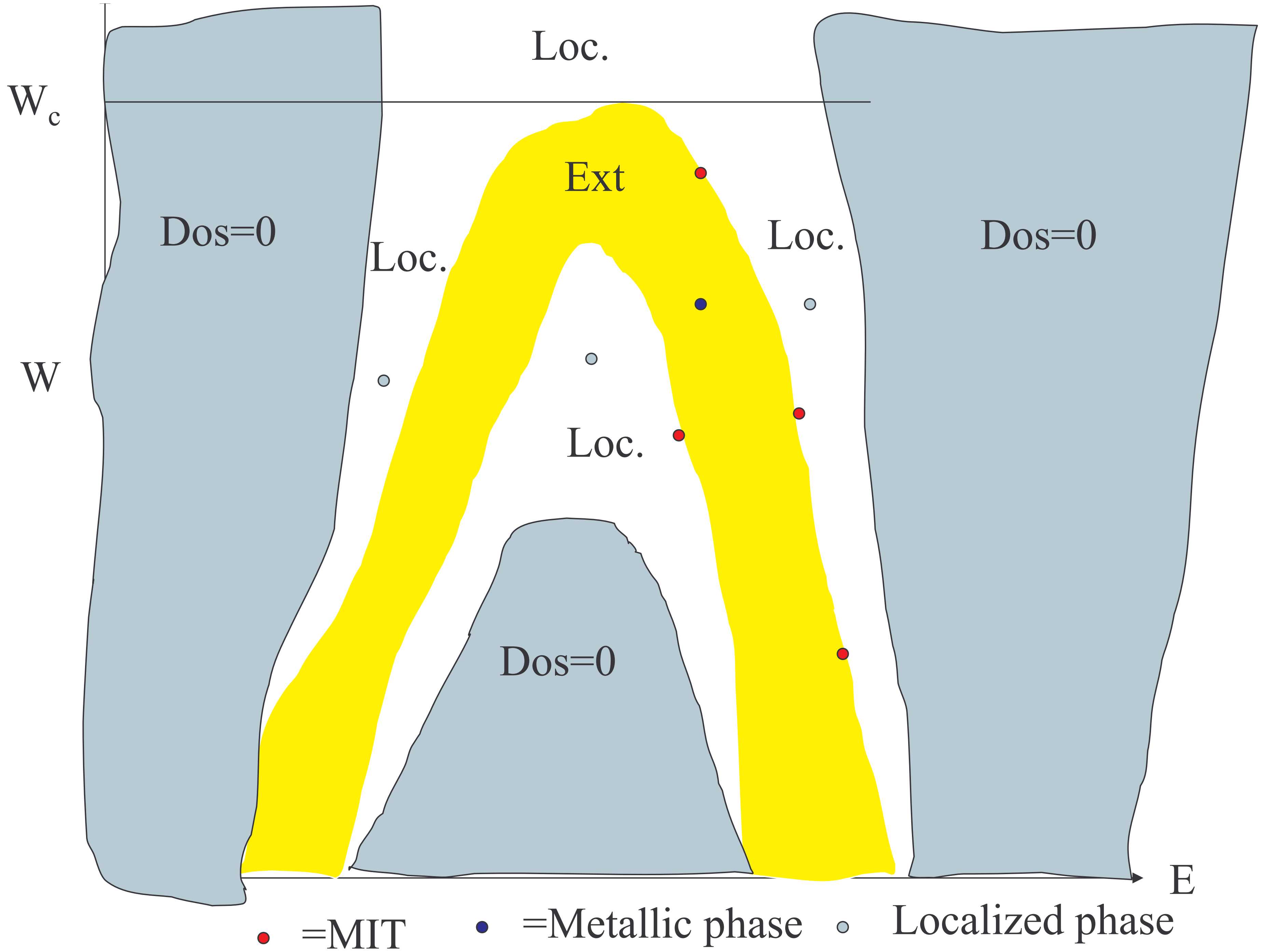


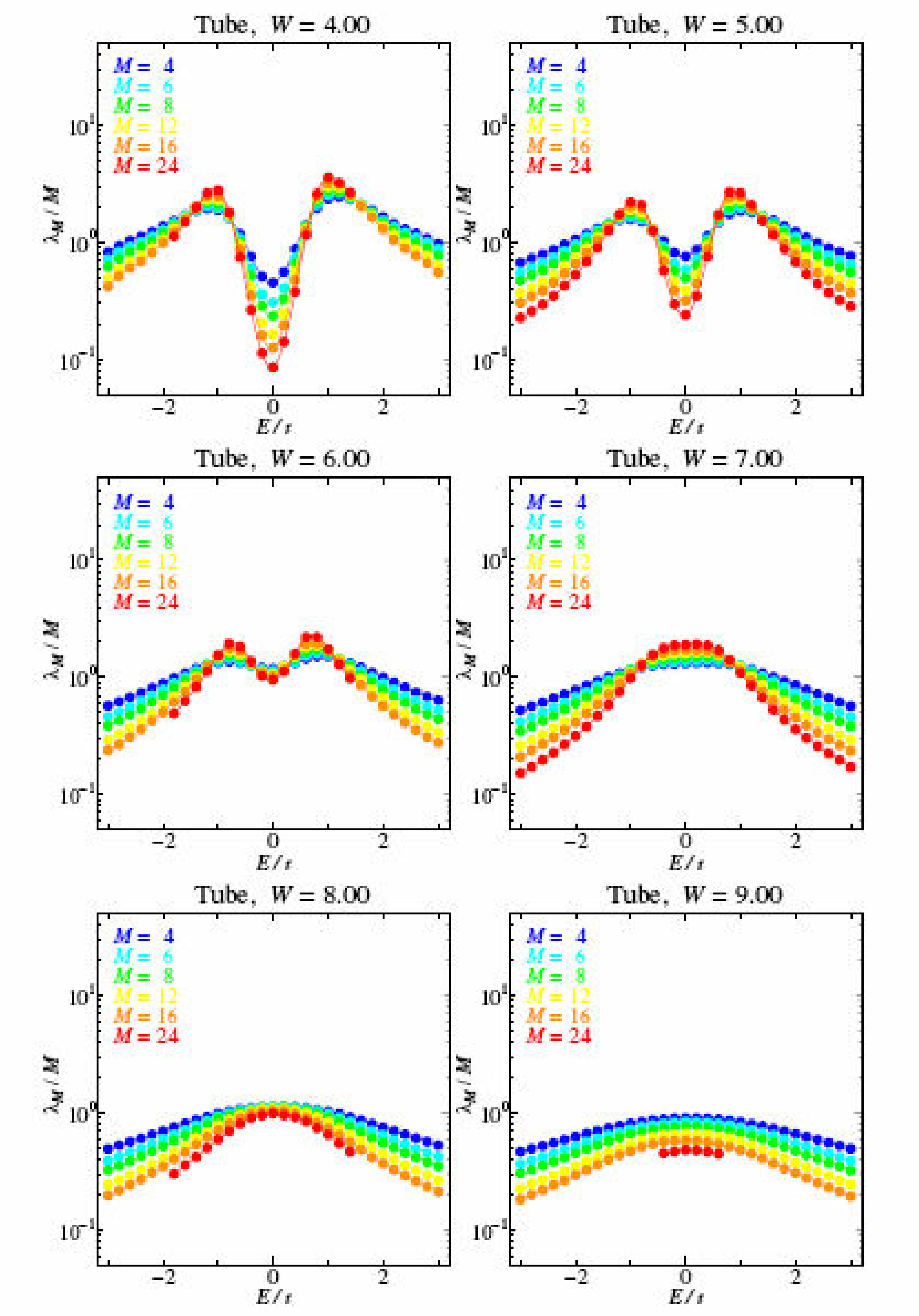
Anderson MIT transition

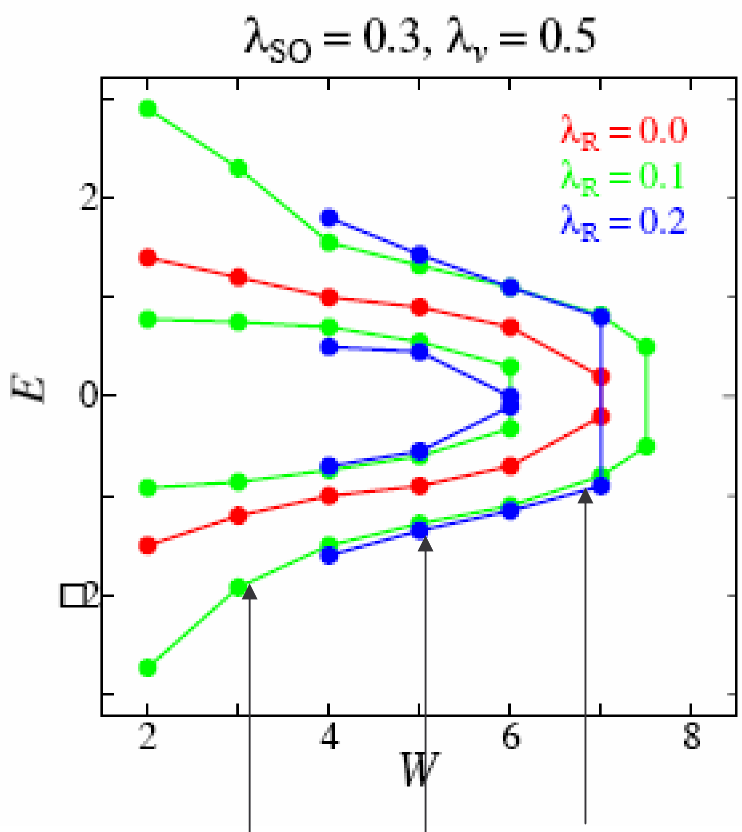
ν=1.57 (GOE 3D)

ν=2.73 (GSE 2D)


$H_{KM+W} = t\sum_{<ij>} c_i^\dagger c_j + i\lambda_{SO} \sum_{<<ij>>} \nu_{ij} c_i^\dagger s^z c_j + i\lambda_R \sum_{<ij>} c_i^\dagger [\mathbf{s}\times\mathbf{d}_{ij}]_z c_i + \lambda_v \sum_i \xi_i c_i^\dagger c_i + \sum_i w_i c_i^\dagger c_i$
W_3
W_2
W_1
W=0
Dos
E


Loc.
W_c
W
Dos=0
Loc.
Ext
Loc.
Dos=0
Loc.
Dos=0
E
=MIT
=Metallic phase
Localized phase





ν=2.7
(symplectic)

ν=1.6
(surprise)

