Ordering and Freezing Transitions in Classical Geometrically Frustrated Antiferromagnets

John Chalker
Physics Department, Oxford University

Work with Tom Pickles and Tim Saunders

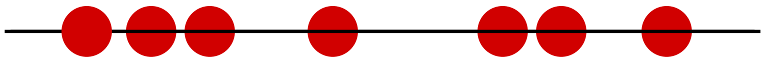
cond-mat/0612458 and *In preparation*

Some Contexts

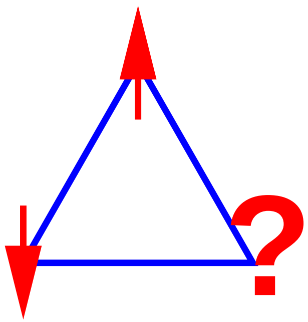
Highly constrained systems

Novel order from
degeneracy + constraints

- Partially filled Landau levels



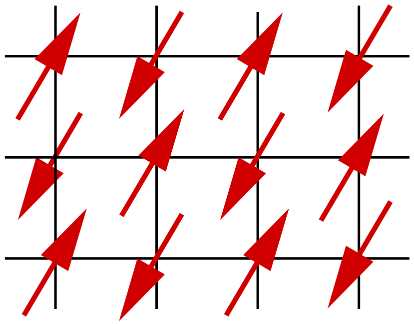
- Frustrated magnets

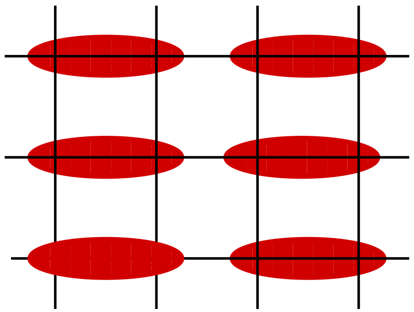


Critical Phenomena

Landau-Ginzburg-Wilson
and beyond

- Deconfined quantum criticality





- Power-law correlated states

  2D: Kosterlitz-Thouless

  3D: Coulomb phases

Outline

- **Geometrically frustrated antiferromagnets**

  Frustration in spin systems

  Models and lattices

  Ground state degeneracy

  Correlations within the ground state manifold

