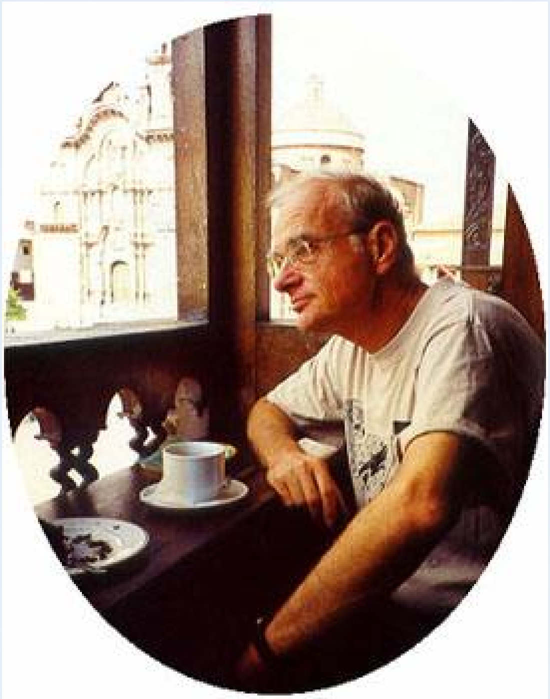


Order and Disorder in Condensed Matter:
From Solutions to Glasses

A Physics Department symposium
celebrating **Baruch Horovitz**'s
60th birthday

B. Horovitz and A. Golub, "Superconductors with broken time-reversal symmetry: Spontaneous magnetization and quantum Hall effects,"
Phys. Rev. B68, 214503 (2003) .

Broken time reversal symmetry (BTRS) in d+id' as well as in d+is superconductors is studied and is shown to yield current carrying **surface states**. We evaluate the temperature and thickness dependence of the resulting spontaneous magnetization and show a marked difference between weak and strong BTRS. We also derive the Hall conductance which vanishes at zero wavevector q and finite frequency w, however at finite q,w it has an unusual structure. The **chirality** of the surface states leads to **quantum Hall effects** for spin and heat transport in d+id' superconductors.

Quantum Hall Physics in Graphene

H.A. Fertig, Indiana University

I. Introduction: Graphene Sheets

II. Basics about Graphene: Dirac Spectrum, Berry's phase, Quantized Hall Effect

III. Edge States of Graphene

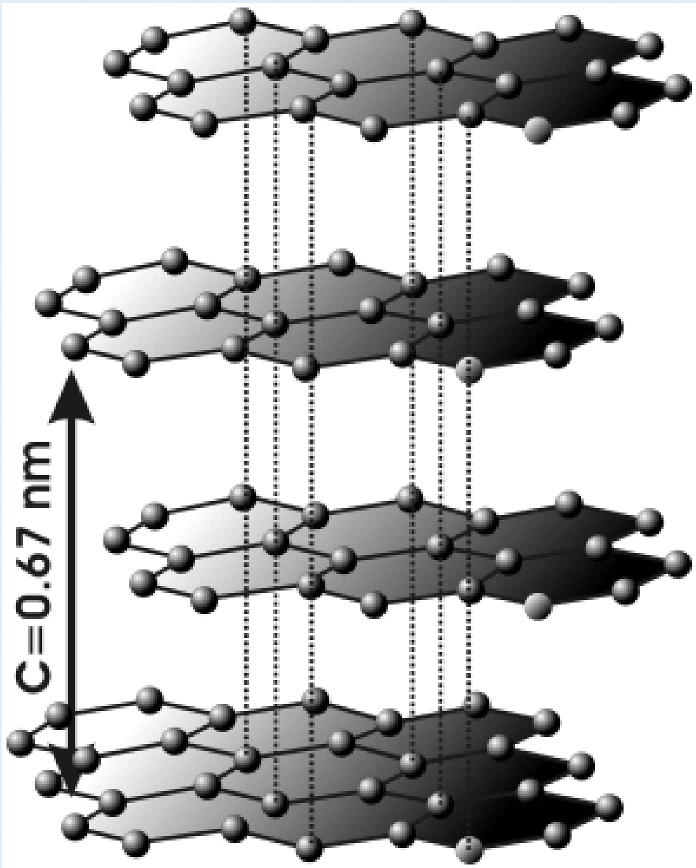
IV. Quantum Hall Ferromagnetism and the Graphene Edge

V. Summary

Collaborator: Luis Brey, CSIC, Madrid

Funding: NSF

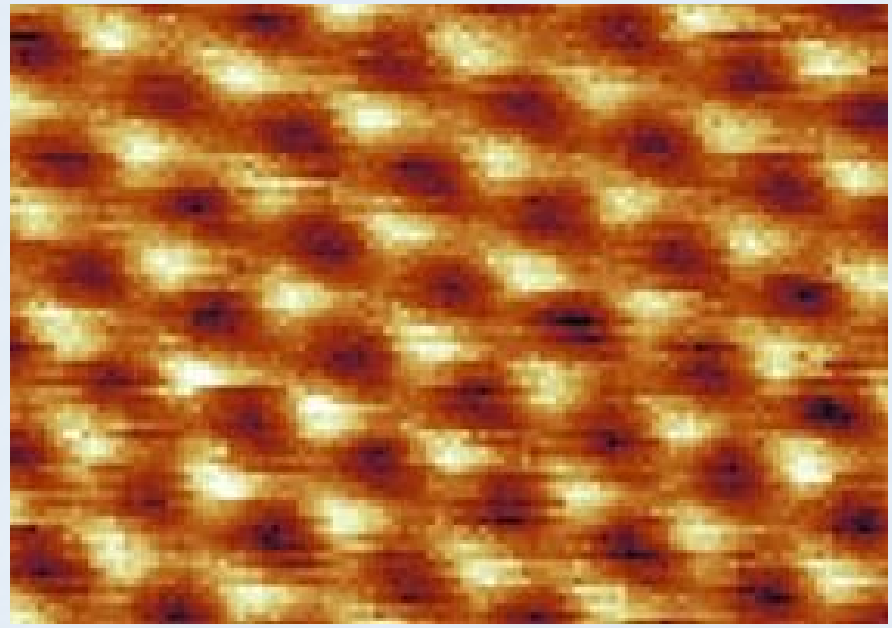
I. Introduction to Graphene



Graphite

- *Graphite* is the most stable form of carbon.
- Consists of sheets of carbon atoms (**graphene)** each with 3 covalently bonded nearest neighbors.
- 4th unbonded electron in p_z orbital allows planes to conduct electricity.
- Weakly bonded layers ⇒ layers easily slide off of one another (dry lubricant; pencils).

AFM image (Geim et al.)
(Single layer flake on SiO_2 substrate)

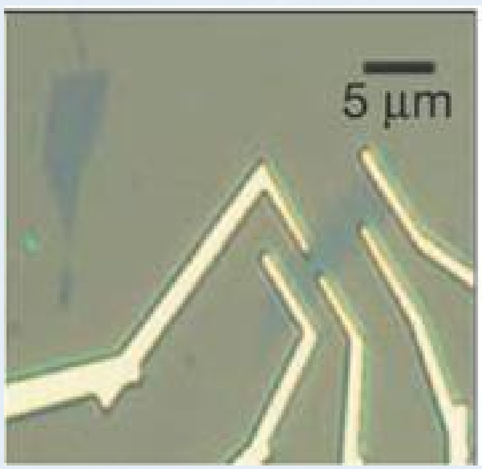


STM image (Geim et al.)

- Single graphene sheets can be isolated by repeated cleavage of graphite crystals (“exfoliation”). Such sheets were presumed to be unstable with respect to buckling, but this appears not to be the case. [Geim and coworkers, Manchester, 2004].

Great potential for electronic devices

- Metallic conductivity $\Rightarrow$ lower power dissipation, higher frequency operation than traditional semiconductors
- High thermal conductivity lowers operating temperatures
- Two dimensional geometry allows use of lithographic techniques

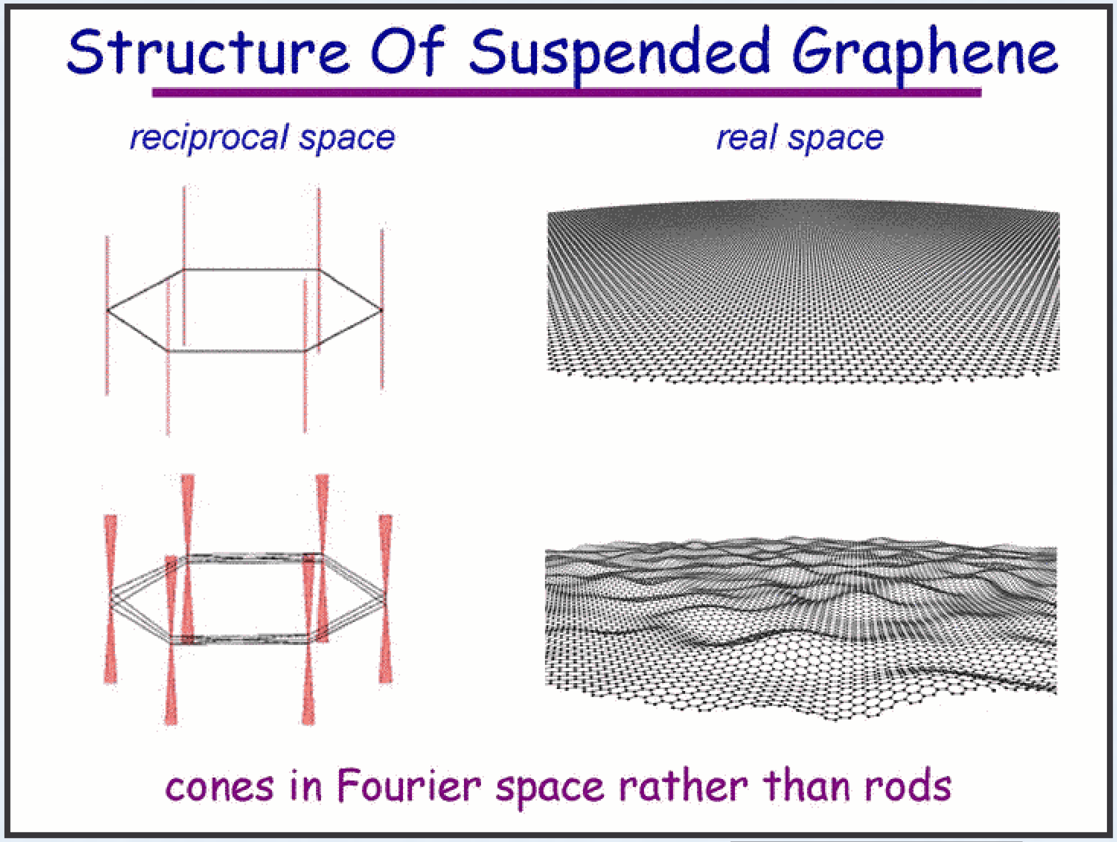


Kim Group, Nature 2005

Geim Group, Nature 2005

- Many groups currently trying to fabricate transistors from graphene

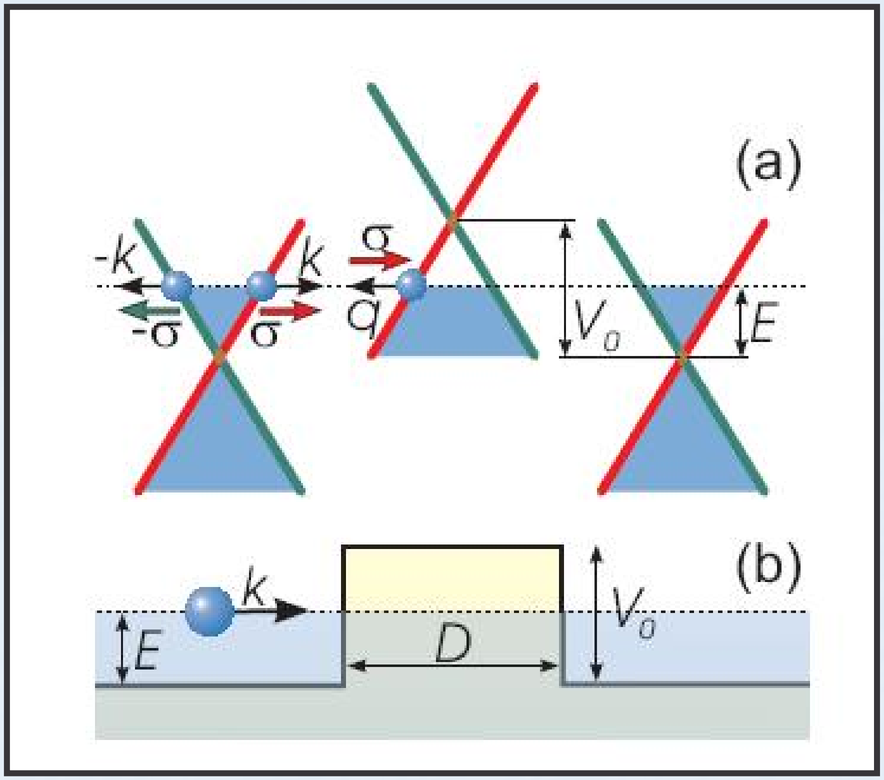
Some Interesting Directions



From A. Geim

- Dirac equation in curved space
- "Effective time-reversal symmetry-breaking"