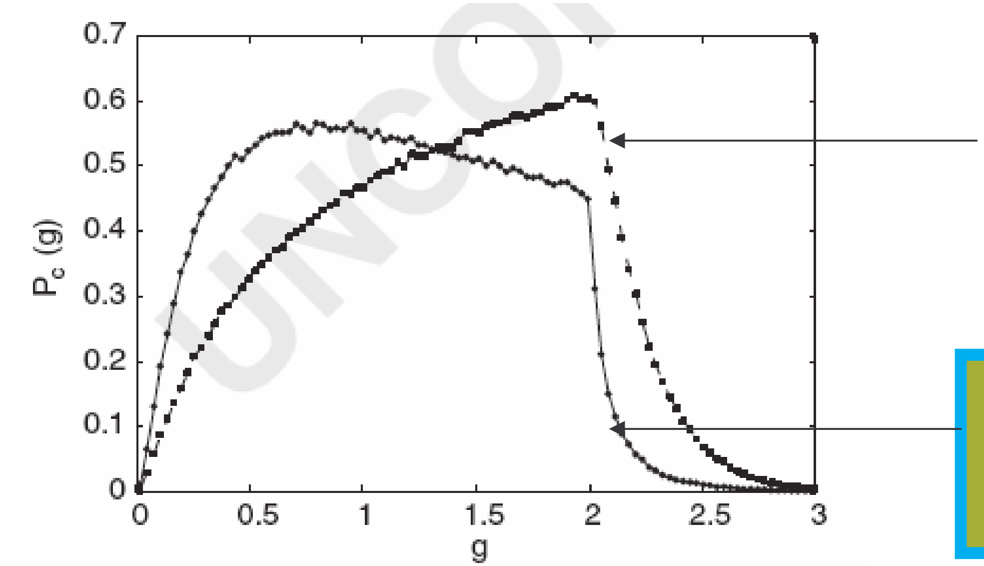
M. Onoda, Y.A & N. Nagaosa
PRL (in press)

Compare graphene Hamiltonian with 2D symplectic (SU(2)) models

$$H_{KM+W} = t\sum_{<ij>} c_i^\dagger c_j + i\lambda_{SO}\sum_{<<ij>>} \nu_{ij} c_i^\dagger s^z c_j + i\lambda_R \sum_{<ij>} c_i^\dagger [\mathbf{s}\times\mathbf{d}_{ij}]_z c_i + \lambda_v \sum_i \xi_i c_i^\dagger c_i + \sum_i w_i c_i^\dagger c_i$$

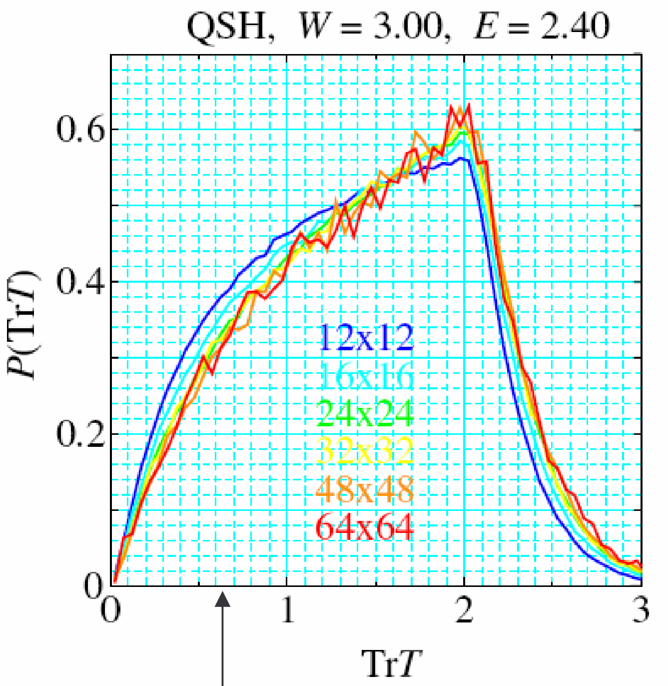
$$H_{GSE} = \sum_i w_i c_i^\dagger c_i - \sum_{<ij>} c_i^\dagger R c_j$$

$$c_i^\dagger = [c_{i\uparrow}^\dagger, c_{i\downarrow}^\dagger], \quad R = 2\times 2 \quad SU(2) \quad matrix$$