- **Phase transitions within the correlated manifold**

  Ordering from the Coulomb phase

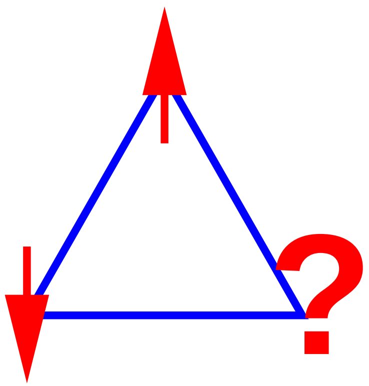
  Spin freezing

Types of frustration in magnetic systems

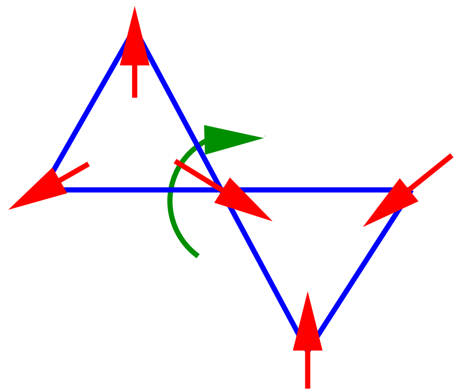
With quenched disorder

- in spin glasses

From competition



From geometry



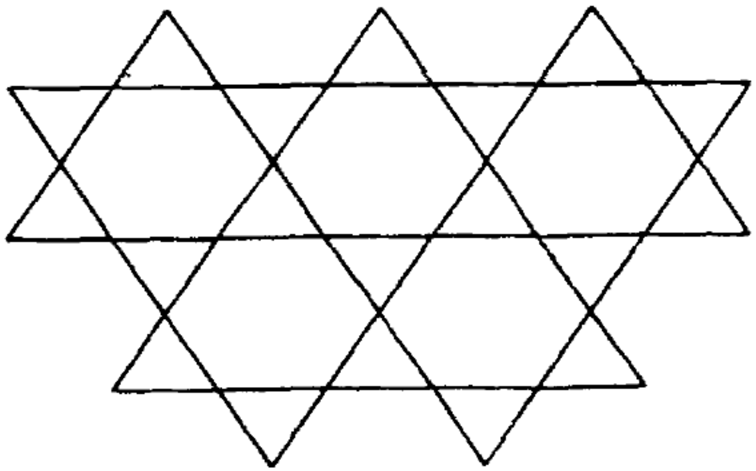
structure $\longrightarrow$ degeneracy

Anderson 1956, Villain 1977

Examples of frustrated lattices

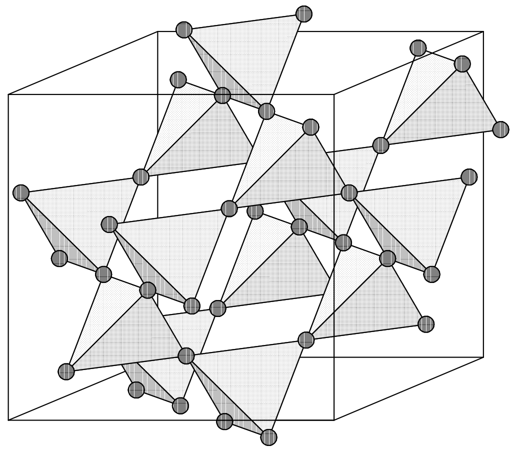
Building block: corner-sharing frustrated units of q spins

2D: kagome lattice



Triangles: $q = 3$

3D: pyrochlore lattice



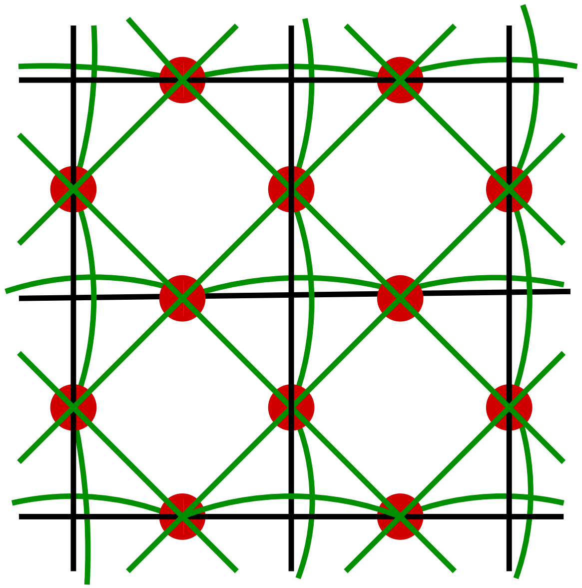
Tetrahedra: $q = 4$

Versions for theorists

In d-dimensions:

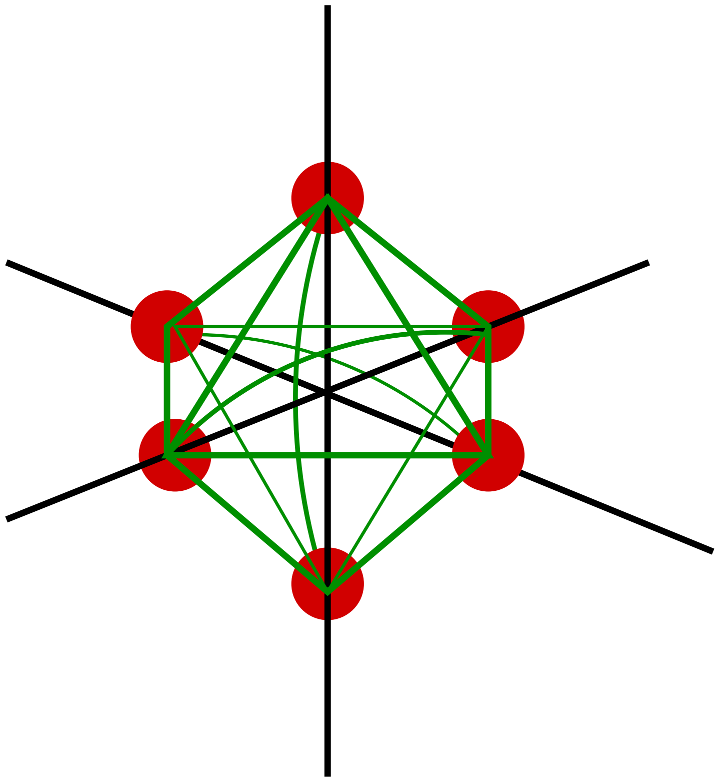
corner-sharing units of $2d$ spins on hypercubic lattice

Two dimensions



Corner-sharing ‘tetrahedra’