- Klein paradox in a solid state system

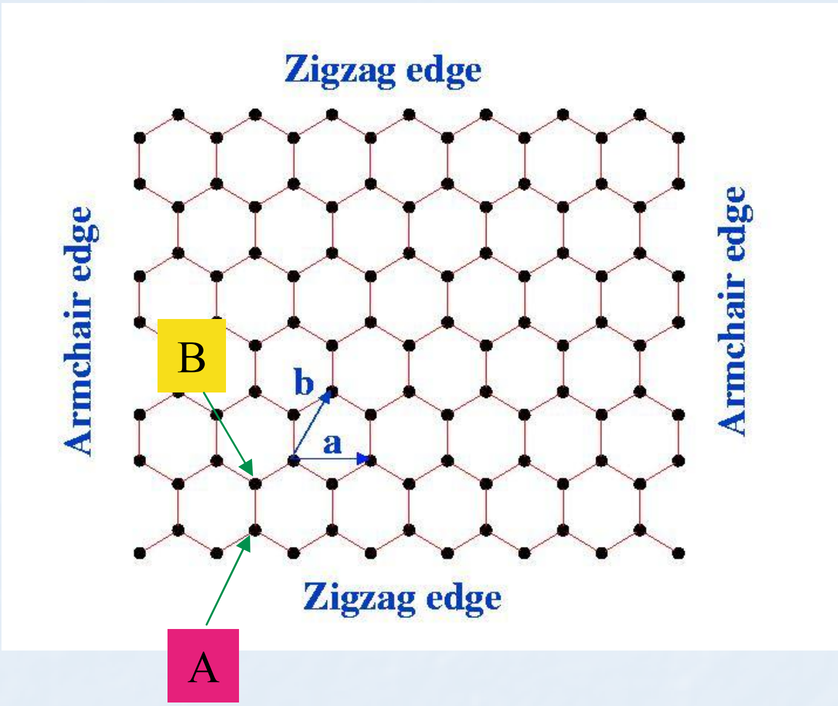


From Katsnelson, Novoselov, and Geim, cond-mat/0604323

- Device concepts
  - Transport

- Electron-electron interactions
  - Screening (Mele and Devincenzo, 1984)
  - Non-Fermi liquid behavior (Guinea and coworkers)

II. Graphene Basics



Honeycomb lattice, two atoms per unit cell

Lattice constant: 2.46Å

Nearest neighbor distance: 1.42Å

Simple tight-binding model for p_z orbitals:

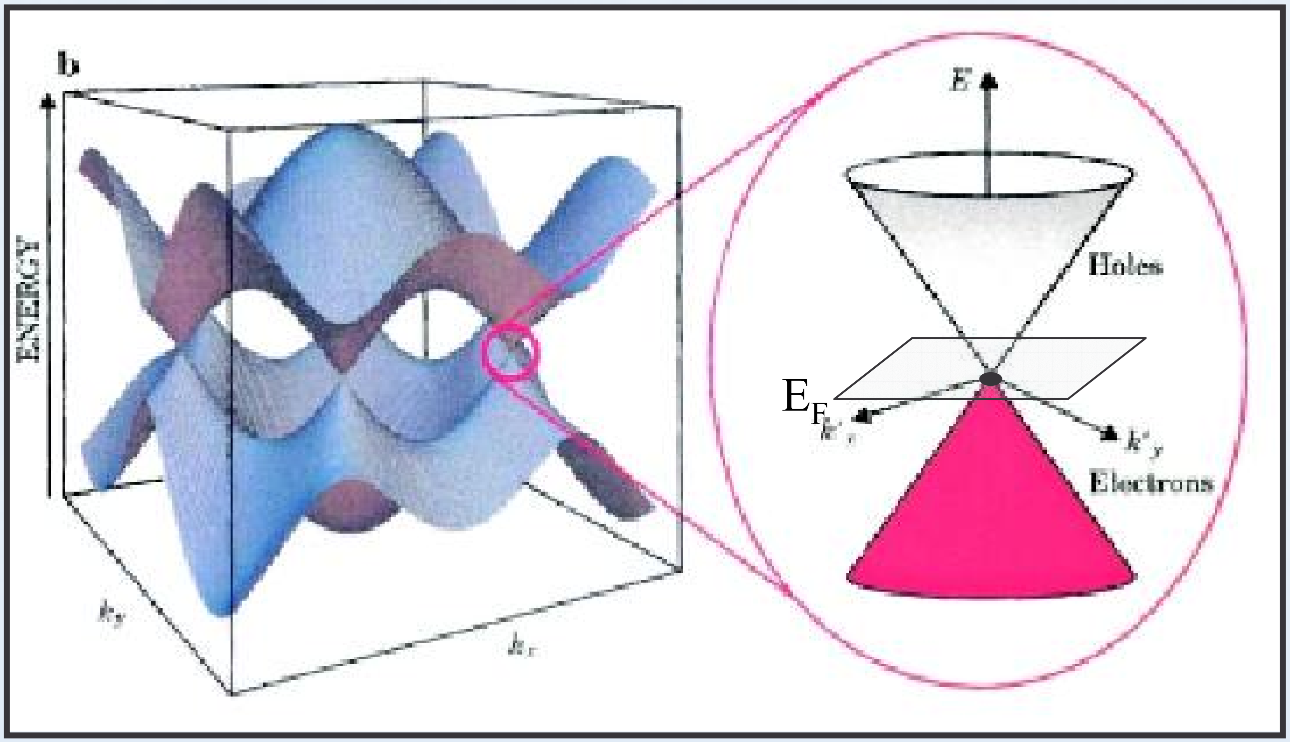
$$H = -t \sum_{n_1 n_2 = n.n.} |n_1\rangle\langle n_2|$$

$$t \approx 2.5\text{-}3 \text{ eV}$$

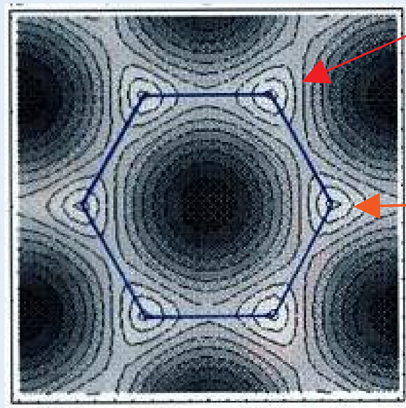
$$-t\begin{pmatrix} 0 & 1+2\cos\frac{k_x a_0}{2}e^{i\frac{\sqrt{3}}{2}k_y a_0} \\ 1+2\cos\frac{k_x a_0}{2}e^{-i\frac{\sqrt{3}}{2}k_y a_0} & 0 \end{pmatrix}\begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix} = \varepsilon \begin{pmatrix} \psi_A \\ \psi_B \end{pmatrix}$$

A and B sublattice sites in unit cell

- For each **k** there are eigenvalues at ±|ε| ⇒ particle-hole symmetry
- Fermi energy at ε=0



Two inequivalent *Dirac points* in Brillouin zone



Valley Index

$$\vec{K}' = \frac{2\pi}{a}\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right) \qquad (\tau = -1)$$

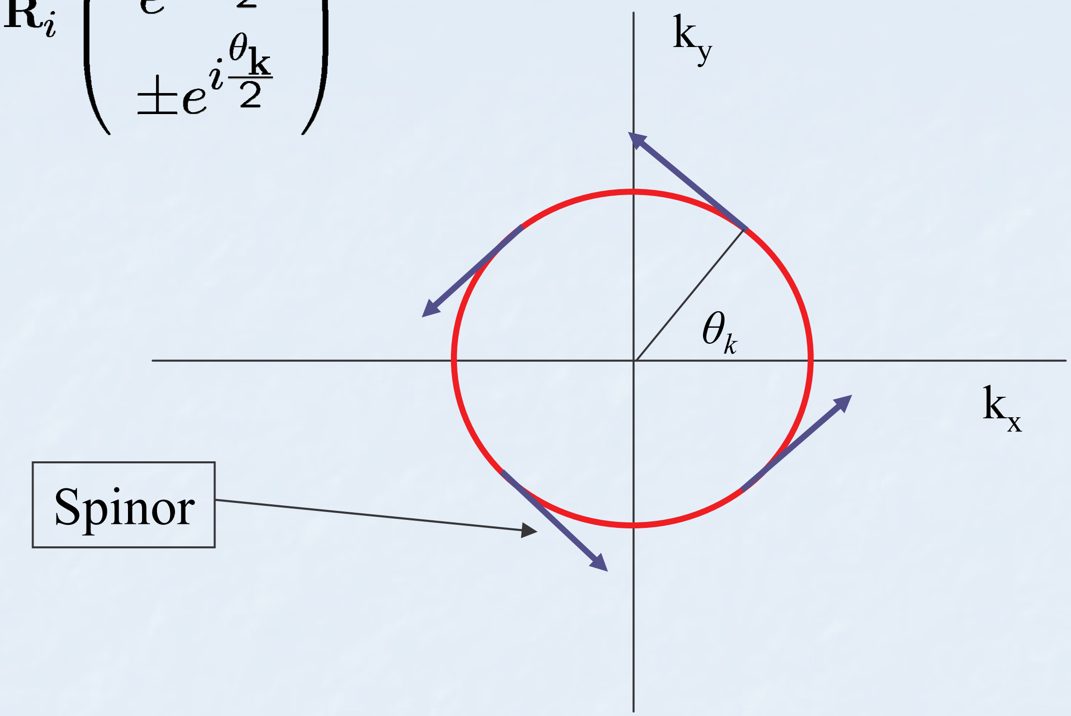
$$\vec{K} = \frac{2\pi}{a}\left(\frac{2}{3}, 0\right) \qquad (\tau = +1)$$

Near: **τ = +1** **τ = -1**

$$H_0 = -a_0 \frac{\sqrt{3}}{2} t \begin{pmatrix} 0 & k_x - ik_y \\ k_x + ik_y & 0 \end{pmatrix} \qquad H_0 = -a_0 \frac{\sqrt{3}}{2} t \begin{pmatrix} 0 & -k_x - ik_y \\ -k_x + ik_y & 0 \end{pmatrix}$$

$$\psi_{(\tau=+,\mathbf{k})}(i) = e^{i\mathbf{K}\cdot\mathbf{R}_i} e^{i\mathbf{k}\cdot\mathbf{R}_i} \begin{pmatrix} e^{-i\frac{\theta_{\mathbf{k}}}{2}} \\ \pm e^{i\frac{\theta_{\mathbf{k}}}{2}} \end{pmatrix} \qquad \psi_{(\tau=-,\mathbf{k})}(i) = e^{i\mathbf{K}'\cdot\mathbf{R}_i} e^{i\mathbf{k}\cdot\mathbf{R}_i} \begin{pmatrix} e^{i\frac{\theta_{\mathbf{k}}}{2}} \\ \mp e^{-i\frac{\theta_{\mathbf{k}}}{2}} \end{pmatrix}$$

$$\psi_{(\tau=+,\mathbf{k})}(i) = e^{i\mathbf{K}\cdot\mathbf{R}_i} e^{i\mathbf{k}\cdot\mathbf{R}_i} \begin{pmatrix} e^{-i\frac{\theta_{\mathbf{k}}}{2}} \\ \pm e^{i\frac{\theta_{\mathbf{k}}}{2}} \end{pmatrix}$$



Spinor for a particle dragged around in k space along a path surrounding Dirac point undergoes 2π rotation

$\Rightarrow$ Berry's phase $e^{2\pi i/2} = -1$

Singularity at k=0 $\Rightarrow$ density of states vanishes as $E_F \rightarrow 0$

$\Rightarrow$ Elastic scattering lifetime diverges in this limit

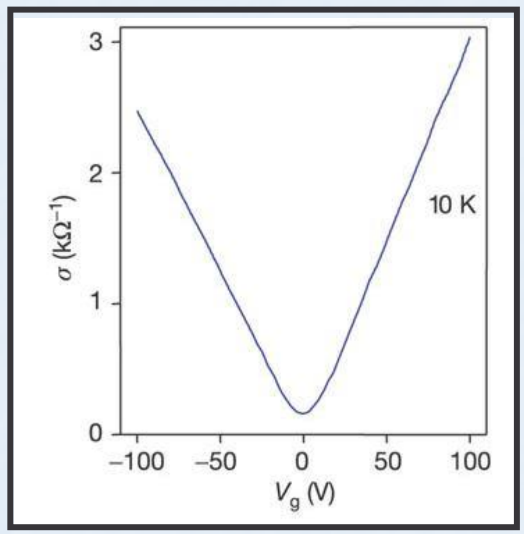
One interesting consequence: minimum metallic conductivity

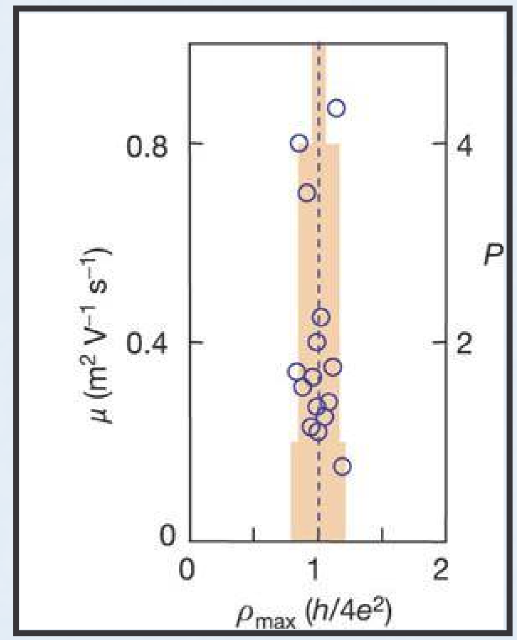
Boltzmann equation: $$\sigma \sim \frac{ne^2 \tau v_F}{\hbar k_F}$$

$$\frac{\tau}{k_F} \sim E_F^{-2}, \quad n \sim E_F^2$$

$\Rightarrow \sigma \to$ const. as $E_F \to 0$

Graphene is a surprisingly good conductor even when undoped!