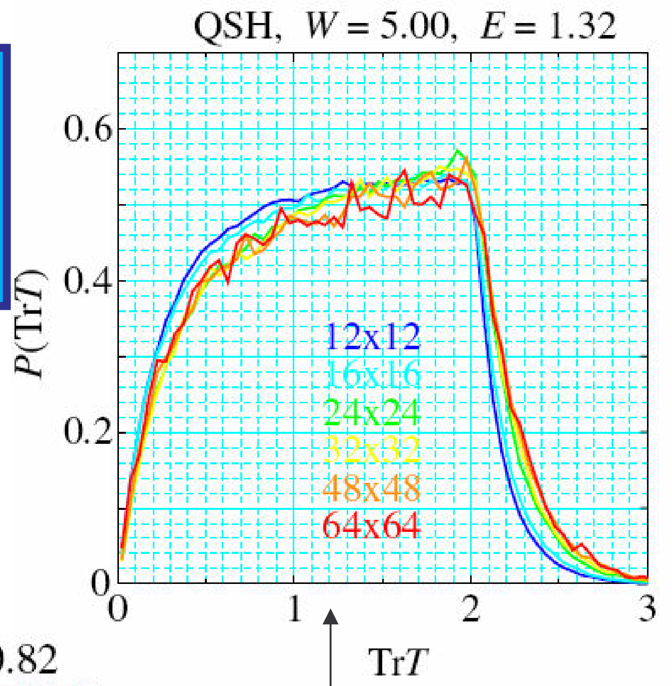


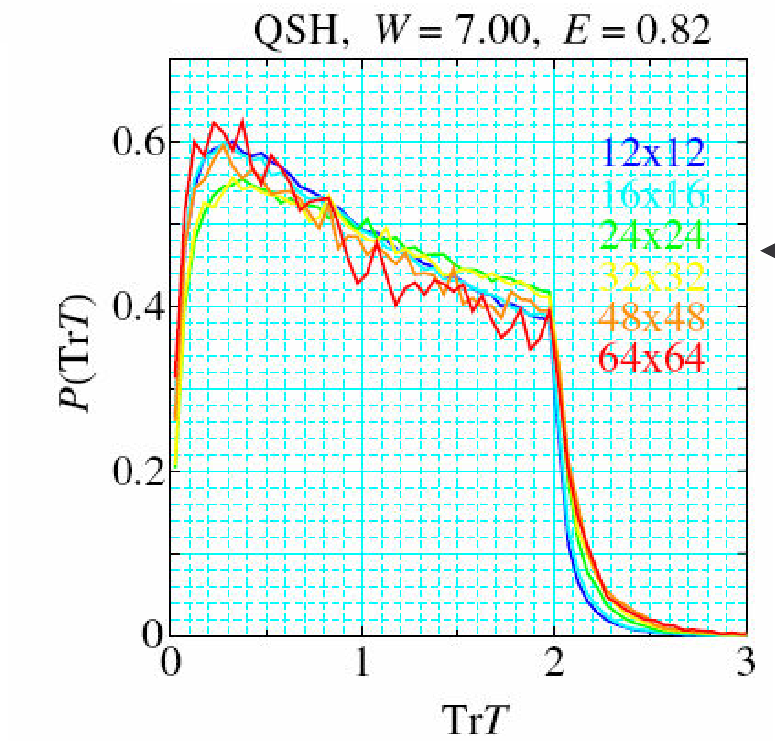
Conductance distribution at the MIT critical point for H_{GSE} (Ohtsuki, Slevin, Kramer)
ν=2.73

Conductance distribution at the IQHE critical point (Ohtsuki, Slevin, Kramer);
ν=2.37 (Huckestein)



Conductance distrib. at 3 different critical points
Note scale inv.





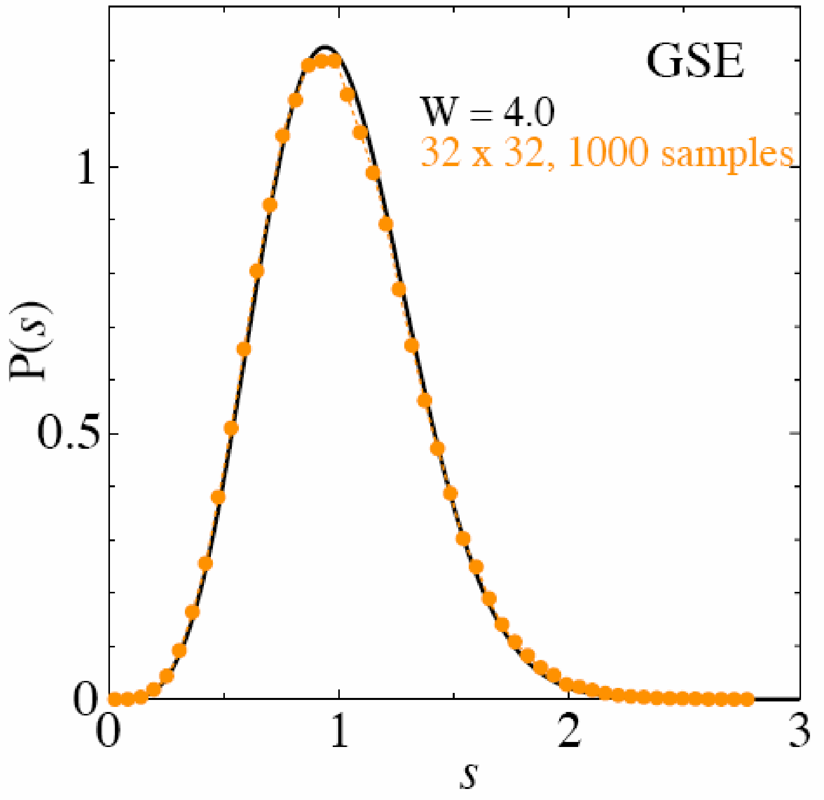
As for GSE (symplectic)

Distinct from symplectic

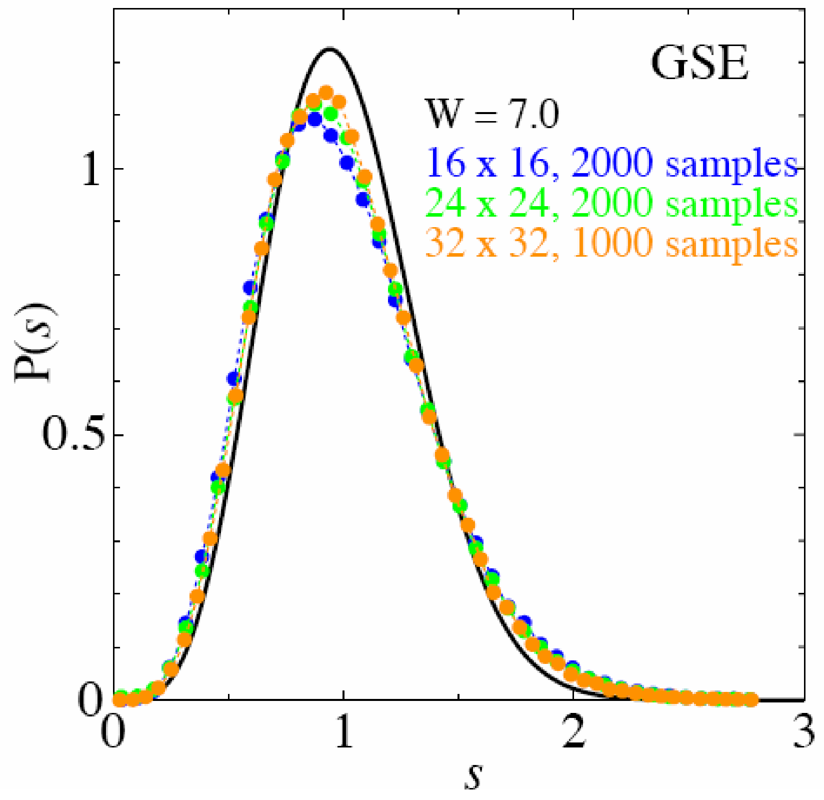
The metallic phase of QSH is identical with GSE.

