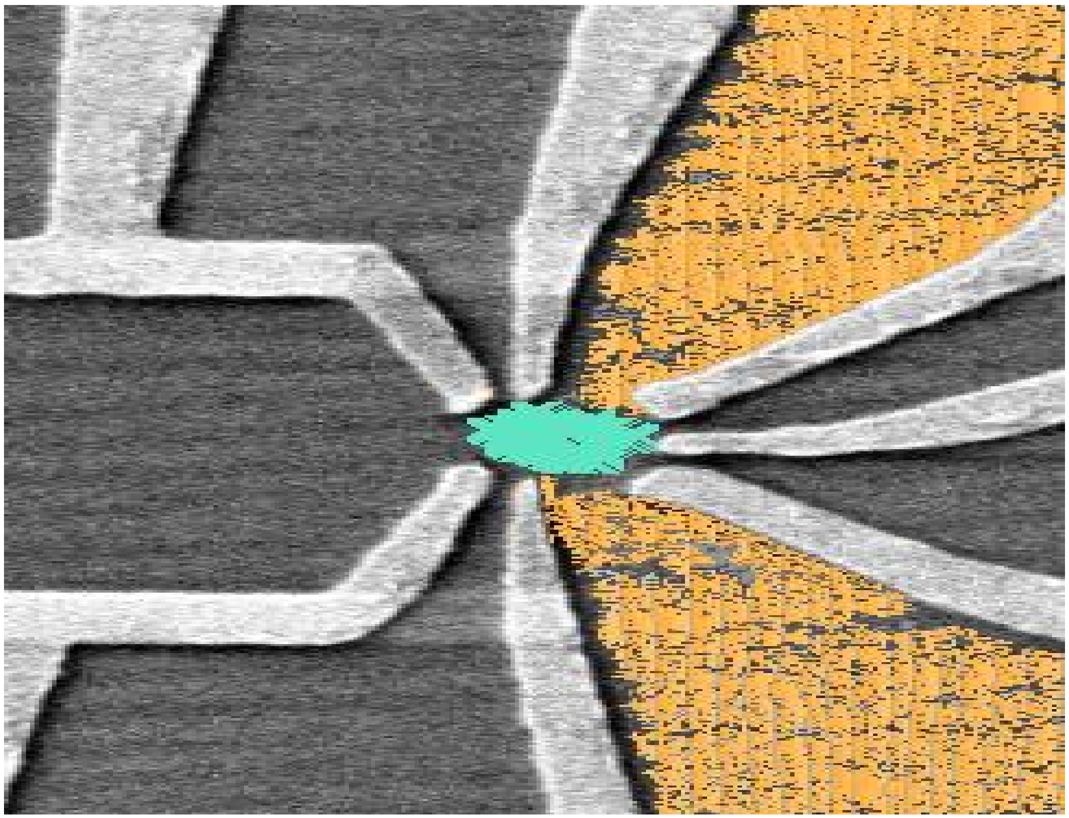
Quantum Impurities Out of Equilibrium



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Non-equilibrium and Strong Correlations

- **Nonequilibrium - poorly understood**

- No unifying theory such as Boltzmann's statistical mechanics
- Many of our standard physical ideas and concepts are not applicable (Scaling? RG? Universality?)
- Non-equilibrium systems are all different- it is unclear what if anything they all have in common.

- **Strongly correlated -in general- poorly understood.**
  - Perturbative approaches fail
  - New degrees of freedom emerge
  - New collective Behavior

Quantum Impurities - Theory and Experiment

Interplay : non-equilibrium and strong correlations

Outline

- **Non-equilibrium and Steady State (Quantum impurities)**
  - **Time dependent description (Keldysh)**
  - **The steady State - open systems**
  - **Time independent description**

    Scattering theory, Lippmann-Schwinger equation
  - **Scattering eigenstates and Non-equilibrium Steady State**

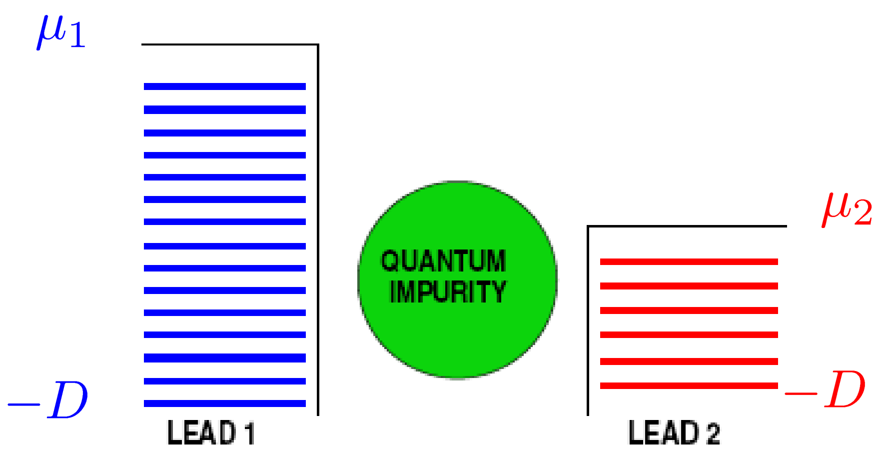
- **Constructing Scattering States**
  - **Traditional Bethe-Ansatz : closed systems** - ***inadequate***
    - **Equilibrium, Thermodynamics**
  - **Scattering Bethe-Ansatz : open systems** - ***new approach (SBA)***
    - **Non-equilibrium Steady States**
    - **Scattering states of electrons off magnetic impurities**

- **Non-equilibrium currents**

- **Dissipation and Entropy Production**

Quantum Impurities out-of-Equilibrium

The quantum impurity *(as seen by a theorist)*



$V = \mu_1 - \mu_2 \ll D$

$$\boxed{H = H_{leads} + H_{imp} + H_{leads-imp}}$$

In the Coulomb blockade regime:

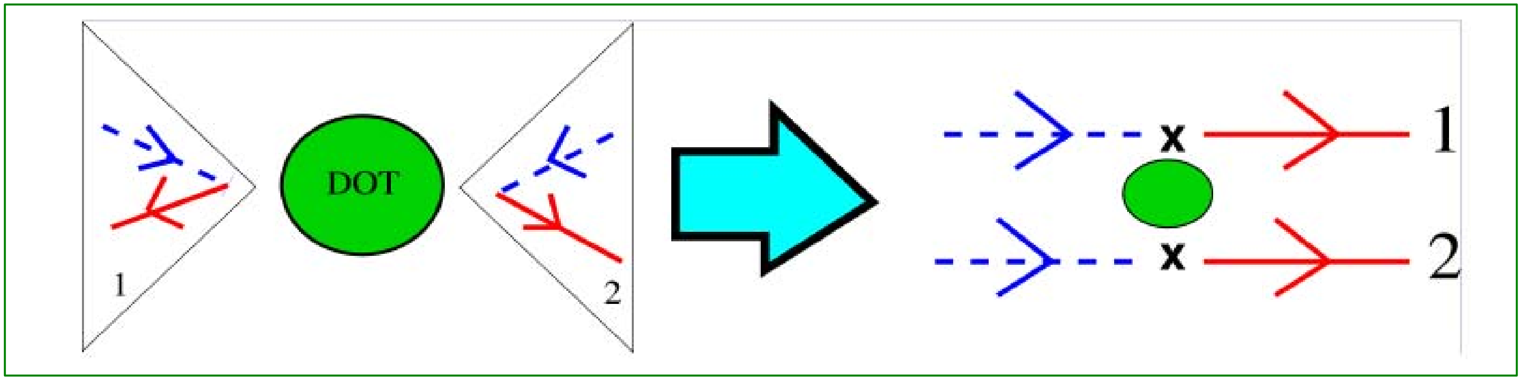
$$H_{qdot} = \sum_{i=1,2} \sum_{\vec{k}a} \epsilon_k c^{\dagger}_{i\vec{k}a} c_{i\vec{k}a} + J \sum_{i\vec{k}a} c^{\dagger}_{i\vec{k}a} (\vec{\sigma})_{aa'} \sum_{i\vec{k}'a'} c_{i\vec{k}'a'} \cdot \vec{S}$$