From Novoselov et al., Nature 2005

Quantum Hall Effect

A. Quantum states in a magnetic field

1. k·P approximation: Write $\psi(\mathbf{r})=e^{i\mathbf{K}^{(\prime)}\cdot\mathbf{r}}\varphi(\mathbf{r})$
2. Peierls substitution:

$$k_\mu \to -i\partial_\mu \to -i\partial_\mu - \frac{e}{c}A_\mu$$

3. Choose gauge:

$$A = (-By,0,0) \Rightarrow \quad b^+ = \frac{\ell}{\sqrt{2}}\left(i\partial_y + y\ell^2 - ik_x\right) \quad \text{with } \ell = \sqrt{\frac{\hbar c}{eB}}$$

$$\sqrt{3}\,\frac{a}{\ell}t\begin{pmatrix} 0 & b \\ b^+ & 0 \end{pmatrix}\begin{pmatrix} \phi_A(\vec{r}) \\ \phi_B(\vec{r}) \end{pmatrix} = \varepsilon\begin{pmatrix} \phi_A(\vec{r}) \\ \phi_B(\vec{r}) \end{pmatrix} \qquad \tau=+1$$

$$\sqrt{3}\,\frac{a}{\ell}t\begin{pmatrix} 0 & -b^+ \\ -b & 0 \end{pmatrix}\begin{pmatrix} \phi_A(\vec{r}) \\ \phi_B(\vec{r}) \end{pmatrix} = \varepsilon\begin{pmatrix} \phi_A(\vec{r}) \\ \phi_B(\vec{r}) \end{pmatrix} \qquad \tau=-1$$

Wavefunctions:

$$\Psi(+,n) = e^{ik_x x}\begin{pmatrix} \pm\phi_{n-1}(y-k_x\ell^2) \\ \phi_n(y-k_x\ell^2) \end{pmatrix} \qquad \Psi(-,n) = e^{ik_x x}\begin{pmatrix} \pm\phi_{n}(y-k_x\ell^2) \\ \phi_{n-1}(y-k_x\ell^2) \end{pmatrix}$$

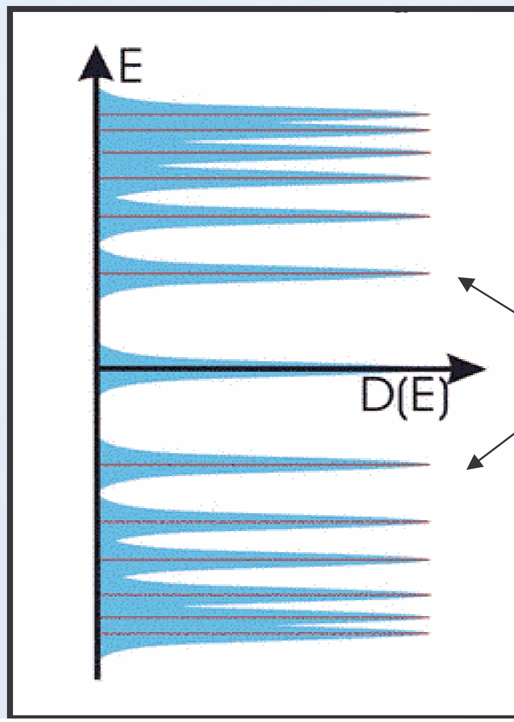
$$\Psi(+,0) = e^{ik_x x}\begin{pmatrix} 0 \\ \phi_0 \end{pmatrix} \qquad \Psi(-,0) = e^{ik_x x}\begin{pmatrix} \phi_0 \\ 0 \end{pmatrix}$$

ϕ_n = harmonic oscillator state

Energies:

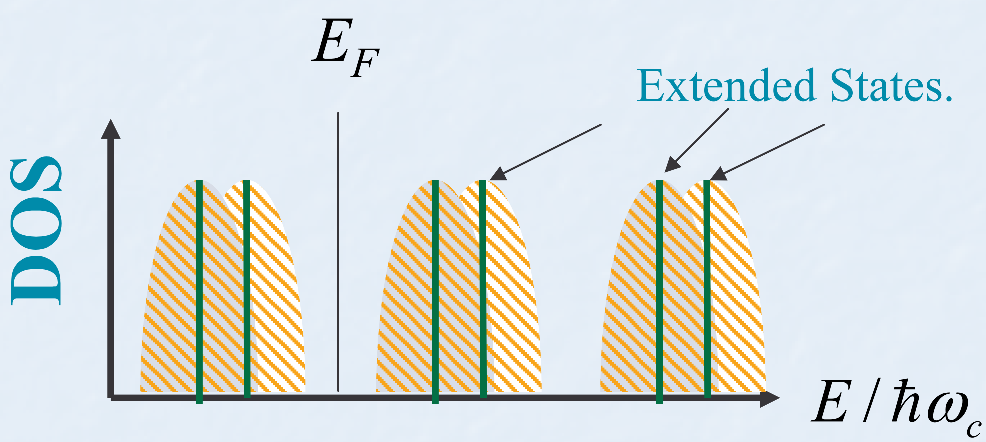
$$\varepsilon(\tau,n) = \pm\sqrt{3\,n}\,\frac{a}{\ell}t$$

With valley and spin indices, each Landau level is 4-fold degenerate



Particle-hole conjugates

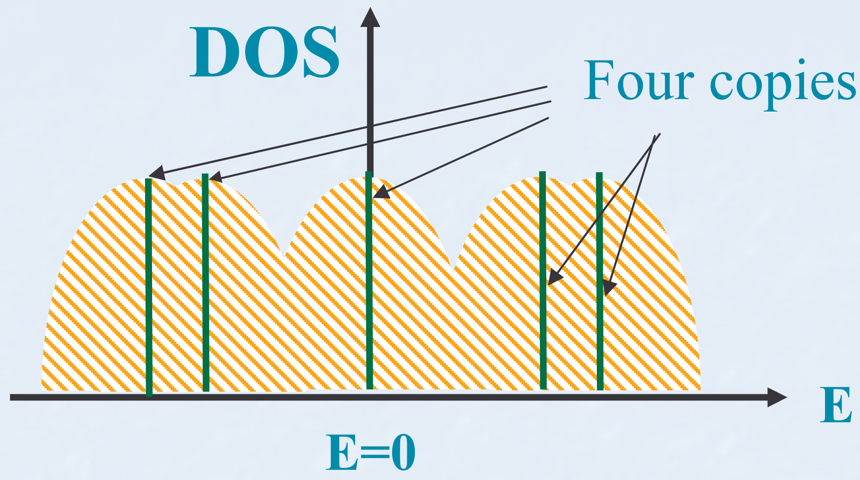
Landau levels have a macroscopic degeneracy $S/(2\pi l^2)$. DOS consists of a set of delta functions. Disorder broads the LL and localize states. There is at least one extended state at the center of each LL. Localized states do not contribute to conduction properties.



$$\sigma_{xy} = \pm \frac{e^2}{h} N$$

N is the number of Landau levels with extended states below the Fermi energy.

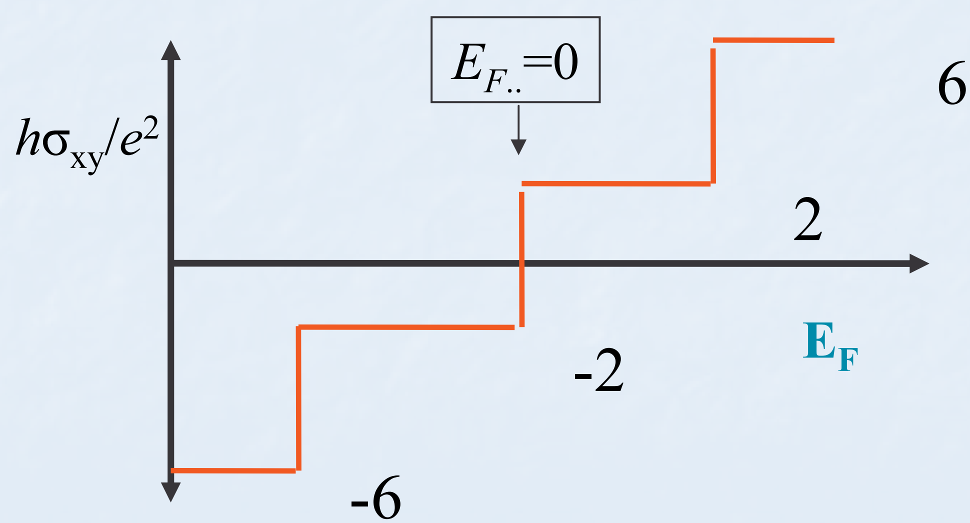
Quantum Hall Effect in Graphene



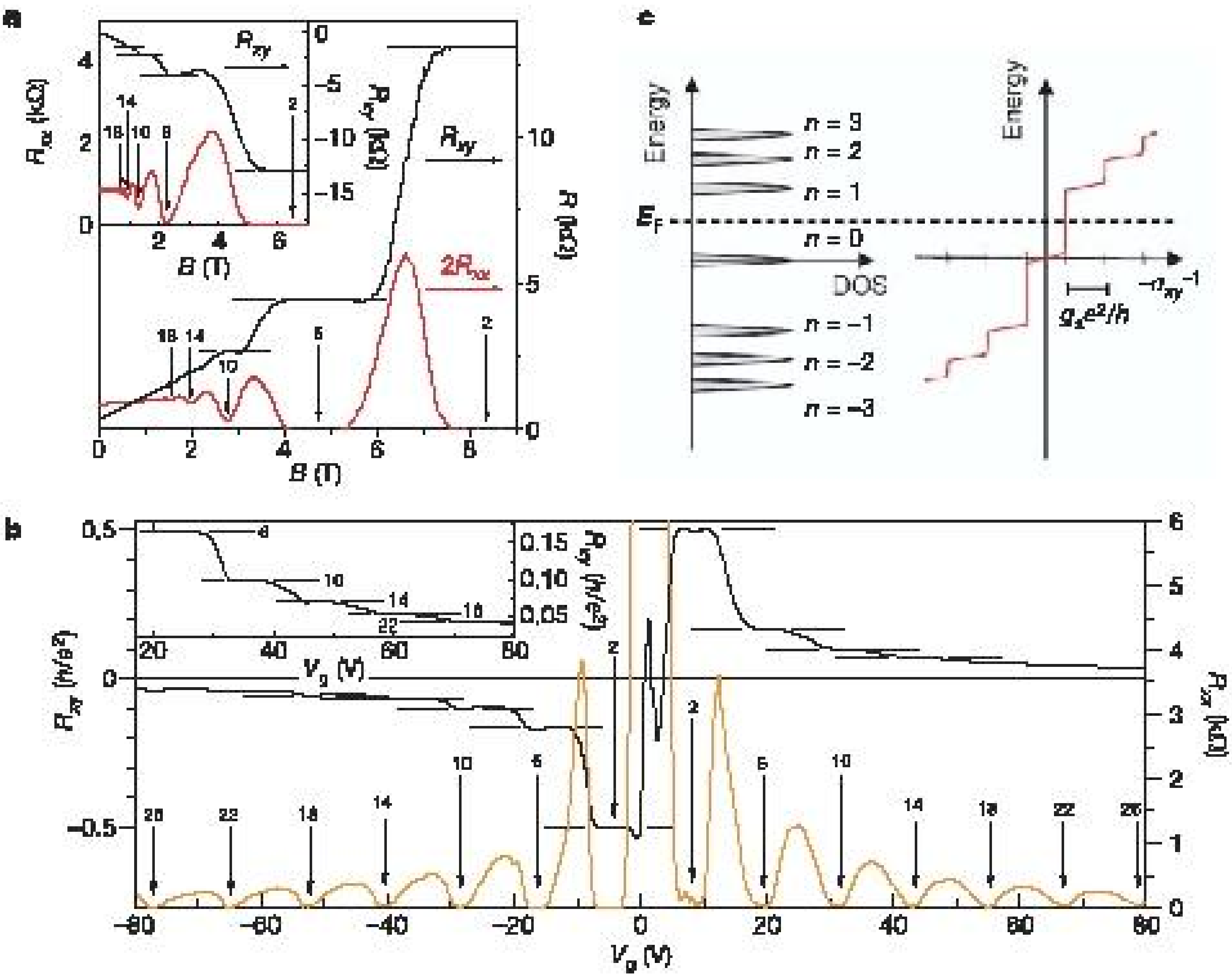
Problem: Within Dirac equation, there are infinitely many occupied Landau levels for any E_F.