1) p(g) is Gaussian and <g>(L)=a+(1/π)log L (Hikami-Larkin-Nagaoka)
2) Nearest level spacing distribution is $p_{GSE}(s)$.

$\lambda_{SO} = 0.3,\ \lambda_R = 0.1,\ \lambda_v = 0.5,\ h_x = 0$

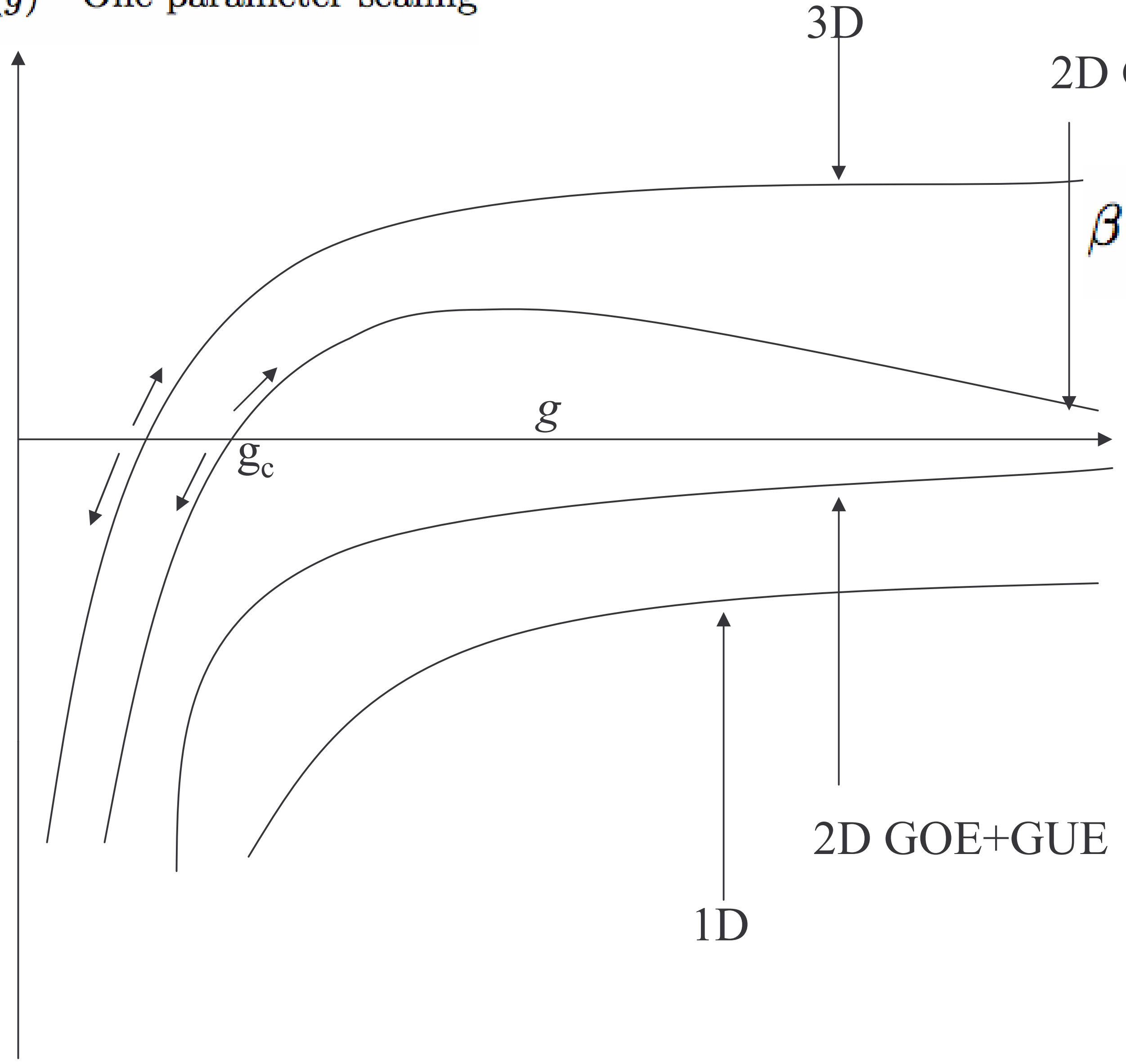


$\lambda_{SO} = 0.3,\ \lambda_R = 0.1,\ \lambda_v = 0.5,\ h_x = 0$



Scaling theory of localization (Abrahams, Anderson, Lichardeillo, Ramakrishnan)