Quantum Impurities out-of-Equilibrium (1d)

Rewrite exactly as 1-d field theory (Affleck, Ludwig, Jones)

$$\psi_{i\epsilon a} \equiv \int d^3k\ \delta(\epsilon_{\vec{k}} - \epsilon)\ c_{i\vec{k}a}$$

$$\psi_{ia}(x) = \int_{-D}^{D} \frac{d\epsilon}{\nu(\epsilon)^{1/2}}\ e^{i\epsilon x}\ \psi_{i\epsilon a}$$



→ {
- low-energy physics, universality
- linearize spectrum, take cut-off to infinty
- 1-d field theory

$$H_{qdot} = -i \int \sum_{i=1,2} \psi_i^\dagger(x)\partial\psi_i(x)\ dx + J(\psi_1^\dagger(0) + \psi_2^\dagger(0))\ \vec{\sigma}\ (\psi_1(0) + \psi_2(0)) \cdot \vec{S}$$

Non-equilibrium: Time-dependent Description

Keldysh

- $t \leq t_o$, system described by: ρ_o
- $t = t_o$, couple leads to impurity
- $t \geq t_o$, evolve with $H(t) = H_o + e^{\eta t} H_1$

For T > 0 :

1. *initial condition* : ρ_o
2. *evolution* : $U(t, t_o) = T\{e^{-i \int_{t_o}^{t} dt' H(t')}\}$
3. *density matrix* : $\rho(t) = U^{\dagger}(t, t_o)\rho_o U(t, t_o)$
4. *non−equil value* : $\langle \hat{O}(t) \rangle = Tr\{\rho(t)\hat{O}\}$

In equilibrium: $lim_{t_0 \to -\infty} U^{\dagger}(t, t_o)\rho_o U(t, t_0) = e^{-\beta H}$

(B. Doyon, N. A. 2005)

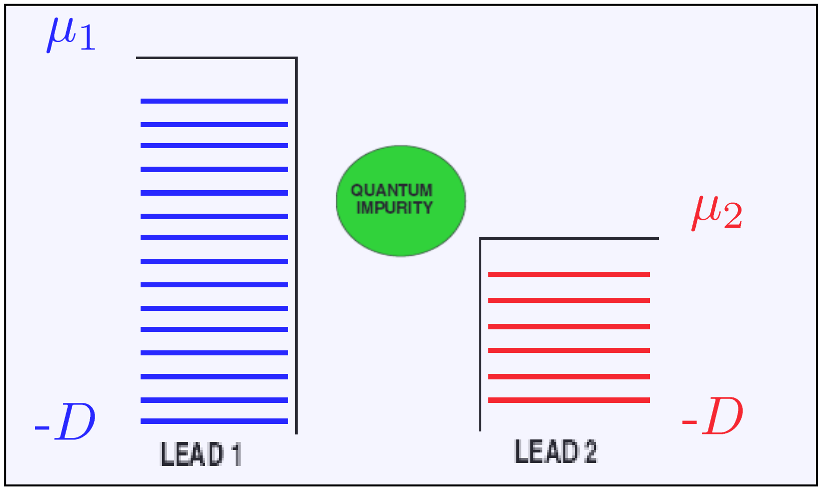
Non-equilibrium: Time-dependent Description

For T=0 :

1. initial condition: $|\phi_o\rangle$
2. evolution: $U(t,t_o) = T\{e^{-i\int_{t_o}^{t} dt' H(t')}\}$
3. evolved state: $|\psi(t)\rangle = U(t,t_o)|\phi_o\rangle$
4. non-equil value: $\langle\hat{O}(t)\rangle = \langle\psi(t)|\hat{O}|\psi(t)\rangle$

The initial condition:

$$|\phi_o\rangle = |\phi_o, V\rangle$$
$$= |bath1\rangle \otimes |bath2\rangle \otimes |\alpha\rangle$$



The Steady State (open system)

When will a steady state occur?

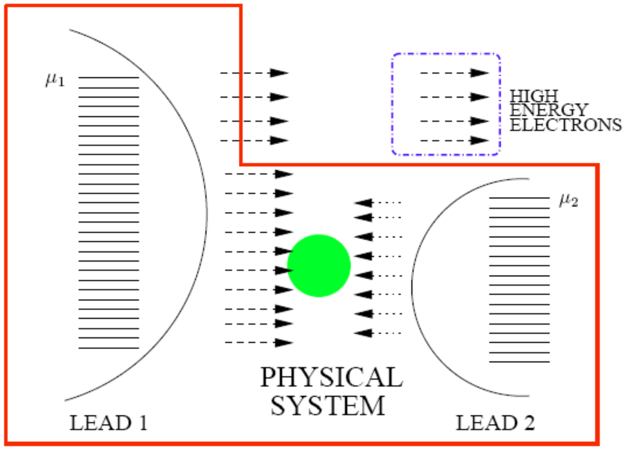
$$\frac{1}{L} \ll \frac{1}{|t_0|} \ll \eta \to 0$$

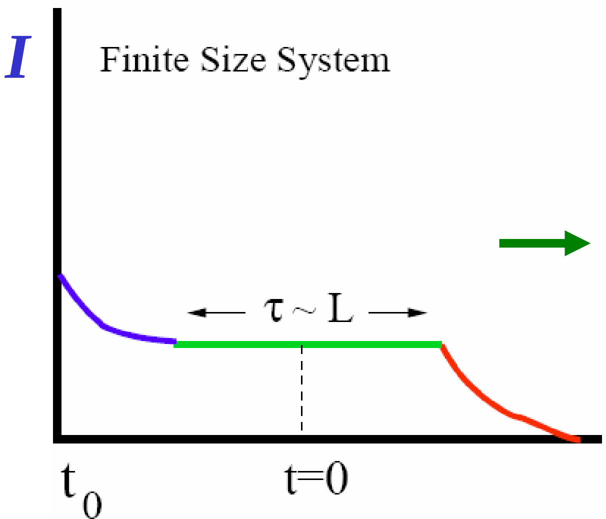
- **Leads good thermal baths, infinite volume limit - open system**

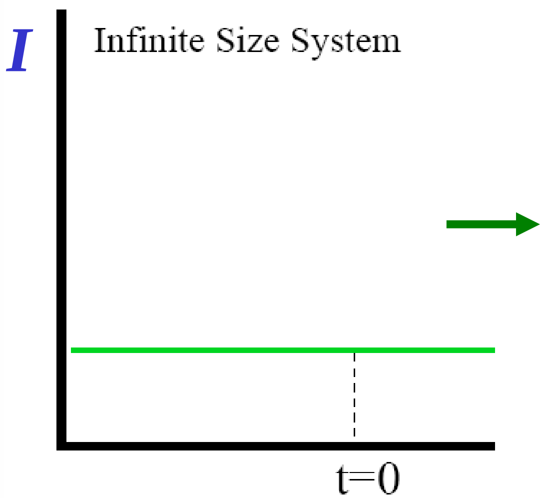
$\Rightarrow \exists \lim_{t_o \to -\infty}$, **no *IR* divergences (*Doyon, NA 2005*)**

Open system:

- Dissipation mechanism
- Time-reversal sym. breaking
- Steady-state non- eq. currents







A steady state ensues

$$\langle \hat{O}(t) \rangle = \langle \hat{O} \rangle$$

The Steady State — time independent description

The limit $\frac{1}{L} \ll \frac{1}{|t_o|} \ll \eta \to 0$ implies:

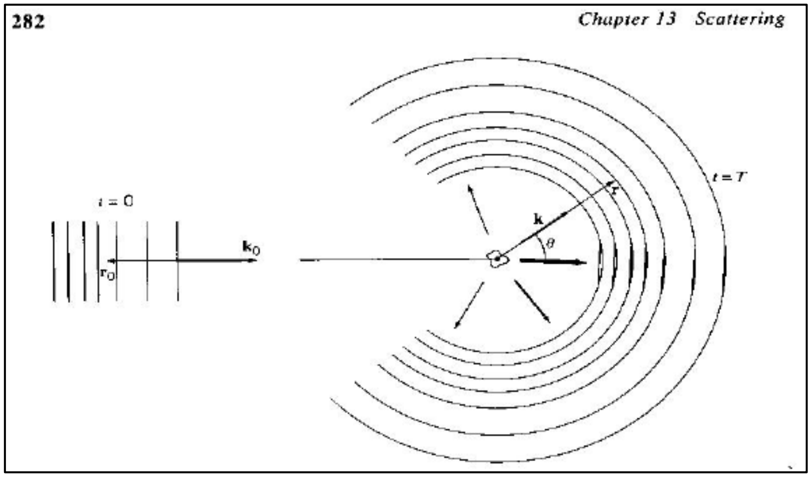
$$\exists\ |\psi, V\rangle_s = U(0, -\infty)|\phi_o, V\rangle$$

fully describes steady-state non-equilibrium

- $|\psi\rangle_s$ **eigenstate** of $H = H_0 + H_1$ (Gellman-Low thm)
- **Lippmann-Schwinger** equation, $|\phi_o\rangle$ -boundary condition

$$|\psi\rangle_s = |\phi_o\rangle + \frac{1}{E - H_0 + i\eta} H_1 |\psi\rangle_s$$

- $|\phi_o\rangle$: Initial condition $\to$ boundary condition
- $|\psi\rangle_s$ **scattering state** - eigenstate on the **infinite line**



from Merzbacher: eigenstate of

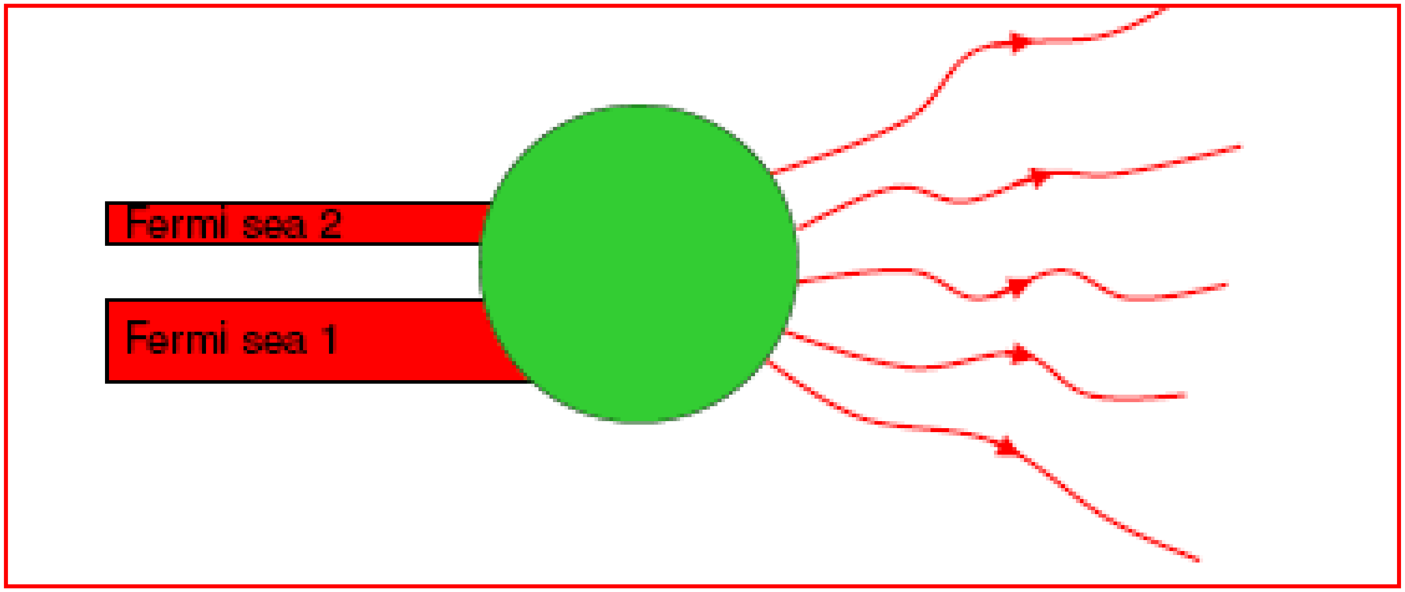
$$H = \frac{1}{2m} p^2 + V(x)$$

with incoming boundary condition

$$\psi(x) \to \phi_o(x) = e^{i\vec{p}\cdot\vec{x}}$$

The scattering state (many body)

Scattering eigenstate determined by *incoming asymptotics*: *the baths*



- **The Scattering State → steady-state non-equilibrium**

For example:

$$I(V) = \langle\, \phi_o, V|\, U^\dagger(0,-\infty)\, \hat{I}\ U(0,-\infty)\,|\phi_o, V\,\rangle \equiv \langle\, T_c\, e^{-i\int_{-\infty}^{0} H(t')dt'}\, \hat{I}(0)\,\rangle$$ **(Keldysh)**

$$= \langle \psi, V\,|\hat{I}\,|\psi, V\rangle_s$$ **(Scattering)**

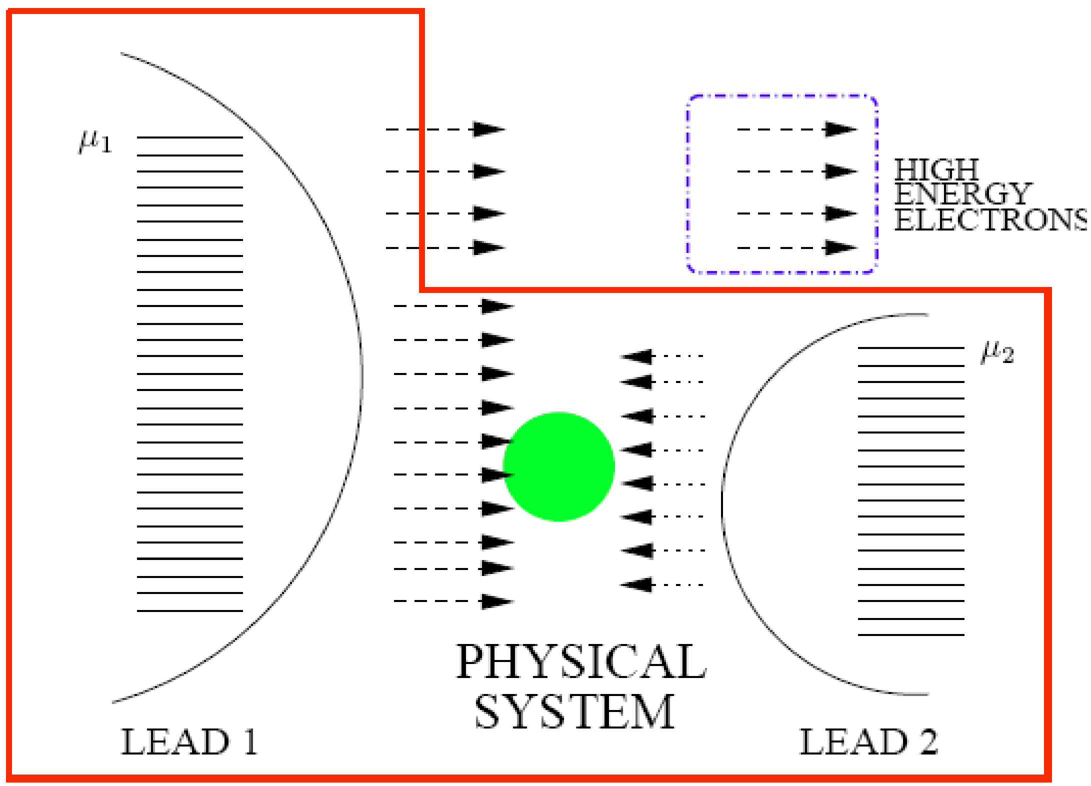
Non-equilibrium Steady-states & Scattering States