Solution: Use particle-hole symmetry. For $E_F=0$, $\sigma_{xy}=0$.

So we expect:



$$\sigma_{xy} = \pm 2\frac{e^2}{h}(2N+1)$$

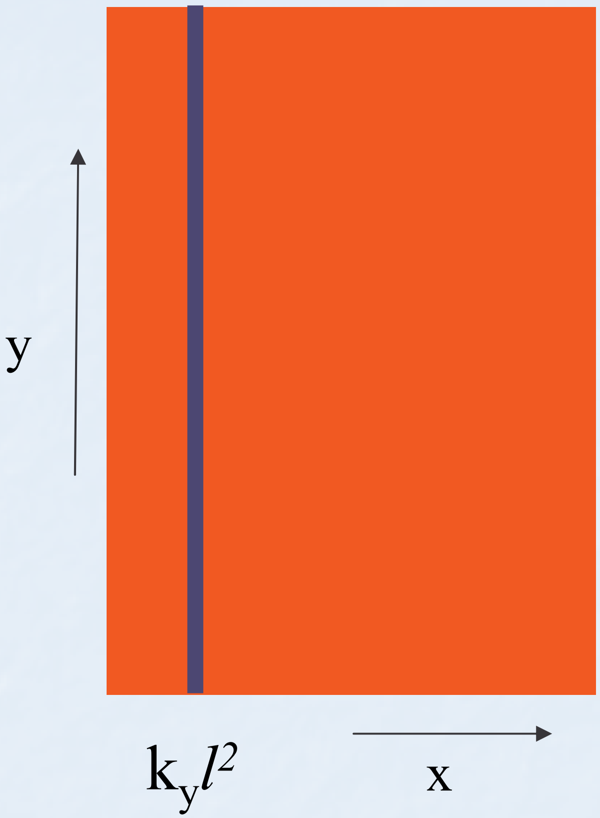


From Zhang et al., Nature (2005)

III. Quantum Hall Effect II: Edge States

- Real samples in experiments are very narrow (.1-1μm) $\Rightarrow$ edges can have a major impact on transport
- Can get a full description of QHE within Dirac equation
- Edge structure can be probed directly via STM at very small length scales. Nothing comparable is possible in standard 2DEG's (GaAs samples, Si MOSFET's)

A. Edge States in Standard 2DEG Systems

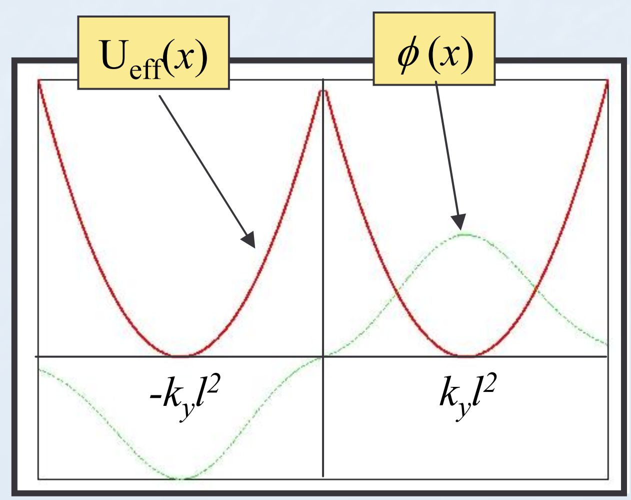


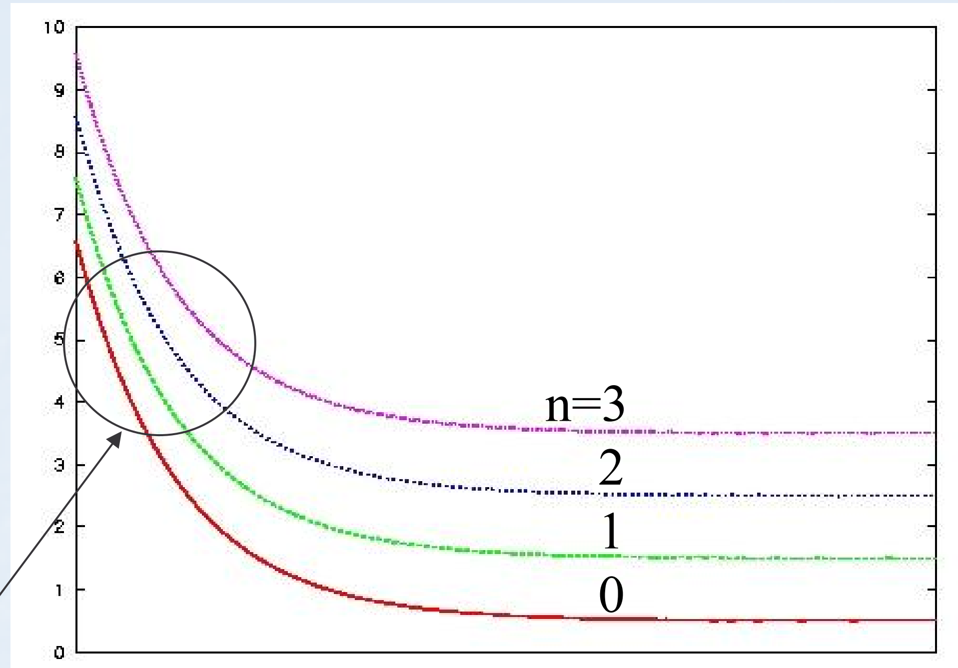
Consider a standard 2DEG, edge at $x = 0$,
infinite in y-direction.
Adopt Landau gauge $\mathbf{A}$=B(0,x,0).
With $\psi(x,y)=e^{ik_y y}\phi(x)$,

$$\left[-\ell^2\partial_x^2 + (x - k_y\ell^2)^2/\ell^2\right]\phi(x) = \frac{\varepsilon}{\hbar\omega_c}\phi(x)$$

Impose boundary condition $\phi(x)$=0

Solve double well problem
and choose antisymmetric
wavefunctions
(Halperin, 1982)





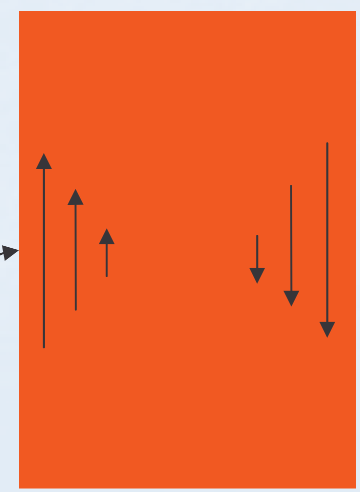
$\frac{\varepsilon}{\hbar\omega_c}$

$k_y l^2$

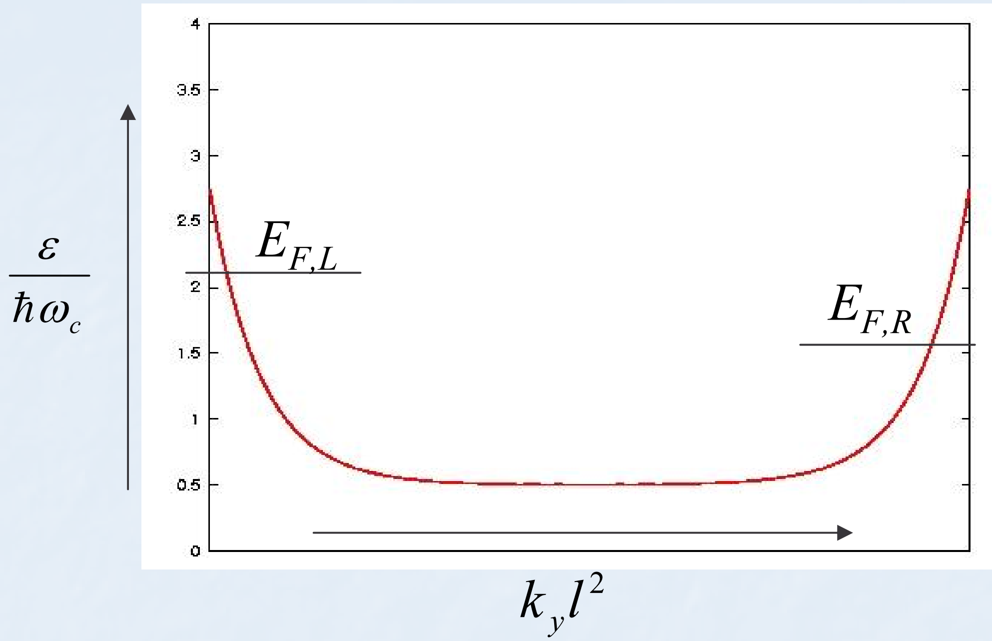
Edge electrons have a group velocity

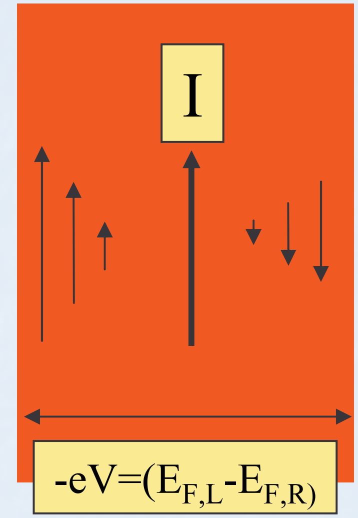
$$v_n(k_y) = \frac{1}{\hbar}\frac{d\varepsilon_n}{dk_y}$$

$\Rightarrow$ Edge currents, even in equilibrium



This explains quantization of the Hall effect.





For each occupied Landau level,

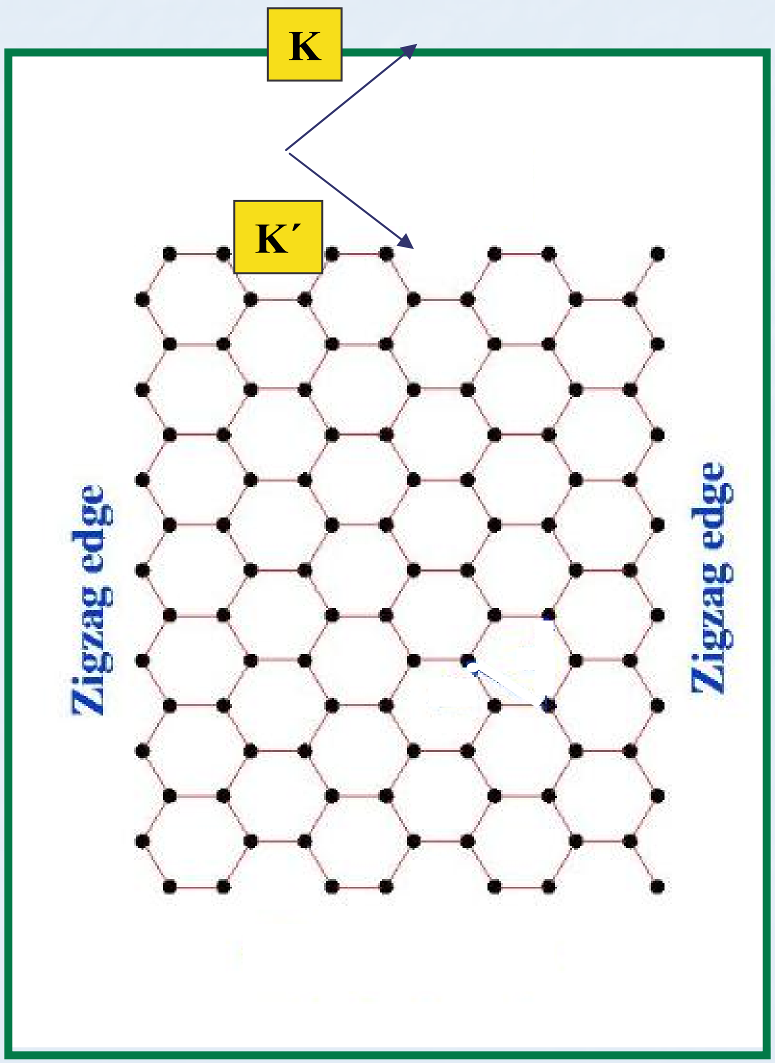
$$I = -e \int v(k_y) \frac{dk_y}{2\pi} = \frac{e}{h} \int_{E_{F,R}}^{E_{F,L}} dE = -\frac{e}{h} \left[E_{F,L} - E_{F,R} \right] = \frac{e^2}{h} V$$