$$\frac{dlogg}{dlogL} = \beta(g) \quad \text{One parameter scaling}$$

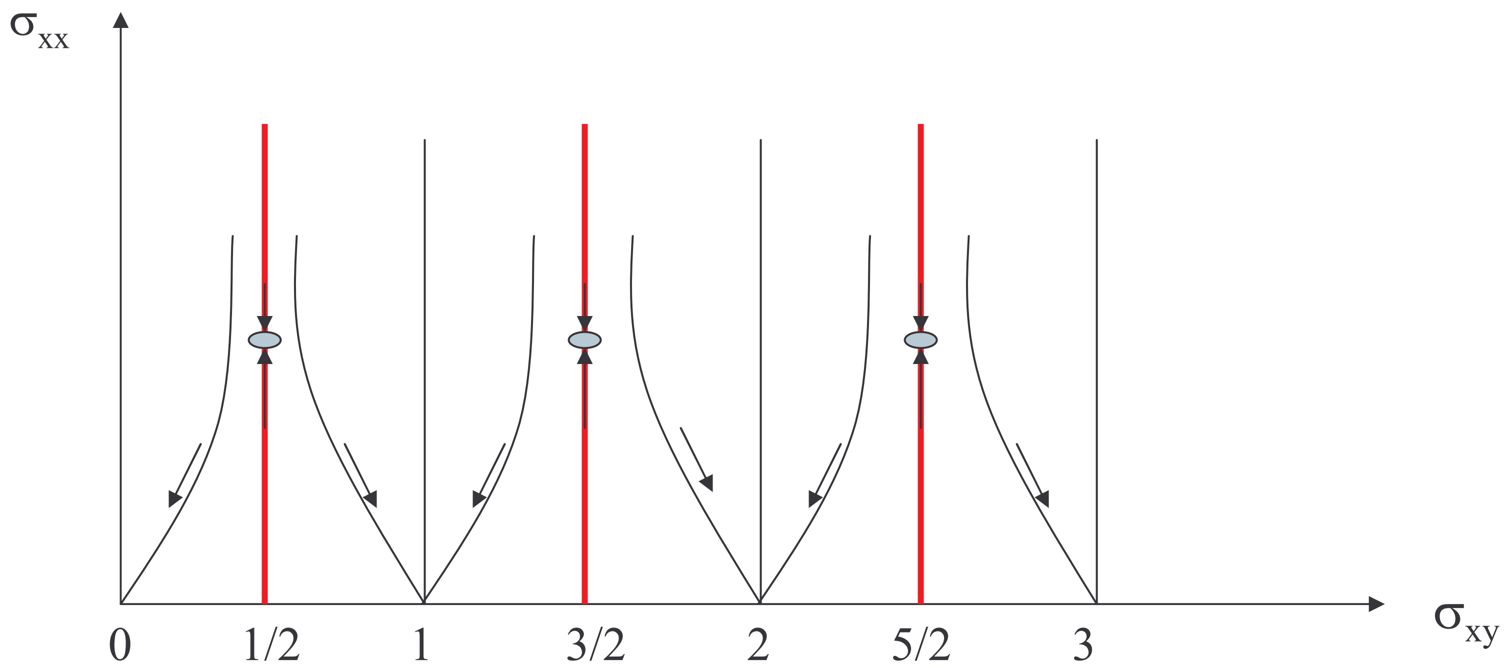


$$\beta(g) \approx \frac{1}{\pi g}$$

Recall the “two-parameter scaling” in the IQHE (Pruisken, Khmelnitzkii)

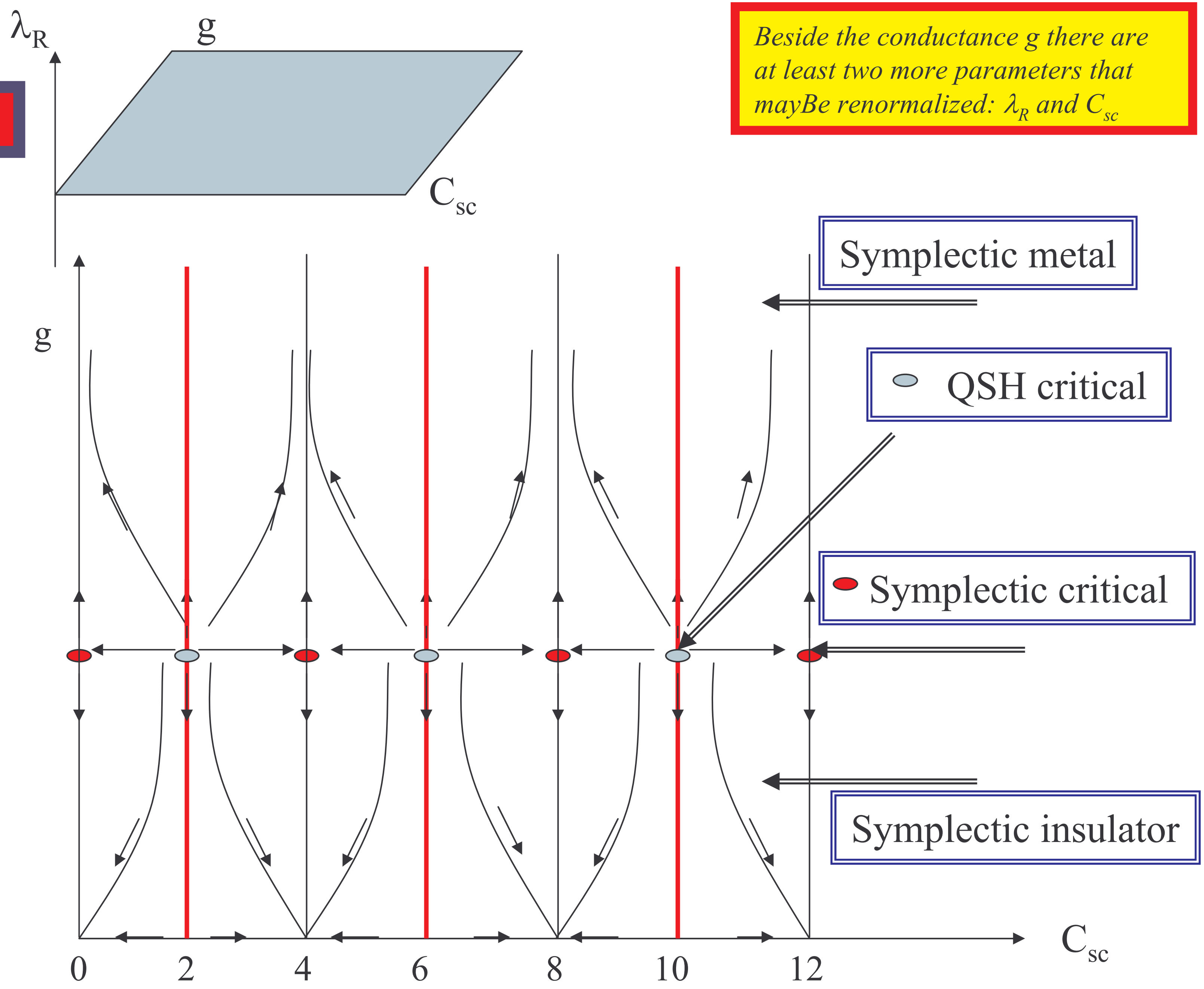
$$\frac{d\sigma_{xx}}{d\log L} = \beta_x(\sigma_{xx}, \sigma_{xy})$$