- **A scattering eigenstate describes all aspects of steady-state non-equilibrium physics:**

  non-equilibrium currents, energy dissipation, entropy production

- **A scattering eigenstate describes the system <u>and</u> its environment**

- ***At $T=0$ need a single scattering state***
- ***At $T>0$ need many scattering states***



Steady State at T>0

- ***For T=0*** $|\phi_o\rangle \xrightarrow{\text{L-S}} |\psi\rangle_s$ $|\phi_o\rangle$ ***g.s. of*** $H_0 - \sum_i \mu_i N_i$

- ***Generally,*** $|\phi_n\rangle \xrightarrow{\text{L-S}} |\psi_n\rangle_s$ ***where*** $|\phi_n\rangle \in \mathcal{H}_o^\perp$

- ***For T>0*** "free leads" boundary conditions: $p_n^o = e^{-\beta E_n^o}/Z_o$

$$\rho_o = \sum_n p_n^o \, |\phi_n\rangle\langle\phi_n| \longrightarrow \begin{cases} \rho_s = \sum_n p_n^o \, |\psi_n\rangle\langle\psi_n|_s \\ \langle\hat{O}\rangle = Tr\rho_s\hat{O} \end{cases}$$

The Scattering Bethe-Ansatz

HOW TO CONSTRUCT $|\psi\rangle_s$, $(for\ T = 0)$? OR ρ_s, $(for\ T > 0)$?

- Keldysh perturbation theory - fails in general (IR div)
- RG is inapplicable, $|\Phi_o\rangle$ highly excited
- Can the Bethe-Ansatz be used?

- **Traditional Bethe-Ansatz - inapplicable**
  - Periodic boundary conditions
  - **Closed System**: Equilibrium, Thermodynamics

- **New technology → Scattering States**
  - Asymptotic Boundary conditions on the infinite line
  - **Open System**: Non-equilibrium, scattering problems

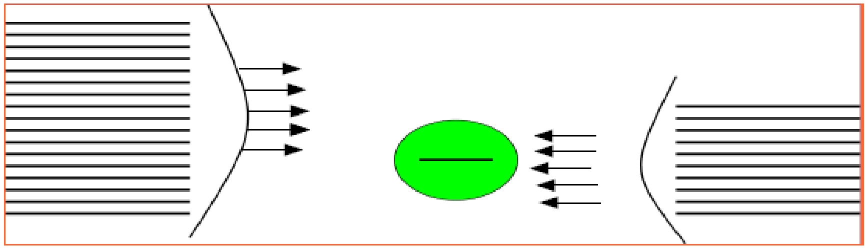
Scattering Bethe-Ansatz

the new technology →

- **consistency of non-eq BC and integrability (YBE)**
- **integrability out-of-equilibrium**

IRL: The Scattering State I

- **The Interacting Resonance Level Model:**



$$H_{IRL} = \sum_{i=1,2\,\vec{k}} \epsilon_k c^\dagger_{i\vec{k}} c_{i\vec{k}} + \epsilon_d d^\dagger d + t \sum_{i=1,2\,\vec{k}} (c^\dagger_{i\vec{k}} d + h.c) + 2U \sum_{i=1,2\,\vec{k}} c^\dagger_{i\vec{k}} c_{i\vec{k}} d^\dagger d$$

- **The hamiltonian :**
  - low-energy physics, c.o. independence
  - field theory, universality, renormalizability

$$H_{IRL} = -i \sum_{i=1,2} \int dx\, \psi^\dagger_i(x) \partial \psi_i(x) + \epsilon_d d^\dagger d + t \sum_i \left(\psi^\dagger_i(0) d + h.c. \right) + 2U \sum_i \psi^\dagger_i(0) \psi_i(0) d^\dagger d$$

***Diagonalize H via* Scattering Bethe-Ansatz:**

diagonalize directly on the infinite line (open system)

1. **construct 1-particle eigenstates (with boundary conditions)**
2. **construct 2-paricle eigenstates ..**

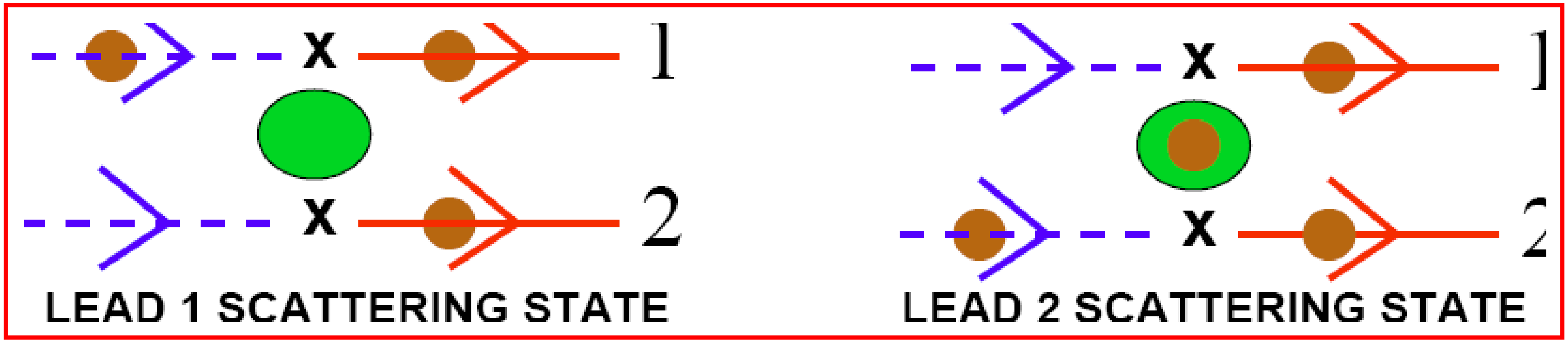
$$H|F_N\rangle = E_N|F_N\rangle \quad N = 1, 2...$$

IRL: The Scattering State II

Single-particle scattering states: $\delta_p = 2tan^{-1}[\frac{t^2}{2(p-\epsilon_d)}]$, phase shift

$$
\begin{aligned}
|1p\rangle &= \int dx e^{ipx}\left[\left([\theta(-x)+\frac{e^{i\delta_p}+1}{2}\theta(x)]\psi_1^\dagger(x)+[\frac{e^{i\delta_p}-1}{2}\theta(x)]\psi_2^\dagger(x)\right)+e_p d^\dagger\delta(x)\right]|0\rangle \\
&\equiv \int dx e^{ipx}\alpha_{1p}^\dagger(x)|0\rangle
\end{aligned}
$$

$$
\begin{aligned}
|2p\rangle &= \int dx e^{ipx}\left[\left([\theta(-x)+\frac{e^{i\delta_p}+1}{2}\theta(x)]\psi_2^\dagger(x)+[\frac{e^{i\delta_p}-1}{2}\theta(x)]\psi_1^\dagger(x)\right)+e_p d^\dagger\delta(x)\right]|0\rangle \\
&\equiv \int dx e^{ipx}\alpha_{2p}^\dagger(x)|0\rangle
\end{aligned}
$$