Three dimensions

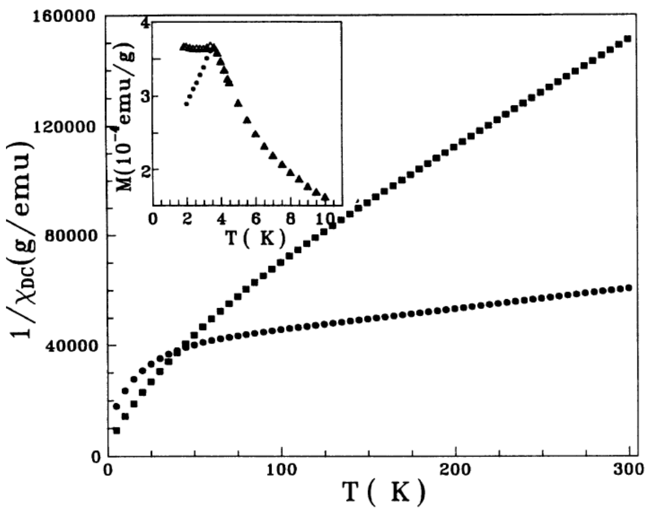


Corner-sharing octahedra

Characteristics of geometrically frustrated antiferromagnets

$\mathrm{SrGa_{12-x}Cr_xO_{19}}$ (SCGO) as an example

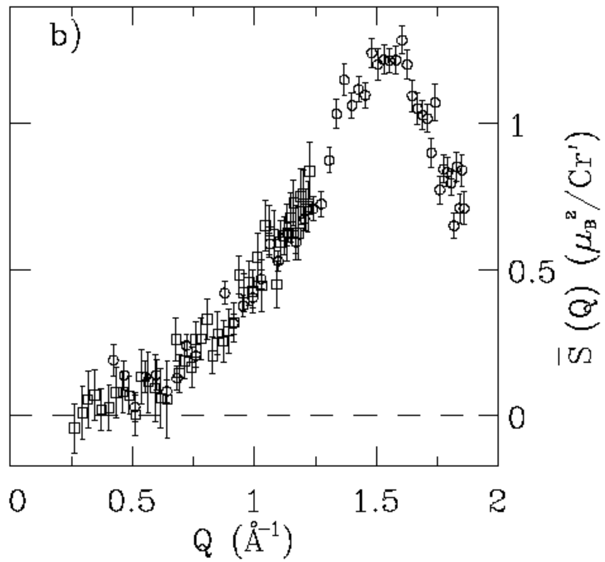
Paramagnetic even for $T \ll |\Theta_{CW}|$



χ^{-1} vs T

Martinez et al, PRB **46**, 10786 (1992)

Strong short-range correlations



Elastic neutron scattering

S.H. Lee et al, Europhys Lett **35**, 127 (1996)

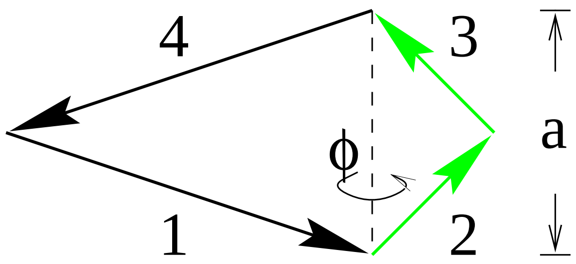
Ground state degeneracy in Heisenberg AFM

Single tetrahedron

$$\mathcal{H} = J \sum_{\text{pairs}} \mathbf{S}_i \cdot \mathbf{S}_j \equiv \frac{J}{2} |\mathbf{L}|^2 + c$$

with

$$\mathbf{L} = \mathbf{S}_1 + \mathbf{S}_2 + \mathbf{S}_3 + \mathbf{S}_4$$



frustration $\longrightarrow$ accidental degeneracy

Full lattice

$$\mathcal{H} = J \sum_{\text{bonds}} \mathbf{S}_i \cdot \mathbf{S}_j \equiv \frac{J}{2} \sum_{\text{units}} |\mathbf{L}_\alpha|^2 + c$$

Total number of degrees of freedom:

$$F = 2 \times (\text{number of spins})$$

Constraints satisfied in ground state:

$$K = 3 \times (\text{number of units})$$

Ground state dimension:

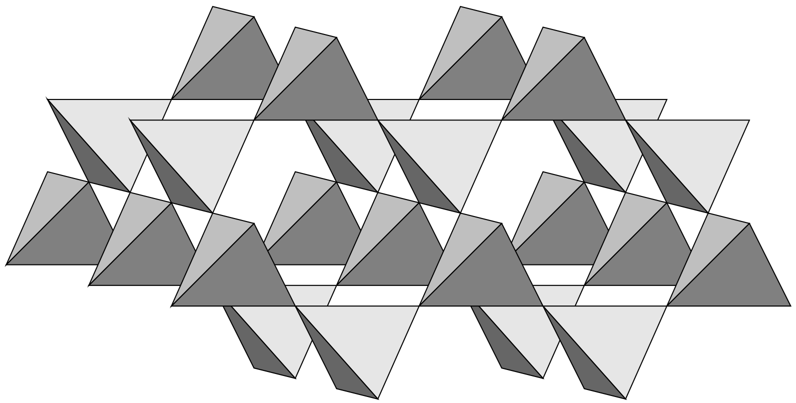
D=F-K

geometric frustration $\longrightarrow$ macroscopic D

Correlations induced by ground state constraints

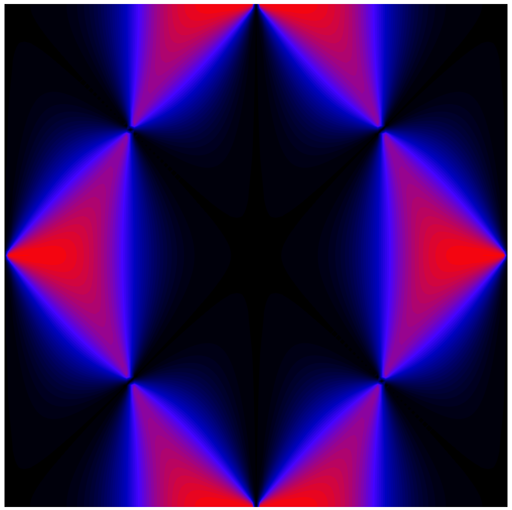
Local constraints

$$\sum_{tet} \mathbf{S}_i = 0$$



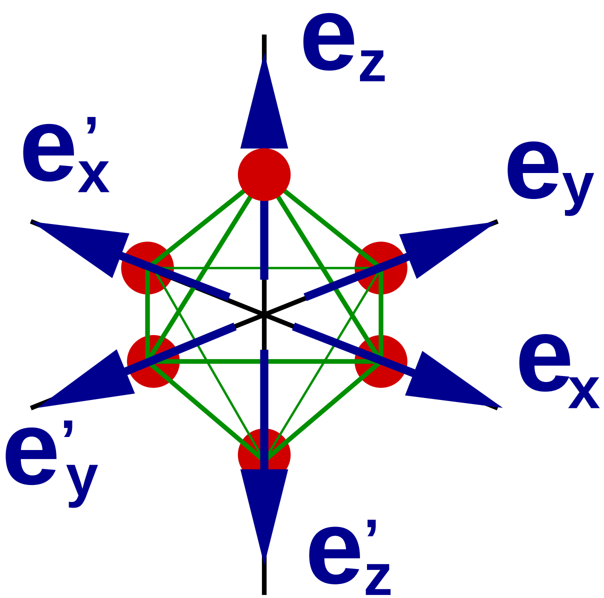
Long range correlations

Sharp structure in
$\langle \mathbf{S}_{-\mathbf{q}} \cdot \mathbf{S}_{\mathbf{q}} \rangle$



Gauge theory of ground state correlations

Youngblood *et al* (1980), Huse *et al* (2003), Hermele *et al* (2004); Henley (2004)



Construct vector fields $\mathbf{B}^a(\mathbf{r}_i)$

from each spin component S_i^a :

$$\mathbf{B}^a(\mathbf{r}_i) = \hat{e}_i S_i^a$$

$$\sum_{unit} S_i^a = 0 \rightarrow \nabla \cdot \mathbf{B}^a = 0$$

Ground state constraint

becomes flux conservation law:

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a$$

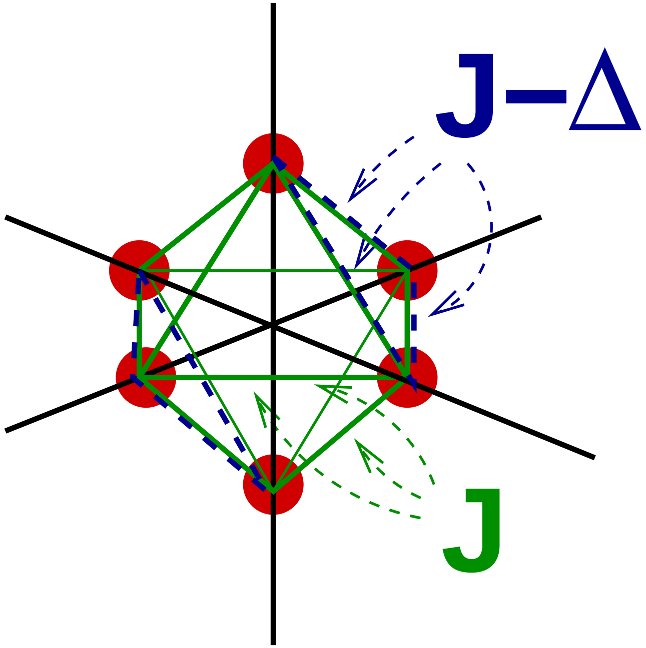
Coarse-grained distribution: $P[\mathbf{B}^a(\mathbf{r})] \propto \exp(-\kappa \int |\mathbf{B}^a(\mathbf{r})|^2)$

Dipolar correlators: $\langle B_i^l(\mathbf{r}) B_j^m(\mathbf{0})\rangle \propto \delta_{lm} \frac{3r_i r_j - r^2 \delta ij}{r^5}$

Ordering transitions from the Coulomb phase

Add interactions to select ordered state

Corner-sharing octahedra strained in $[111]$ direction



Constrain to Coulomb phase: $J \to \infty$

Study statistical mechanics vs T/Δ

Continuum description:

$$P[\mathbf{B}^a(\mathbf{r})] \propto \exp(-\mathcal{H})$$

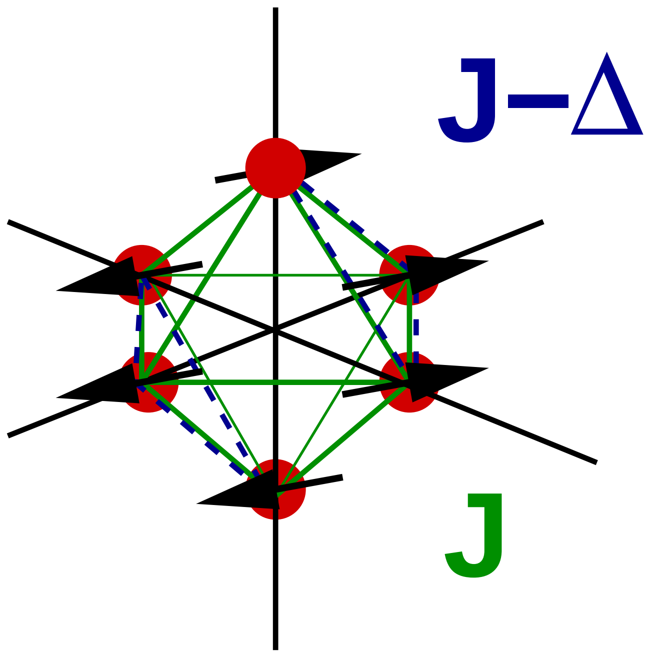
$$\mathcal{H} = \int \mathrm{d}^3\mathbf{r} \{ \kappa \sum_a \left[|\mathbf{B}^a_\perp(\mathbf{r})|^2 + t\,|B^a_{\|}(\mathbf{r})|^2 + c|\nabla \times \mathbf{B}^a(\mathbf{r})|^2 \right] + u \left[\sum_a |\mathbf{B}^a(\mathbf{r})|^2 \right]^2 \}$$

$$t \propto (T - T_c)$$

Ordering transitions from the Coulomb phase

Add interactions to select ordered state

Corner-sharing octahedra strained in $[111]$ direction



Constrain to Coulomb phase: $J \to \infty$

Study statistical mechanics vs T/Δ

Continuum description:

$$P[\mathbf{B}^a(\mathbf{r})] \propto \exp(-\mathcal{H})$$

$$\mathcal{H} = \int \mathrm{d}^3\mathbf{r} \{ \kappa \sum_a \left[|\mathbf{B}^a_\perp(\mathbf{r})|^2 + t\, |B^a_{\|}(\mathbf{r})|^2 + c|\nabla \times \mathbf{B}^a(\mathbf{r})|^2 \right] + u \left[\sum_a |\mathbf{B}^a(\mathbf{r})|^2 \right]^2 \}$$

$$t \propto (T - T_c)$$

Critical behaviour

Flux condensation transition

$$\mathcal{H} = \int \mathrm{d}^3\mathbf{r}\{\kappa \sum_a \left[|\mathbf{B}_\perp^a(\mathbf{r})|^2 + t\,|B_\parallel^a(\mathbf{r})|^2 + c|\nabla \times \mathbf{B}^a(\mathbf{r})|^2\right] + \ldots\}$$