$$\frac{d\sigma_{xy}}{d\log L} = \beta_y(\sigma_{xx}, \sigma_{xy})$$



Speculation!!!

Beside the conductance g there are at least two more parameters that mayBe renormalized: λ_R and C_{sc}



VOLUME 82, NUMBER 17 PHYSICAL REVIEW LETTERS 26 APRIL 1999

Quantum Hall Plateau Transitions in Disordered Superconductors

V. Kagalovsky,[1,2] B. Horovitz,[1] Y. Avishai,[1] and J. T. Chalker[3]
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[2]*The Technological College of Beer-Sheva, School of Engineering, Beer-Sheva 84100, Israel*
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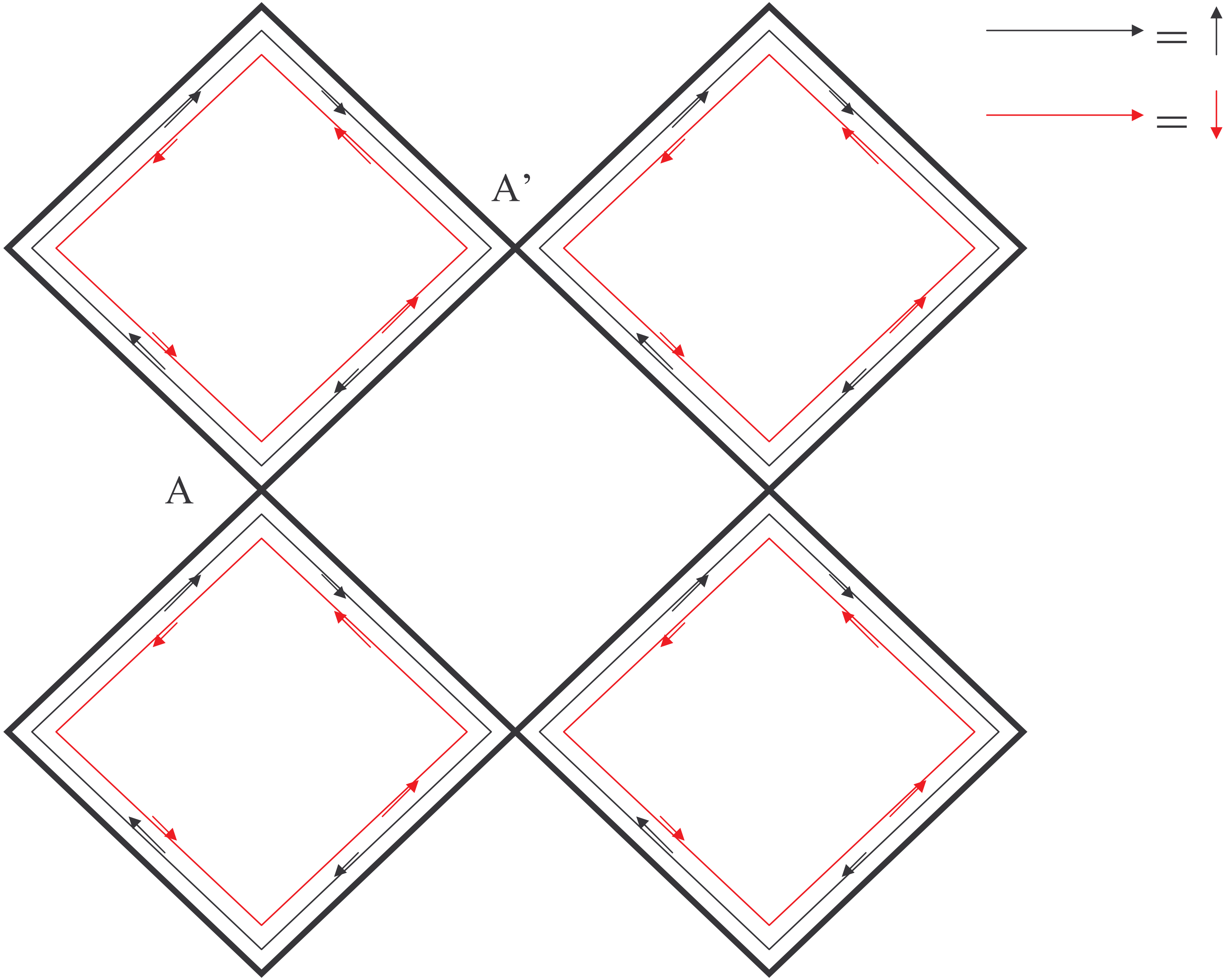
(Received 24 November 1998)