The scattering states satisfy incoming bc: $\alpha_i(x) = \psi_i(x)$ for $x < 0$

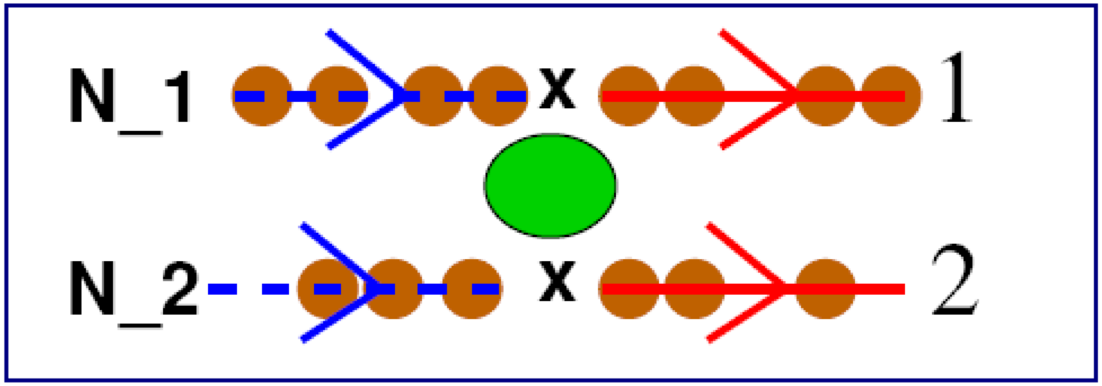
IRL: The Scattering State III

Multi-particle scattering state: *N_1 lead-1, N_2 lead-2*

$$|\{p\}\rangle_s = \int dx\, e^{i\sum_j^N p_j x_j} e^{i\sum_{j<l}^N \Phi(p_j,p_l) sgn(x_j - x_l)} \Pi_u^{N_1} \alpha_1^\dagger(x_u) \Pi_v^{N_2} \alpha_2^\dagger(x_v)|0\rangle$$

with

$$e^{2i\Phi(p_i,p_j)} \equiv S(p_i,p_j) = \frac{i - \frac{U}{2}\frac{p_i - p_j}{p_i + p_j - 2\epsilon_d}}{i + \frac{U}{2}\frac{p_i - p_j}{p_i + p_j - 2\epsilon_d}}$$



- $|\{p\}\rangle_s$ **scattering eigenstate for any choice of** $\{p\}$
- **impose non-eq boundary conditions to determine** $\{p\}$

- Why not choose $\{p\}$ Fermi-Dirac distribution?

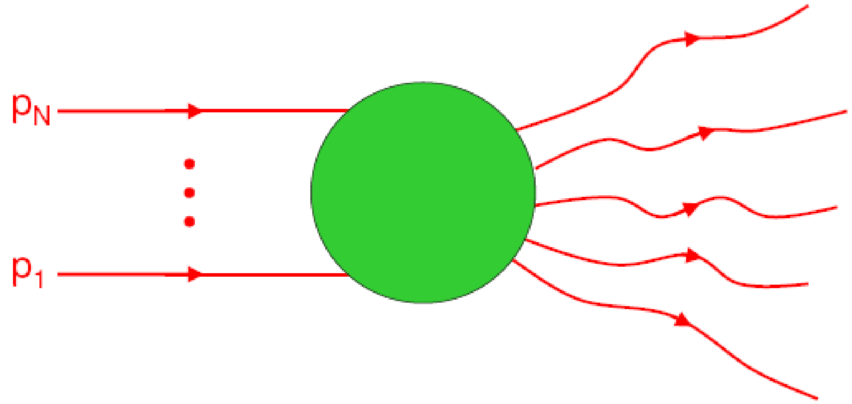
The Boundary Conditions I (The Bethe Basis)

Bethe basis in $\mathcal{H}_o$

$|\{p\}\rangle_o$

- $|\{p\}\rangle_s$ eigenstate for any $p_1 \cdots p_N$

$x_i < 0$ ←



- $\{p\}$ - BA momenta (not Fock momenta)
- Choice of BA momenta: determined by problem.

Non-eq BC: *far from impurity → 2 free leads*

Fock Basis in $\mathcal{H}_o$

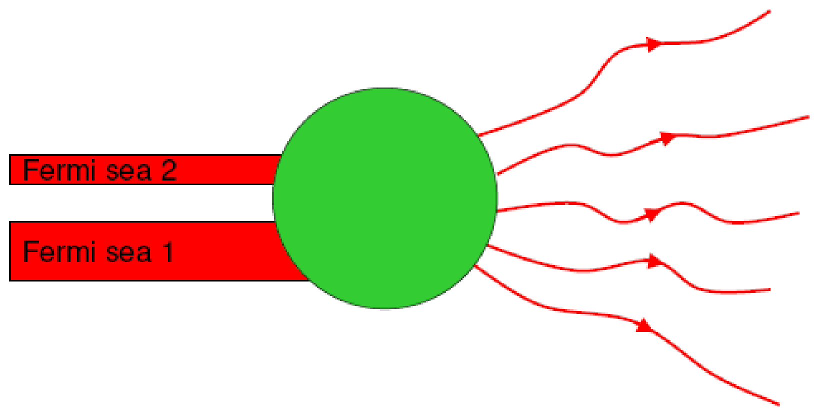
BC: $|\phi_o\rangle$

The ground state of:

$H_0 - \sum_i \mu_i N_i$

in the Bethe Basis

$x_i < 0$ ←



- Momentum distributions in leads - need to solve TBA eqns

boundary condition → $\rho_1(p), \rho_2(p)$

The Boundary Conditions II

Turn the BA equations into integral equations for : ρ_1, ρ_2

•Non-eq BC → momentum distributions $\rho_1(p),\ \rho_2(p)$:

– TBA eqns with upper cut-offs $k_o^j = k_o(\mu^j)$, lower cut-off, D:

$$\rho_1(p) = \frac{1}{2\pi}\theta(k_o^1 - p) - \sum_{j=1,2}\int_{-D}^{k_o^j} \mathcal{K}(p,k)\rho_j(k)\,dk$$

$$\rho_2(p) = \frac{1}{2\pi}\theta(k_o^2 - p) - \sum_{j=1,2}\int_{-D}^{k_o^j} \mathcal{K}(p,k)\rho_j(k)\,dk$$

with: $$\mathcal{K}(k,p) = \frac{U}{\pi}\frac{\epsilon_d - k}{(k+p-2\epsilon_d)^2 + \frac{U^2}{4} + (k-p)^2}$$

- **TBA eqns describe the free leads on a ring (in the Bethe basis)**
- **For U=0 distributions reduce to Fermi-Dirac distributions**

Comment:

These TBA eqns valid for: $\epsilon_d \geq 0$ **otherwise, eqns more complicated**

IRL: Current & Dot Occupation

- Current and dot-occupation:

$$\hat{I} = \frac{i}{\sqrt{2}} t \sum_{j=1,2} (-1)^j (\psi_j^\dagger(0) d - h.c)$$

$$\hat{n}_d = d^\dagger d$$

- Expectation values: $\hat{I}, \hat{n}_d$ in Scattering State: $|\{p\}\rangle_{L\to\infty}^{\mu_1,\mu_2}$

$$\langle I \rangle_s^{\mu_1,\mu_2} = \int dp \, [\rho_1(p) - \rho_2(p)] \frac{\Delta^2}{(p-\epsilon_d)^2 + \Delta^2}$$