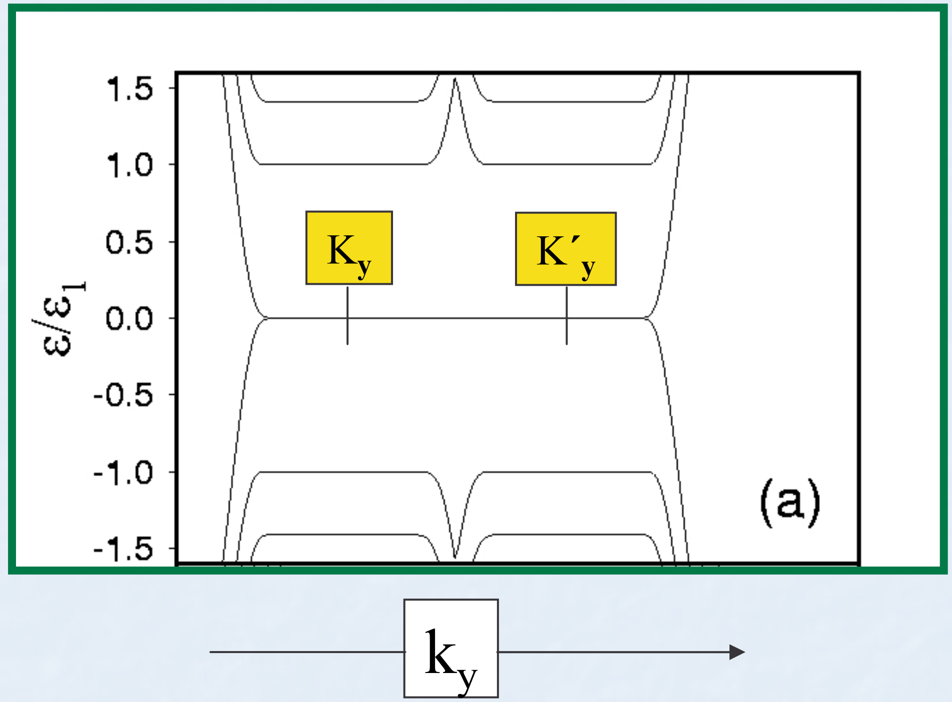
Hall resistance: $R_H = V/NI = h/Ne^2$

with N the number of edge state pairs crossed by E_F

B. Edge States in Graphene

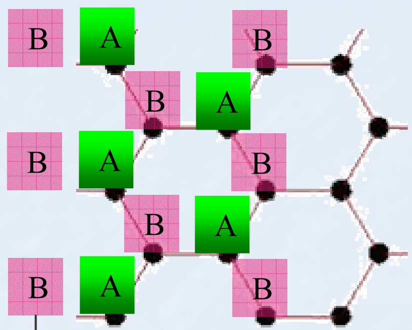
(1) Tight-binding results, zigzag edge





What happened to the n=0 edge states in the center?

Edge states from Dirac equation:



Boundary condition:
$\phi_B(x=0)=0$

Missing row of atoms all from same sublattice

Acting on wavefunctions *twice* with Hamiltonian gives scalar equations:

$$\frac{2\gamma^2 a_0^2}{\ell^2} b^\dagger b \varphi_B = \varepsilon^2 \varphi_B$$

$$\varphi_A = b\varphi_B/\varepsilon$$

$$\frac{2\gamma^2 a_0^2}{\ell^2} b b^\dagger \varphi'_B = \varepsilon^2 \varphi'_B$$

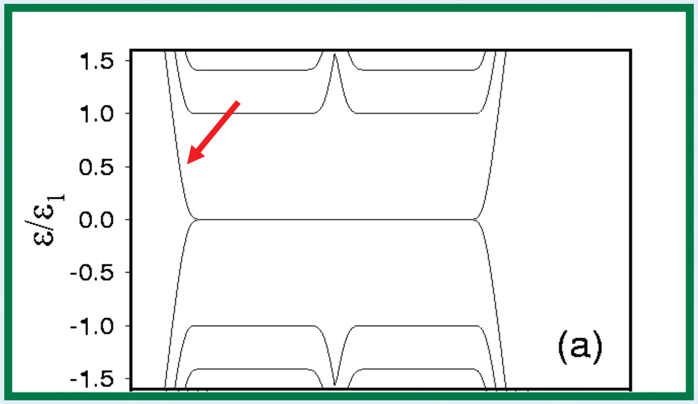
$$\varphi'_B = -b\varphi'_A/\varepsilon$$

$$\gamma = \frac{\sqrt{3}}{2} t$$

- For high Landau levels, problem essentially identical to GaAs case
- For n=0, **K** and **K′** wavefunctions behave differently

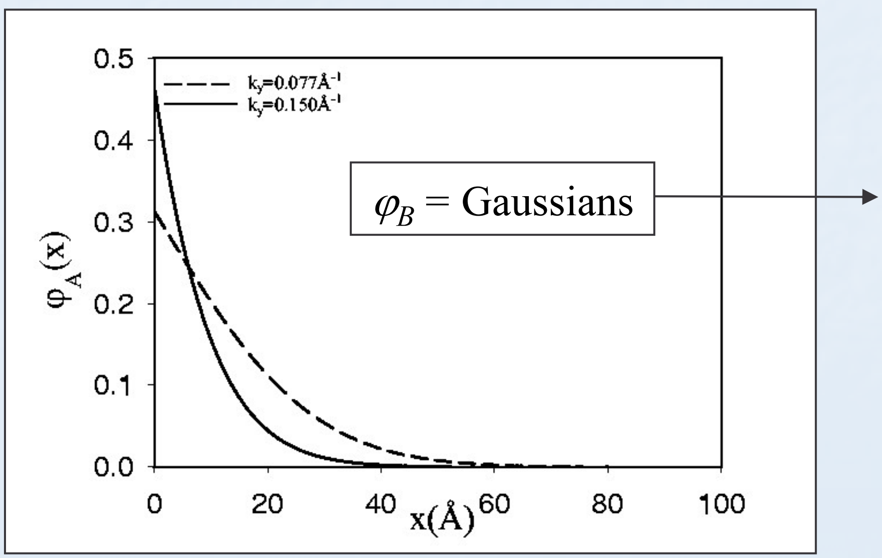
$$\frac{2\gamma^2 a_0^2}{\ell^2} b^\dagger b \varphi_B = \varepsilon^2 \varphi_B$$

$$\varphi_A = b\varphi_B/\varepsilon$$

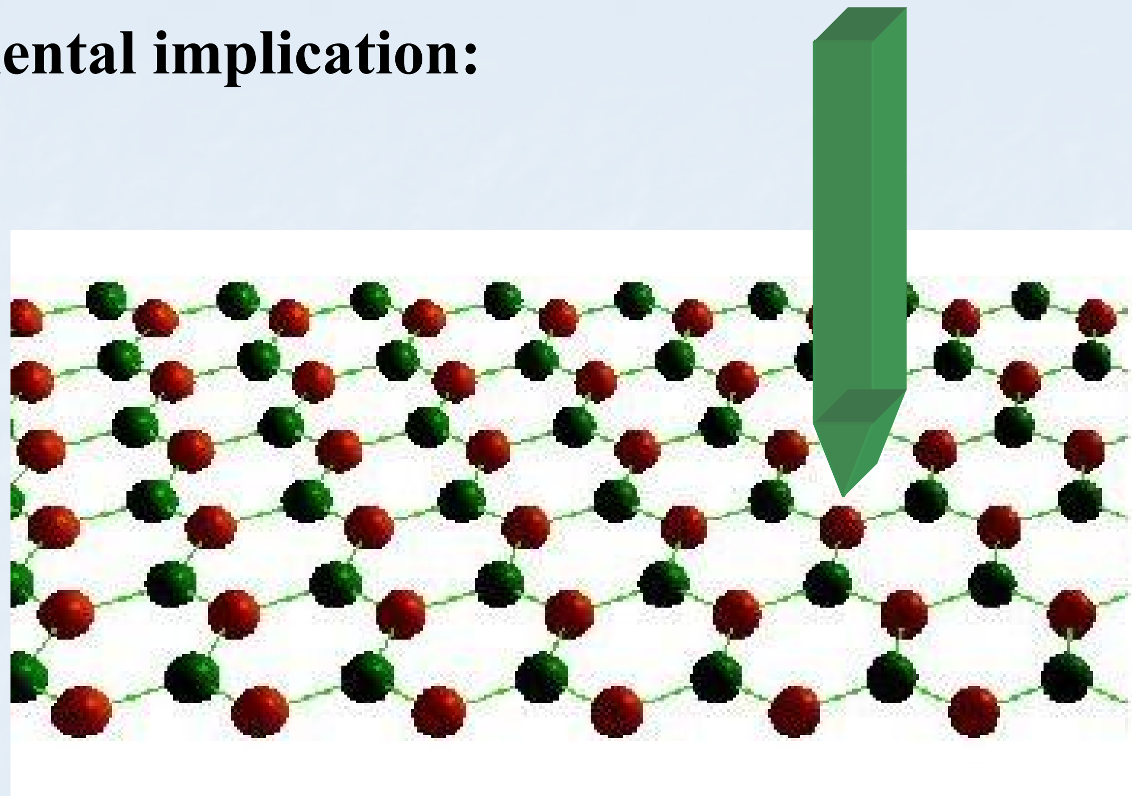


K-valley: Current Carrying Edge States

- For $k_y l^2$ not too close to edge, φ_B very similar to a bulk LLL state. Only significant difference is at x=0.
- Since b annihilates an LLL state, φ_A strongly localized near x=0!
- When $k_y l^2$ close to edge, tunneling between A and B becomes significant and hole-like and particle-like states disperse.



Possible experimental implication:

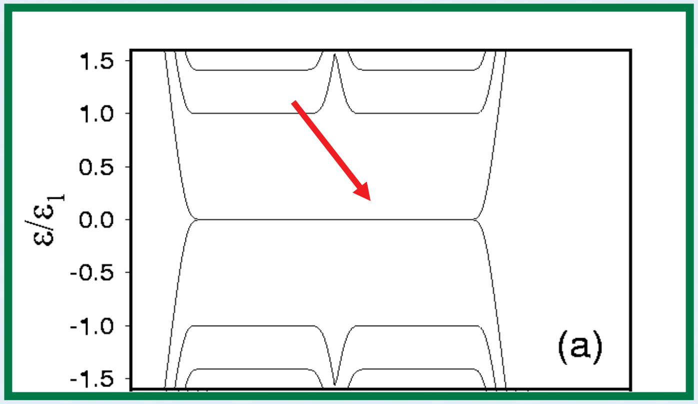


May be able to observe oscillations when injecting electrons with an STM tip near a zigzag edge.
Oscillation period depends on distance from edge.