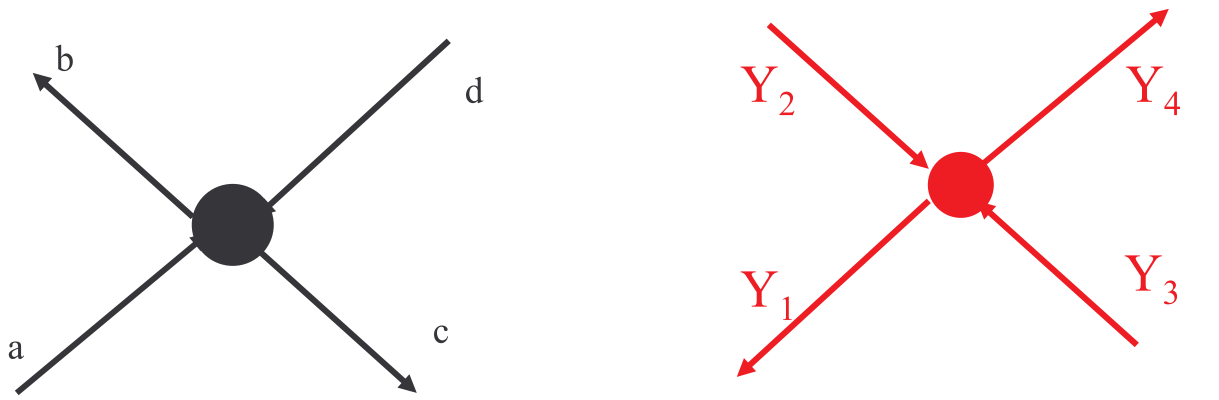
We study a delocalization transition for noninteracting quasiparticles moving in two dimensions, which belongs to a new symmetry class. This symmetry class can be realized in a dirty, gapless superconductor in which time-reversal symmetry for orbital motion is broken, but spin-rotation symmetry is intact. We find a direct transition between two insulating phases with quantized Hall conductances of zero and two for the conserved quasiparticles. The energy of the quasiparticles acts as a relevant symmetry-breaking field at the critical point, which splits the direct transition into two conventional plateau transitions. [S0031-9007(99)09003-1]

Employ the Chalker-Coddincton Network model

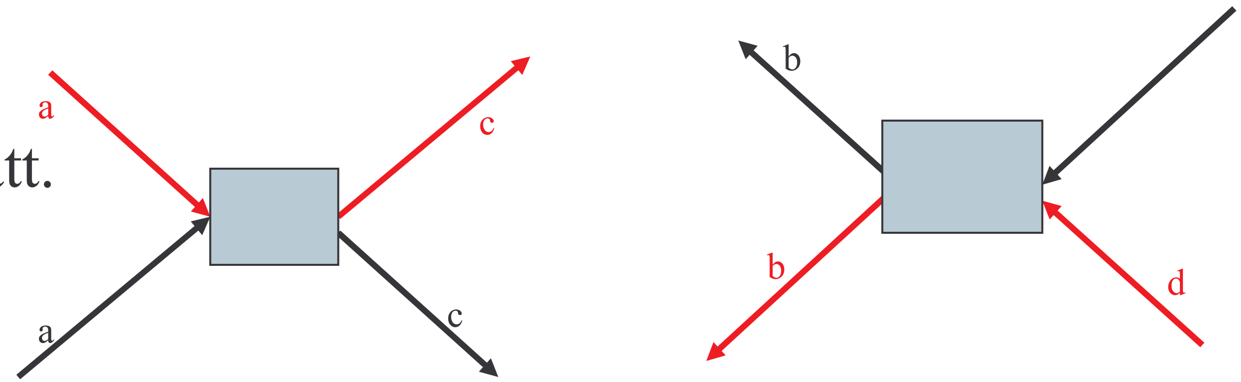
Network model for helical liquid (a single Kramer pair along the edge)



Pot. Scat.
CC



Spin Scatt.



Victor Kagalovsky & Y.A - work in progress

Summary:

1. Band insulators, spin non-conservation, problems defining spin current.
2. TR invariance, Z_2 topology in clean systems.
3. Disordered systems, spin Chern number (reservation by Fu & Kane).
4. QSH, C=4n+2, odd number of edge Kramer pairs.
5. SHI, C=4n, even number of edge Kramer pairs.
6. Localization problem: SHI+metallic QSH= symplectic ensemble.
7. QSH critical - Complicated RG flow pattern in g,C,λ_R parameter space.
8. Network model?

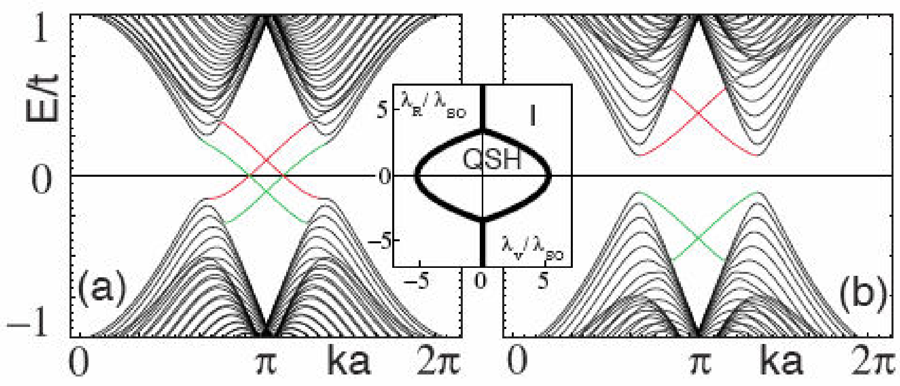


FIG. 1: Energy bands for a one dimensional "zigzag" strip in the (a) QSH phase $\lambda_v = .1t$ and (b) the insulating phase $\lambda_v = .4t$. In both cases $\lambda_{SO} = .06t$ and $\lambda_R = .05t$. The edge states on a given edge cross at $ka = \pi$. The inset shows the phase diagram as a function of λ_v and λ_R for $0 < \lambda_{SO} \ll t$.

Thank you for your attention

ありがとう　ございました

Merci beaucoup

Viellen Danke

תודה רבה