$\Delta = t^2/2$

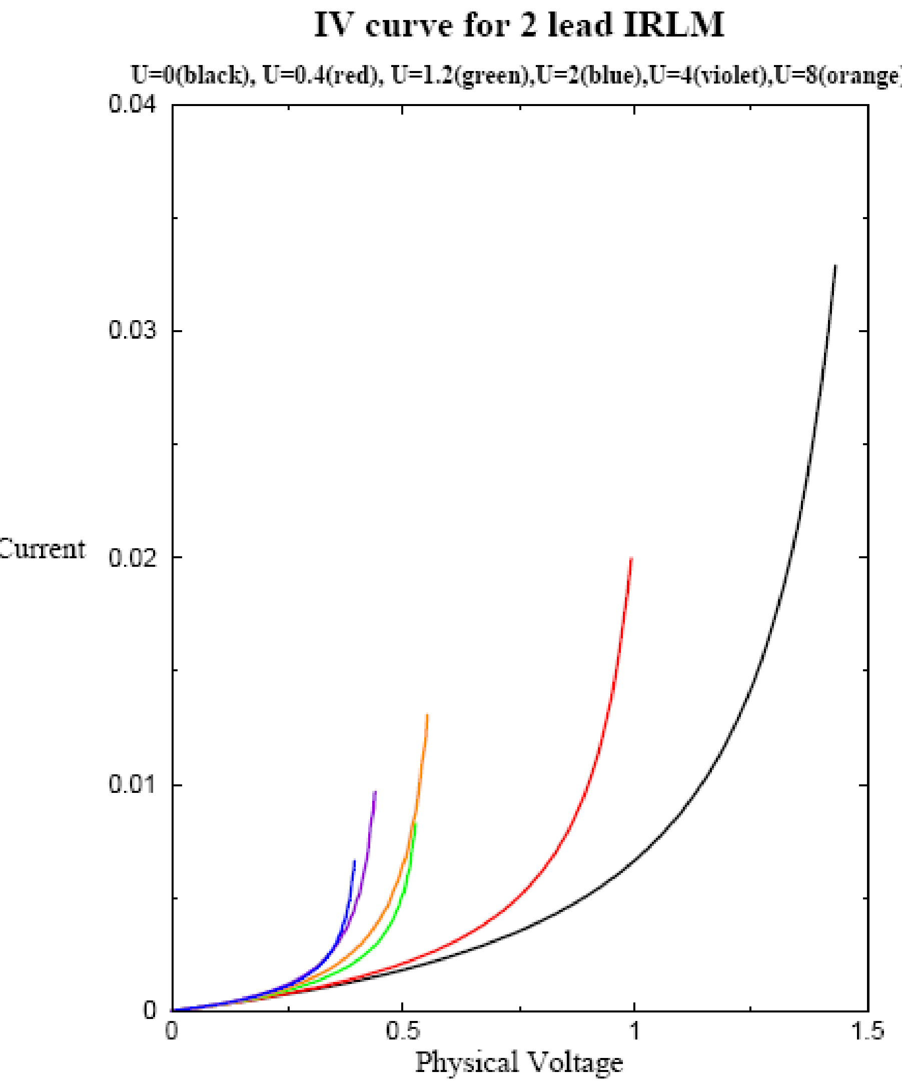
$$\langle n_d \rangle_s^{\mu_1,\mu_2} = \int dp \, [\rho_1(p) + \rho_2(p)] \frac{\Delta}{(p-\epsilon_d)^2 + \Delta^2}$$

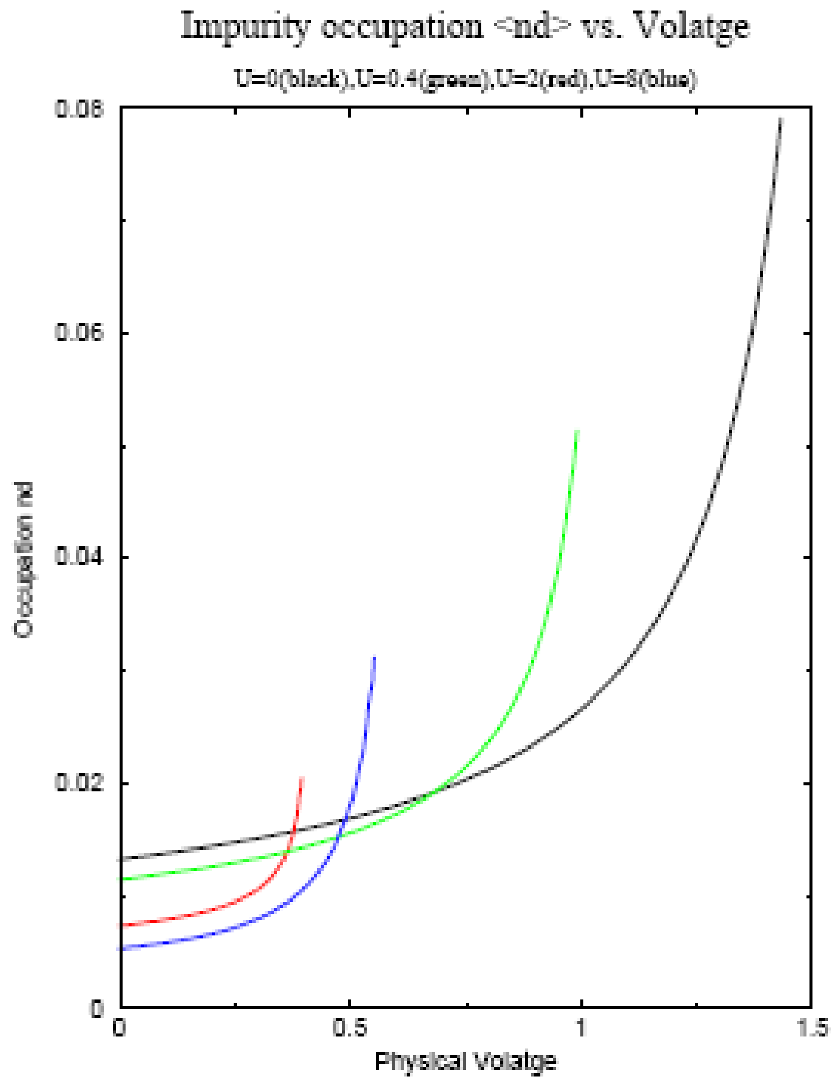
- **For U=0, Landauer expressions (from the scattering eigenstate!)**

- **For U>0, in the Bethe-Anzatz basis, expressions look "simple":**
  - Excitations undergo phase shifts only
  - Choice of momenta incorporates interactions and boundary conditions

IRL: Current vs. Voltage

- **Compute Exactly current and dot occupation as a function of Voltage:**





Sung Po Chao

- Can easily generalize to finite temperature case
- **Universality out of equilibrium**: change in **D** can be compensated by change in **U** and Δ

Traditional vs Scattering BA

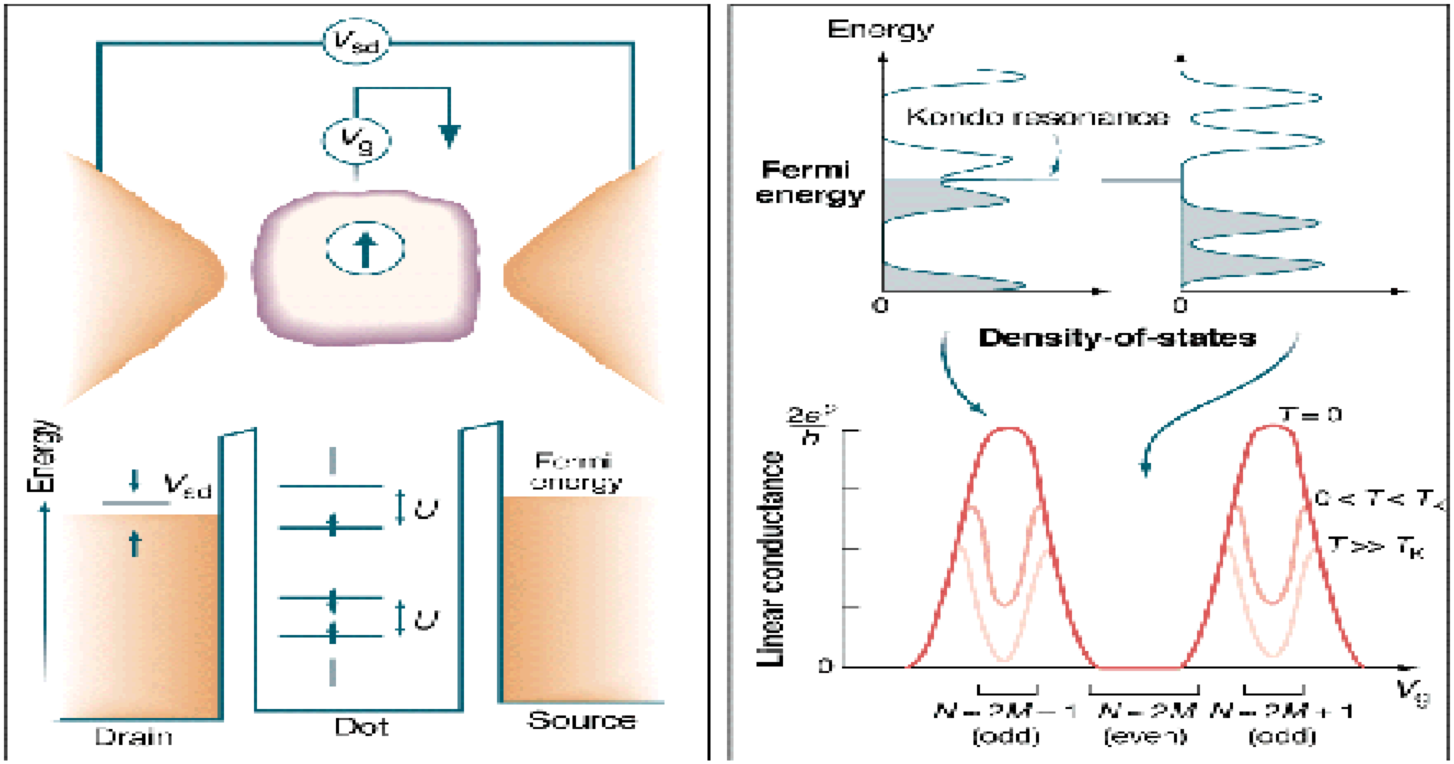
The construction of $|\psi\rangle_s$ is an example of the SBA approach:

	SBA	TBA
System	Infinite	Finite
Boundary condition	asymptotic (open)	periodic
Wavefunctions	used explicitly	not used
Thermodynamics	difficult	easy
Scattering Properties	possible	not possible
Nonequilibrium Generalization	Yes	No

More applications:

- **Scattering S-matrix of electrons off magnetic impurities**
  - *elastic and inelastic cross sections*
- **Calculation single particle Green's functions, spectral functions**
  - *finite temperature resistivity (resistance minimum)*

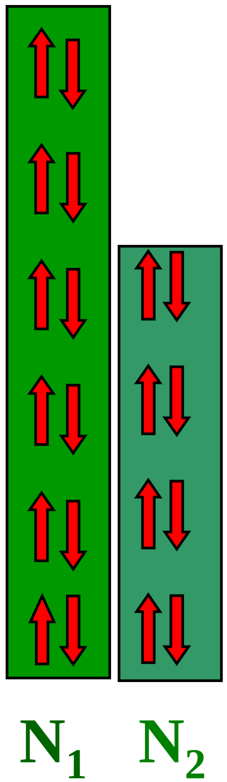
The Kondo Impurity Out of Equilibrium



Inoshita:Science 24 July 1998: Vol. 281. no. 5376, pp. 526 - 527

- Can control the number of electrons on the dot using gate voltage
- For odd number of electrons- quantum dot acts like a quantum impurity (Kondo, Interacting Resonant Level Model)
- Quantum impurity models exhibit new collective behaviors such as the Kondo effect

The Kondo Impurity out-of- Equilibrium



Leads - $SU_{spin}(2) \times SU_{flavor}(2)$

$$H_o = -i \int \psi^\dagger_{ia} \partial \psi_{ia} \, dx$$

$$i \; = \; 1,2 \;\; leads$$
$$a \; = \; \pm 1/2 \; spin$$

Interaction

$$H_I = (\psi_1^\dagger(0) + \psi_2^\dagger(0)) \, \vec{\sigma} \, (\psi_1(0) + \psi_2(0)) \cdot \vec{S}$$

$$N_1 = N_0 + \Delta N/2$$ *Number of electrons in lead 1*

$$N_2 = N_0 - \Delta N/2$$ *Number of electrons in lead 2*

$$N_e = N_1 + N_2 = 2N_0$$ *Total number of electrons*

$$\Delta N = VL/2\pi$$

Kondo Impurity: The Scattering State I

- **The leads-dot interaction:**

$$H_I = (\psi_1^\dagger(0) + \psi_2^\dagger(0))\, \vec{\sigma}\, (\psi_1(0) + \psi_2(0)) \cdot \vec{S}$$

- **Induces the S-matrices: (satisfy YBE out of equilibrium)**
  - *Electron-electron (same as for the 2-channel Kondo)*

$$S^{ij} = P_f^{ij} S_K^{ij} = P_f^{ij} P_s^{ij}$$

  - *Electron-impurity*

$$S^{j0} = \begin{pmatrix} (S_K^{j0} + 1_s^{j0})/2 & (S_K^{j0} - 1_s^{j0})/2 \\ (S_K^{j0} - 1_s^{j0})/2 & (S_K^{j0} + 1_s^{j0})/2 \end{pmatrix}_j \qquad S_K^{j0} = \frac{I_s^{j0} - icP_s^{j0}}{1 - ic}$$

- **The scattering state is constructed with these S-matrices**

Kondo Impurity: The Scattering State II

The scattering state:

$$|\Psi\rangle_s = \int dx \sum_Q \theta(x_Q) e^{i\sum_j k_j x_j} (S_Q A)^{\alpha_1\ldots\alpha_N}_{a_1\ldots a_N} \psi^\dagger_{a_1\alpha_1}(x_1)\cdots\psi^\dagger_{a_N\alpha_N}(x_N)|0\rangle$$

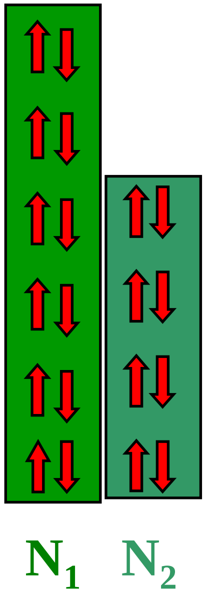
The Bethe-momenta $\{k\}$:

$$k_\delta^\pm = k_\delta \pm i\Lambda c/2, \qquad \delta = 1\cdots N_2$$

$$k_j, \qquad j = 1\cdots \Delta N$$

$$e^{i2k_\delta L} = \prod_{\gamma=1}^{N_0} \frac{\chi_\gamma + ic}{\chi_\gamma - ic} \qquad \delta = 1,\ldots,N_2 \qquad (2-strings)$$

$$e^{ik_j L} = \prod_{\gamma=1}^{N_0} \frac{\chi_\gamma + i\frac{c}{2}}{\chi_\gamma - i\frac{c}{2}} \qquad j = N_2+1,\ldots,N_1 \; (1-strings)$$



The spin momenta $\{\chi_\gamma\}$ ***are determined from:***

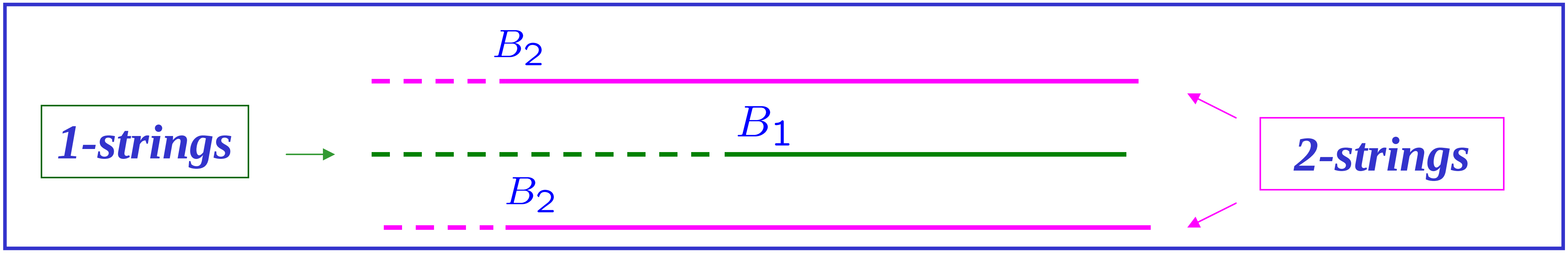
$$-\prod_{\beta=1}^{N_0} \frac{\chi_\gamma - \chi_\beta + ic}{\chi_\gamma - \chi_\beta - ic} = \left(\frac{\chi_\gamma + i\frac{c}{2}}{\chi_\gamma - i\frac{c}{2}}\right)^{N_1-N_2} \left(\frac{\chi_\gamma + ic}{\chi_\gamma - ic}\right)^{N_2}$$