K′-valley: Dispersionless Edge States

$$\frac{2\gamma^2 a_0^2}{\ell^2} b b^\dagger \varphi'_B = \varepsilon^2 \varphi'_B$$

$$\varphi'_B = -b\varphi'_A/\varepsilon$$

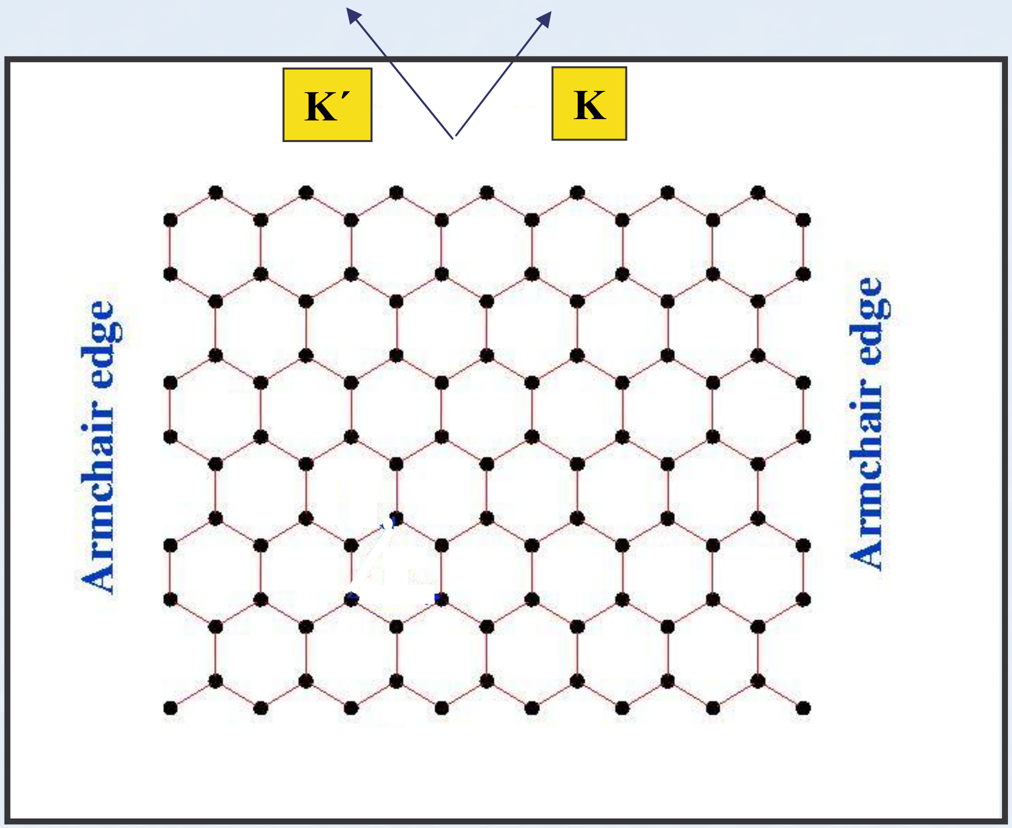


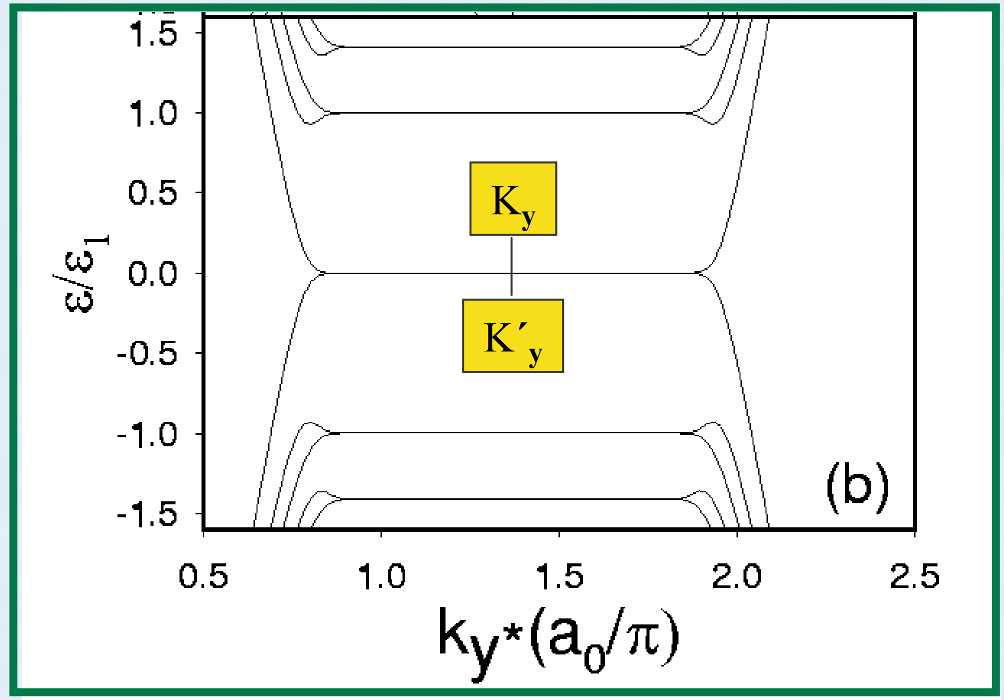
- Equations satisfied for φ'_B=0 and φ'_A a LLL state, even when guiding center is off edge of system ⇒ φ'_A the tail of a Gaussian, and $\varepsilon = 0$

- These states carry no current!

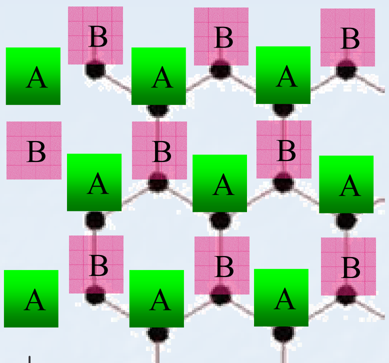
⇒ n=0 Hall conductance is half as large as steps for larger n

(2) Tight-binding results, armchair edge





Two sets of edge states for n>0, one for n=0.



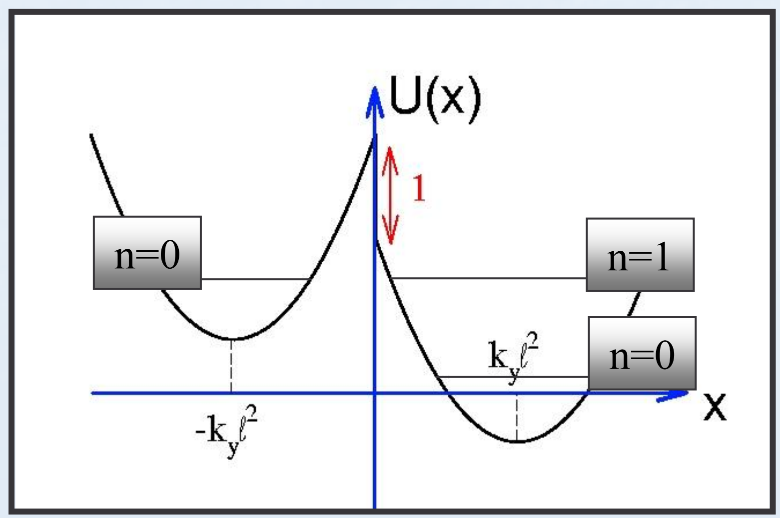
Boundary conditions:
$\phi_B(x=0)+\phi'_B(x=0)=0$
$\phi_A(x=0)+\phi'_A(x=0)=0$
Admixes valleys

Missing row of atoms with equal number of atoms from both sublattices

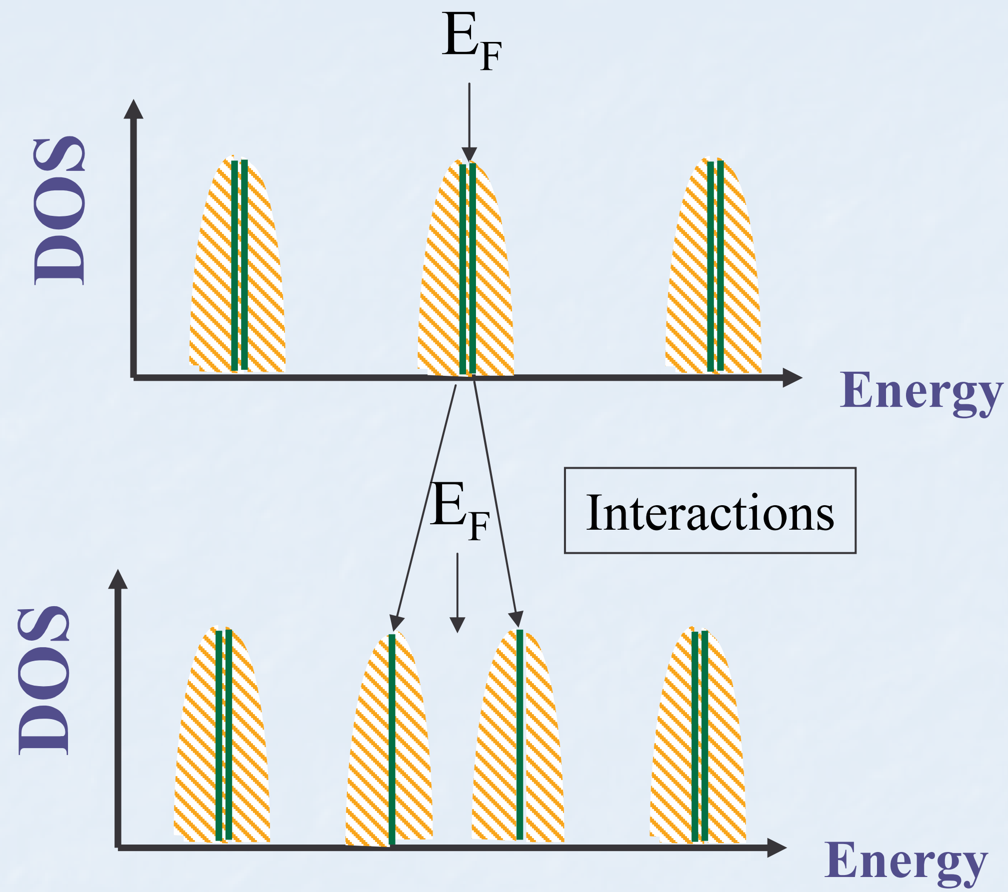
$$\varphi_A = b\varphi_B/\varepsilon$$
$$\varphi'_A = -b^\dagger\varphi'_B/\varepsilon$$
$$\Rightarrow \quad \partial_x\varphi_B(x=0) + \partial_x\varphi'_B(x=0) = 0$$

Trick: define wavefunction
for $-\infty < x < \infty$:

$$\Psi(x) = \varphi_B(x)\theta(x) + \varphi'_B(-x)\theta(-x)$$



IV. Quantum Hall Ferromagnetism and the Graphene Edge



- Exchange tends to force electrons into the same level even when bare splitting between levels is small
- Renormalizes gap to much larger value than expected from non-interacting theory (even if bare gap is zero!)

This does happen in graphene (Zhang et al., 2006).

- Plateaus at ν=0,±1.
- System is a *pseudospin ferromagnet*.

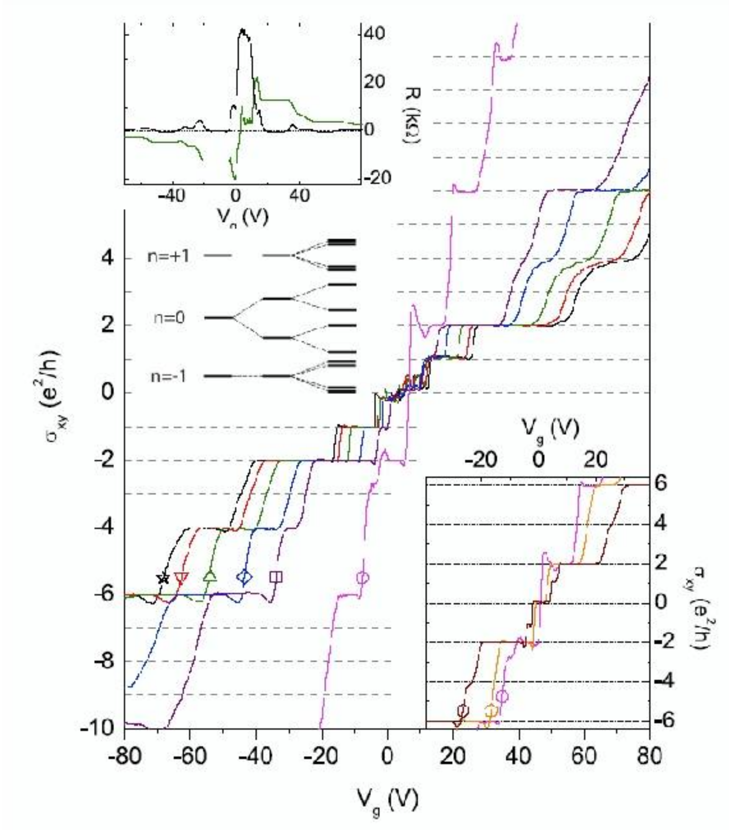
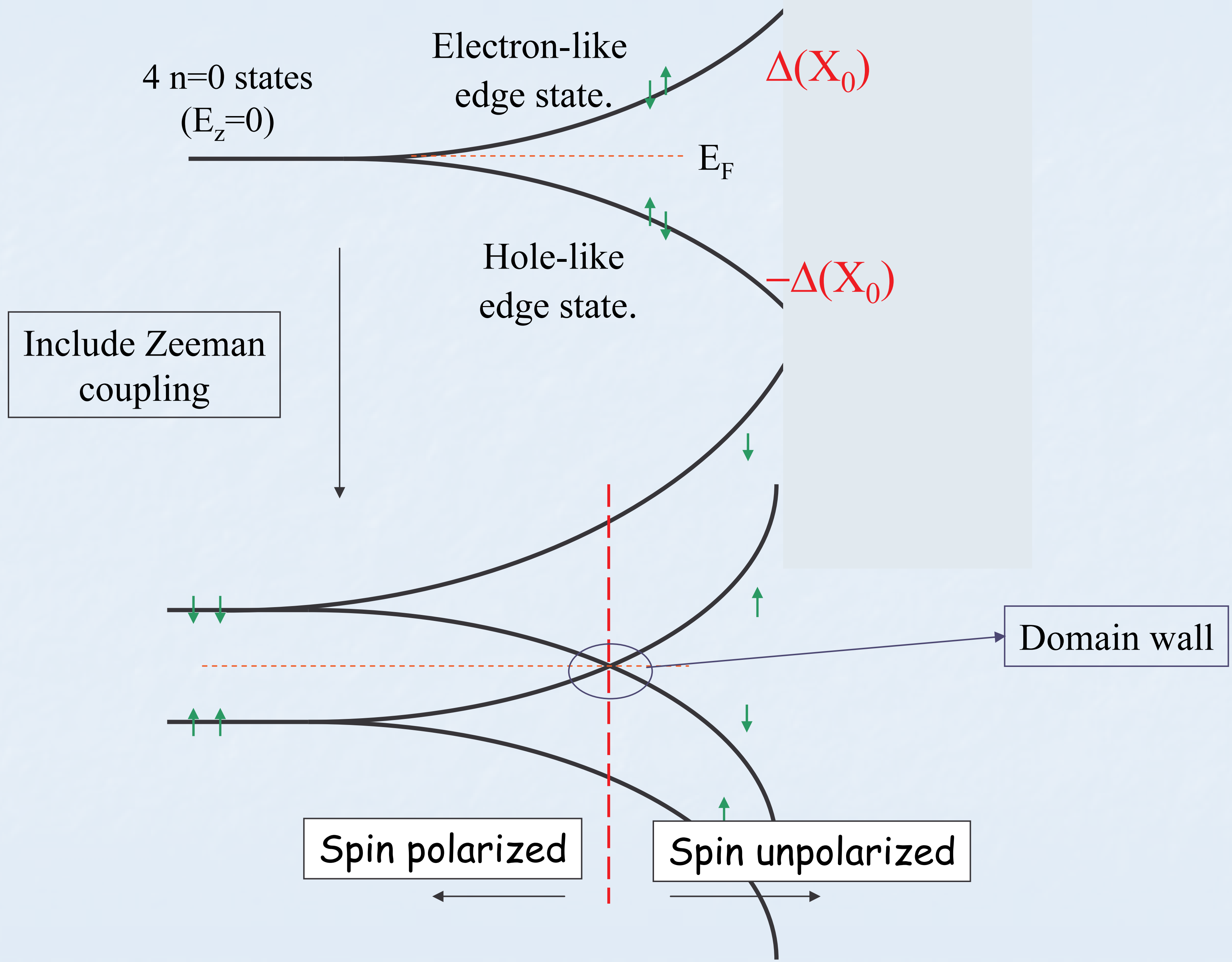