Reduction to essentials: $\mathbf{B}^a = \nabla \times \mathbf{A}^a$ **with** $\nabla \cdot \mathbf{A}^a = 0$

Two modes $A^a_{\mathrm{crit}}, A^a_{\mathrm{noncrit}}$ **one critical. Rescale:** $qA^a_{\mathrm{crit}}(\mathbf{q}) = \varphi^a(\mathbf{q})$

Effective theory

$$\mathcal{H}_{\mathrm{eff}} = \int \mathrm{d}^3\mathbf{q} \sum_a \kappa\left[(1-t)\frac{q_\parallel^2}{q^2} + t + cq^2\right]|\varphi^a(\mathbf{q})|^2 + u\int\left[\sum_a |\varphi^a|^2\right]^2$$

Non-analytic dispersion $q_\parallel^2/q^2$ **equivalent to dipolar interactions**

Long-range forces $\longrightarrow$ **upper critical dimension is** $d_u = 3$

— hence critical behaviour (almost) mean field

[cf Larkin and Khemlnitskii, 1969]

Spin freezing in geometrically frustrated antiferromagnets

Freezing in experiment

Observed at low T

$$T_{\rm F} \ll |\Theta_{\rm CW}|$$

Possible source of disorder

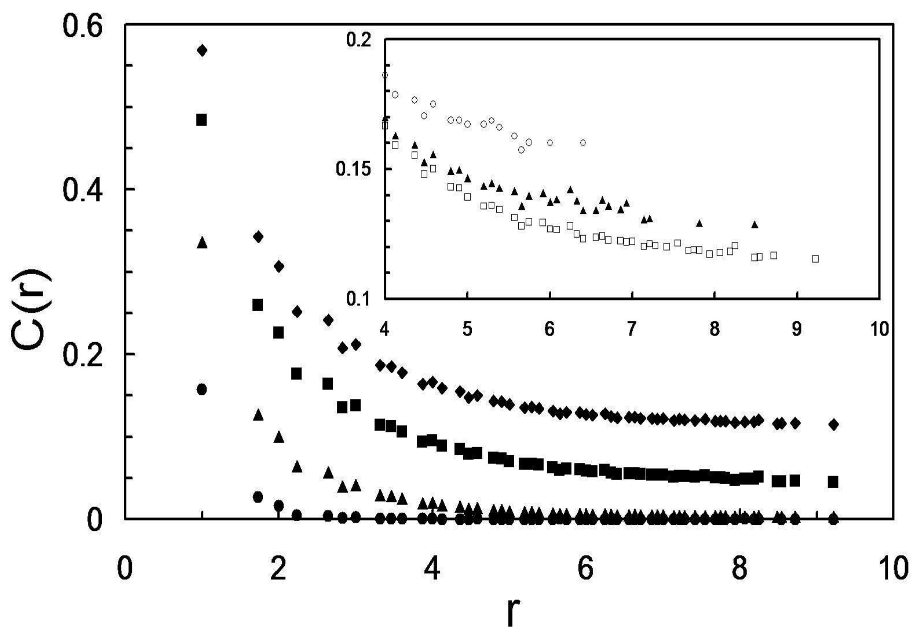
random strains modulating exchange

$$\mathcal{H} = \sum J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j$$

$$[J_{ij}]_{\rm av} = J \qquad [(J_{ij} - J)^2]_{\rm av} = \Delta^2$$

— with $\Delta \ll J$

Spin glass order in simulations



$$C(\mathbf{r}) = [\langle \mathbf{S}(\mathbf{0}) \cdot \mathbf{S}(\mathbf{r}) \rangle_1 \langle \mathbf{S}(\mathbf{0}) \cdot \mathbf{S}(\mathbf{r}) \rangle_2]_{\rm av}$$

Summary

Groundstates with ideal geometric frustration have:

- Macroscopic degeneracy
- Dipolar correlations

— Constitute a Coulomb phase

Ordering from this manifold of states:

- In clean system: non-standard critical behaviour
- In random system: spin freezing

— New class of spin glass?