Kondo: The Scattering State III

The 1-string and 2-string densities $\rho_1(p)$ $\rho_2(p)$ ***satisfy:*** ***(Andres Jerez)***

$$\rho_1^-(p) + \rho_1^+(p) = \frac{\Delta N e^{ipB_1}}{2ch(p/2)} + \rho_2^-(p)\frac{e^{ik(B_1-B_2)}}{2ch(p/2)}$$

$$\rho_2^-(p) + \rho_2^+(p)(1 + e^{-|p|}) = \frac{1}{2}N_2 e^{-|p|/2} e^{ipB_2} + \rho_1^-(p) e^{-|p|/2} e^{i(B_2-B_1)}$$



With:

$$(mag.\ field)\ h \sim e^{\pi B_2}$$

$$(voltage)\ V \sim e^{-\pi(B_1-B_2)}$$

Thus:

$$\rho_i^{\pm}(p) = \rho_i^{\pm}(p; V)$$

Kondo: The Current (in progress)

The Current:

$$\hat{I} = \psi_1^\dagger(\epsilon)\psi_1(\epsilon) - \psi_1^\dagger(-\epsilon)\psi_1(-\epsilon) = \vec{J}_x \cdot \vec{S}$$

Evaluated in the scattering state:

$$\begin{aligned}\langle\, \hat{I}\, \rangle_{neq}^{V} &= V \int dp \, \left(\rho_1^{+}(p;V) e^{-|p|/2} + \rho_2^{+}(p;V) e^{-|p|} \right) \\ &= V \sum_{n=0}^{\infty} (-1)^n \rho_1^{-}(i(2n+1)\pi) e^{(2n+1)B_1}\end{aligned}$$

I-V curves? – please stay tuned.

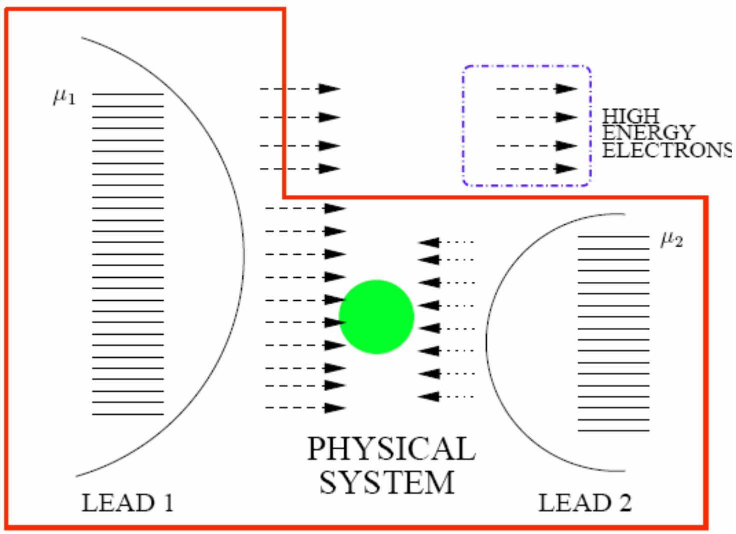
Entropy production I

Non-equilibrium currents dissipate heat into environment:

$$\delta Q_i = dE_i - \mu_i dN_i$$

- **Scattering state describes system +environment**
- **Dissipation mechanism: electrons reaching infinity**
- **Lost high energy electrons generate entropy (entanglement)**

"Discontinuous System"



Recall: currents ~ 1
leads ~ L → infty

$$\frac{dE_1}{dt} \equiv \langle \frac{d\hat{E}_1}{dt} \rangle_s = \langle i[\hat{H}, \hat{H}_{01}] \rangle_s = -\langle I_E \rangle_s$$

$$\frac{dN_1}{dt} \equiv \langle \frac{d\hat{N}_1}{dt} \rangle_s = \langle i[\hat{H}, \hat{N}_1] \rangle_s = -\langle I_N \rangle_s$$

Entropy production II

- **The rate of entropy production "thermodynamically" defined:**

- discontinuous system:

$$\sigma \equiv \frac{dS}{dt} = \frac{1}{T_1}\frac{\delta Q_1}{dt} + \frac{1}{T_2}\frac{\delta Q_2}{dt} = \left(\frac{1}{T_1} - \frac{1}{T_2}\right)\langle I_E\rangle_s + \left(\frac{\mu_1}{T_1} - \frac{\mu_2}{T_2}\right)\langle I_N\rangle_s$$

Note: i) **entropy and energy are defined w.r.t quasi –equil, (L ~ infty)**
ii) **maximal entropy per lead**
iii) **no entropy production in dot**

- **Can compute rate explicitly**
- **"Microscopic" derivation – entanglement (?):**

from: $\rho_s \equiv \rho_{12} = \sum_n p_n^0 |\psi_n\rangle\langle\psi_n|$

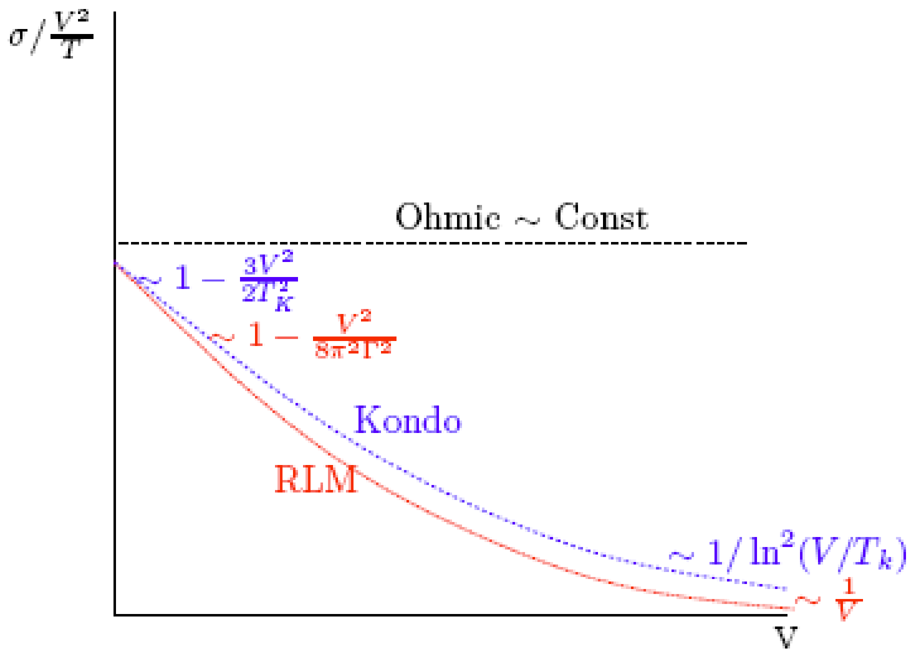
define: $\rho_1 = Tr_2\, \rho_s, \quad \rho_2 = Tr_1\, \rho_s$

then: $\sigma = \frac{1}{T_1}\frac{d}{dt} Tr_1\, \rho_1 \ln \rho_1 + \frac{1}{T_2}\frac{d}{dt} Tr_2\, \rho_2 \ln \rho_2$

- **Considering:** $I_N, I_E = I_N, I_E(\Delta T, \Delta \mu)$
  - **Out-of-equilibrium thermo-electric (Onsager) relations**
  - **Quantum-dot refrigerator**

Entropy Production: Effects of Correlations

How does the Kondo effect manifest itself?



- The RLM describes the Kondo model at Strong coupling
- Stronger correlations suppress entropy production
- To measure: perform spectroscopy of emerging electrons

Conclusions

- **Showed:**

  Scattering States with non-eq BC – Steady State

- **Computed:**

  The Steady State Current: IRLM at $T = 0$

- **Exact results:**

  A strongly correlated impurity system out of equilibrium

- **Many generalizations and applications:**

  Non-equilibrium Impurity Problems:

  - **Non-equilibrium in other impurity models:**
    Kondo, Anderson, Multichannel versions
  - **Non-equilibrium at $T > 0$, $T_1 \neq T_2$, B**
  - **Thermal Currents, spin currents**
  - **More leads: non-equilibrium DOS**

  Scattering Problems:

  - **Inclusive, exclusive scattering amplitudes**
  - **Inelastic scattering amplitudes $T > 0$**

  More ambitious:

  - **Non-equilibrium description of *bulk* systems**
  - **Non-equilibrium RG**