FIG 2. (color online) σ_{xy}, as a function of V_g at different magnetic fields: 9 T (circle), 25 T (square), 30 T (diamond), 37 T (up triangle), 42 T (down triangle), and 45 T (star). All the data sets are taken at $T = 1.4$ K, except for the $B = 9$T curve, which is taken at $T = 30$ mK. Left upper inset: R_{xx} and R_{xy} for the same device measured at $B = 25$ T. Left lower inset: a schematic drawing of the LLs in low (left) and high (right) magnetic field. Right inset: detailed σ_{xy} data near the Dirac point for B = 9 T (circle), 11.5 T (pentagon) and 17.5 T (hexagon) at $T = 30$ mK.

Consequences for edge states:



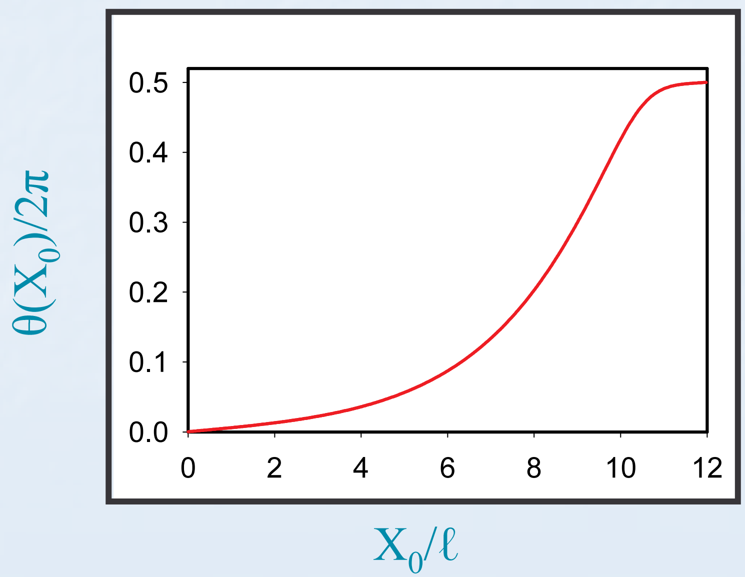
Description of the domain wall:

$$|\Psi\rangle = \prod_{X_0<L}\left[\cos\frac{\theta(X_0)}{2}C^{+}_{+,X_0,\uparrow} + \sin\frac{\theta(X_0)}{2}C^{+}_{-,X_0,\downarrow}\right]C^{+}_{-,X_0,\uparrow}|0>$$

$$X_0 \to -\infty \quad \theta = 0; \quad X_0 \to L \quad \theta = \pi$$

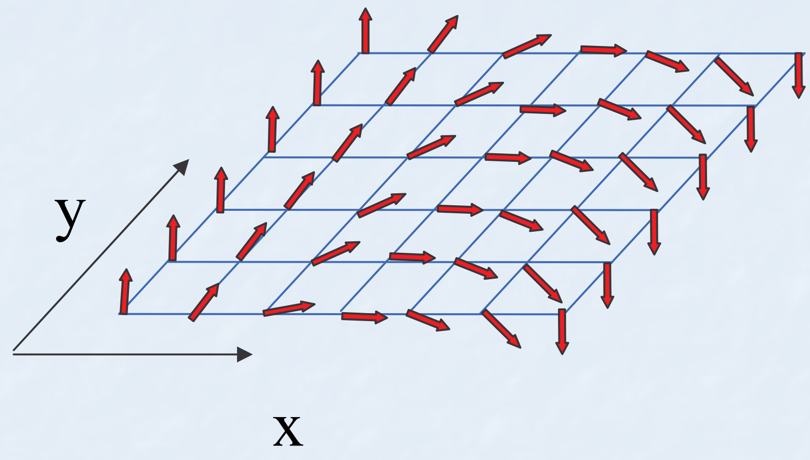
$$E = \pi\ell^2 \rho_s \sum_{X_0<L}\left(\frac{d\theta}{dX_0}\right)^2 + \sum_{X_0<L}\left(E_z - \Delta(X_0)\right)\cos\theta(X_0)$$

Pseudospin stiffness

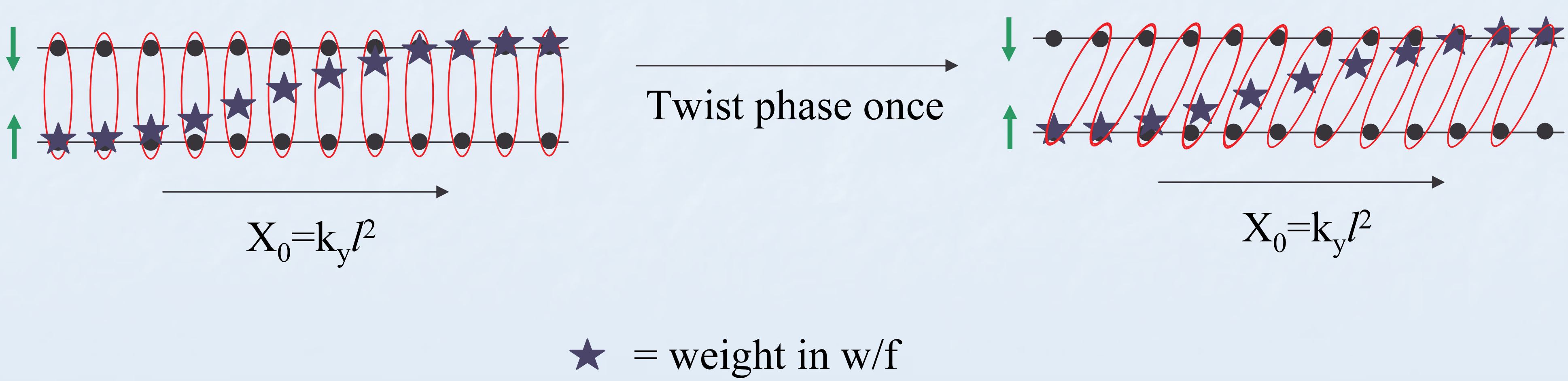


Result of minimizing energy. Width of domain wall set by strength of confinement.

Properties of the domain wall



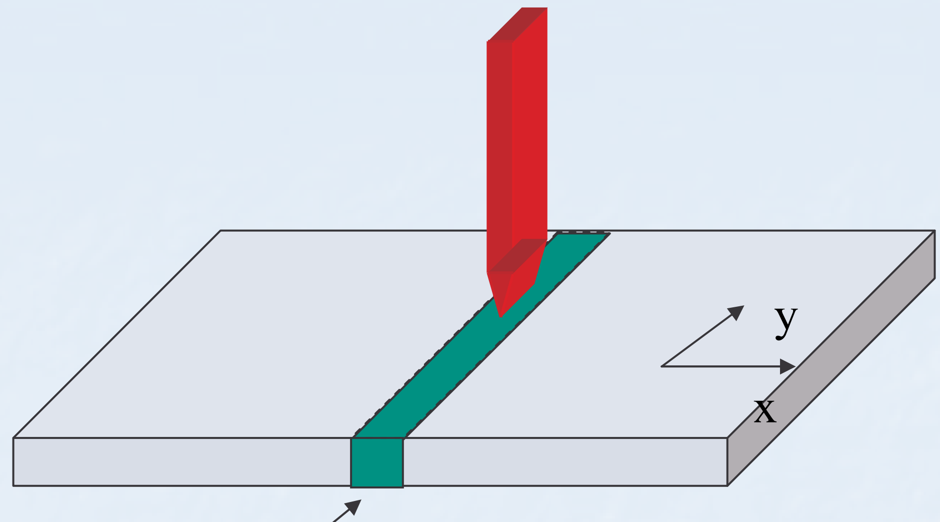
1. Broken U(1) symmetry ⇒ gapless collective mode
2. Spin-charge coupling ⇒ gapless charged excitations!



3. Tunneling from STM tip: power law IV

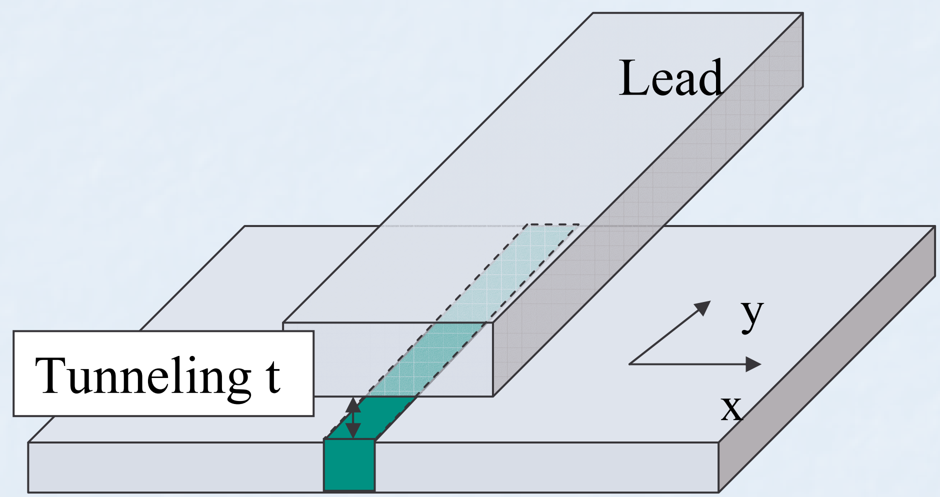
⇒ not a Fermi liquid!

Power law exponent a function of confinement potential



4. Tunneling from a bulk lead:

possibility of a quantum phase transition (into 3D metal).



$$\frac{dt^2}{dl} = -(\kappa - 2)t^2 \quad \kappa = (x + 1/x)/2 \quad x = 4\pi\sqrt{\rho/\Gamma}$$

ρ = U(1) spin stiffness

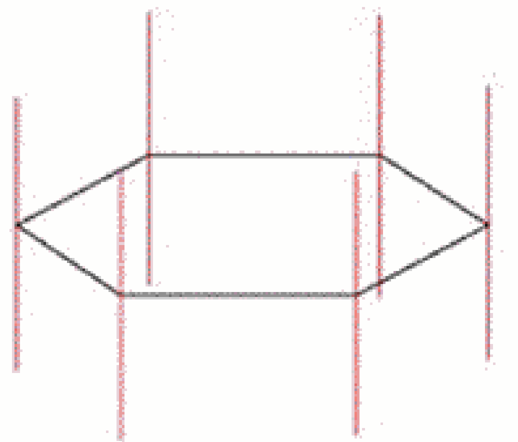
Γ ~ confinement potential

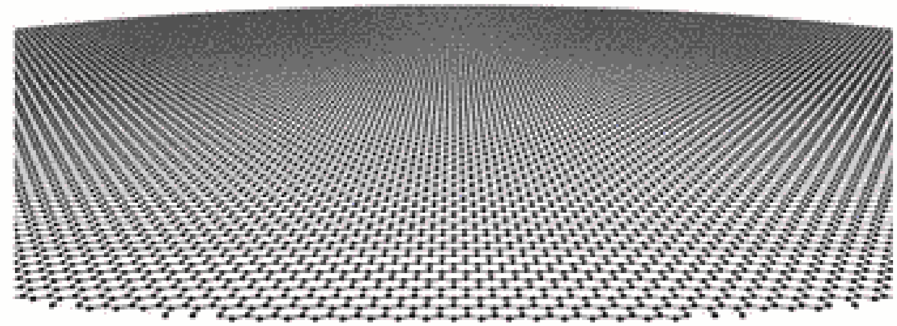
Summary

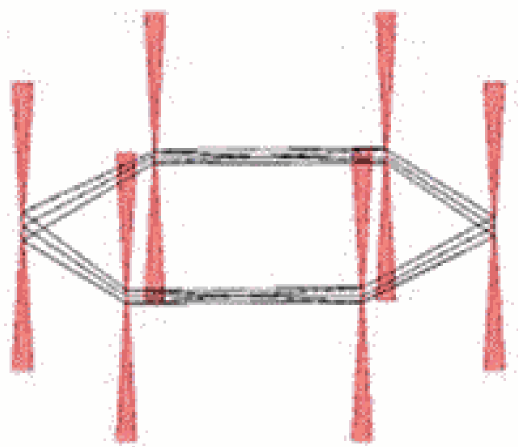
- Graphene: a new and interesting material both for fundamental and applications reasons.
- Single particle states reflect Berry's phase.
- Unusual quantized Hall effect due to Berry's phase.
- Edge states: Lowest Landau level contributes only half the the Hall conductivity compared to other Landau levels.
- Zigzag edges: Current-carrying and dispersionless edge states
- Armchair edges: Admixture of two valleys.
- Graphene is a pseudospin ferromagnet.
- ν=0 edge supports a coherent domain wall with gapless excitations.
- Coherent domain wall has unusual properties when probed by tunneling.

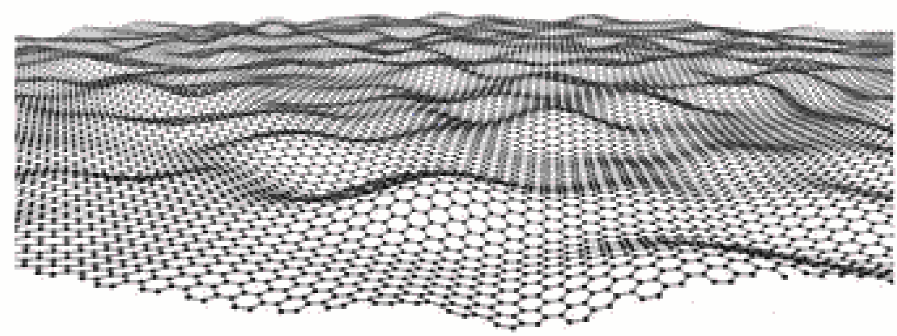
Structure Of Suspended Graphene

reciprocal space | *real space*

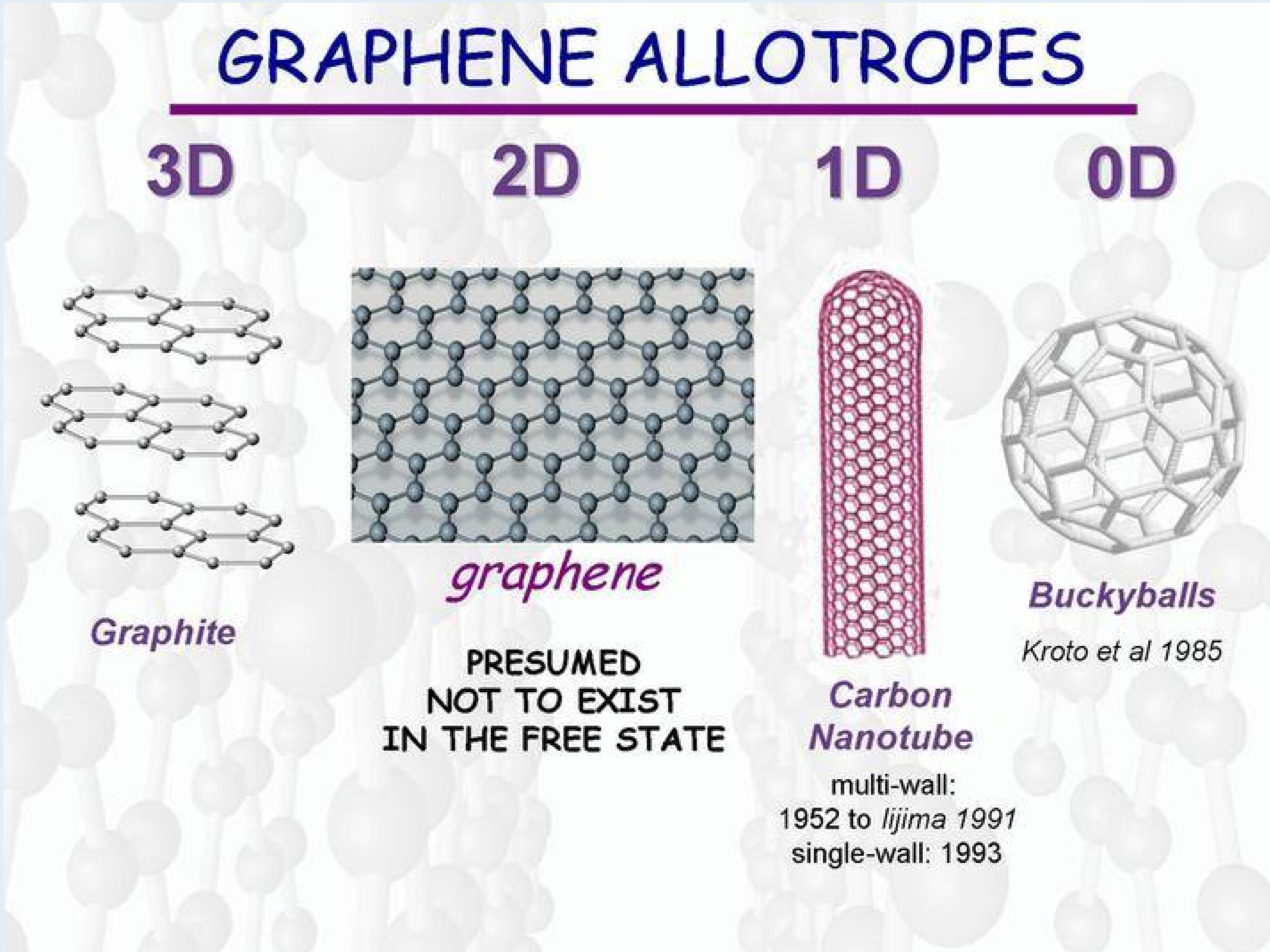








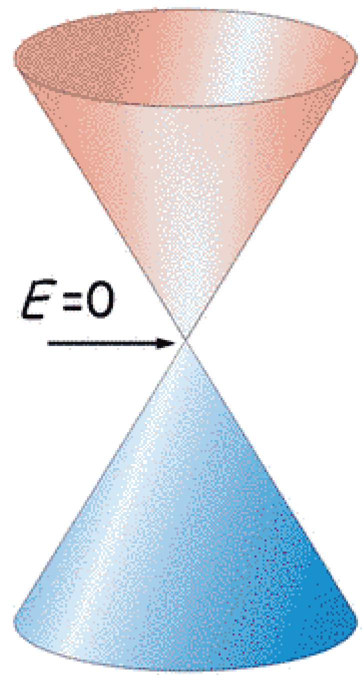
cones in Fourier space rather than rods



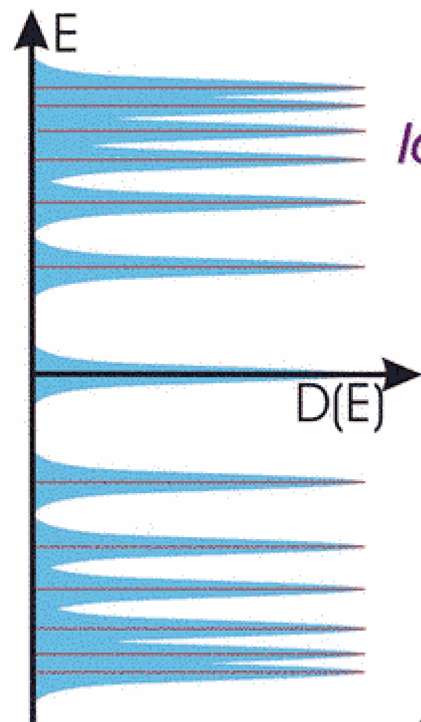
GRAPHENE ALLOTROPES
3D
2D
1D
0D
graphene
Graphite
PRESUMED NOT TO EXIST IN THE FREE STATE
Carbon Nanotube
multi-wall: 1952 to Iijima 1991
single-wall: 1993
Buckyballs
Kroto et al 1985

quantization of Dirac fermions

$E = \hbar v_F k$



$$E = \pm\sqrt{2\hbar v_F^{\,2} eBN}$$

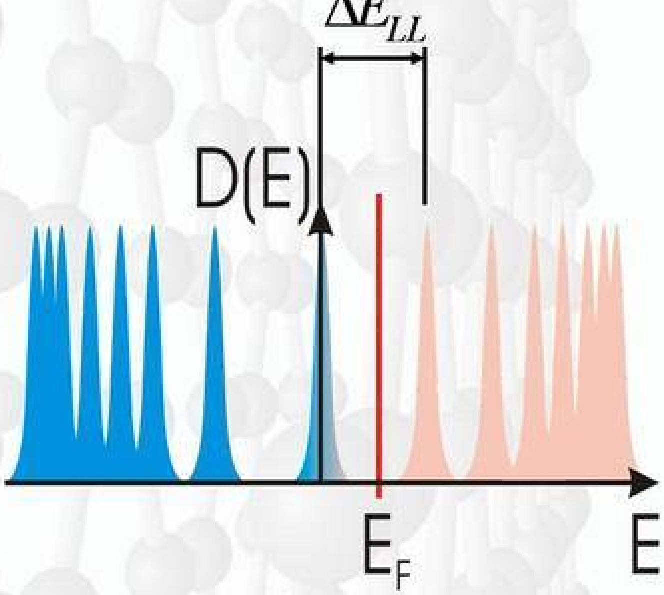


lowest Landau level at ZERO energy

$\Delta n = 4B/\varphi_0$

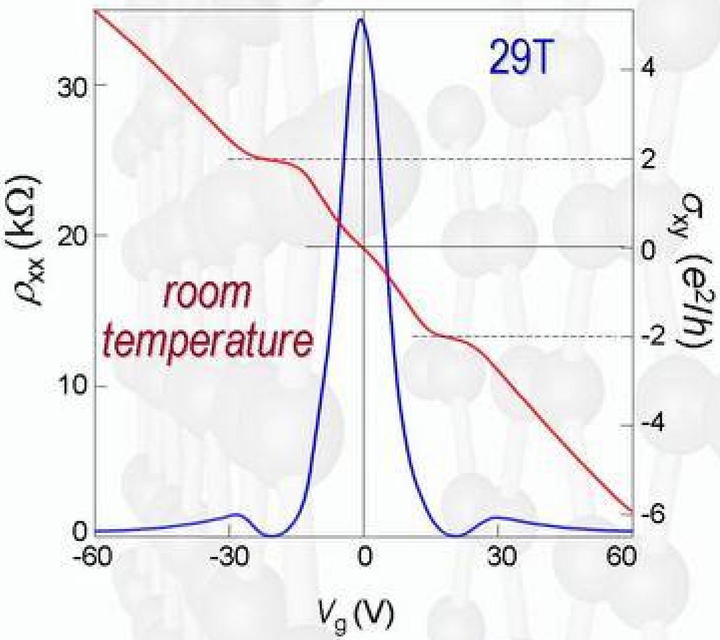
McClure 1956
Semenoff 1984
Haldane 1988
Gonzalez 1993
.....
Gusynin 2005; Peres 2006
.....

room-temperature QHE



$$\Delta E_{LL} = v_F \sqrt{2e\hbar B}$$

$$\Delta E_{LL}(K) = 420\sqrt{B(T)}$$



previously, only below 30K