Enslaved and Locked: on periodic forcing in time and space

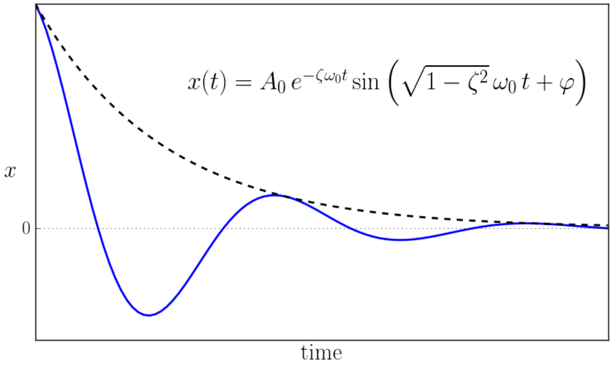
Yair Mau

Department of Physics
Ben-Gurion University of the Negev

14 March 2013

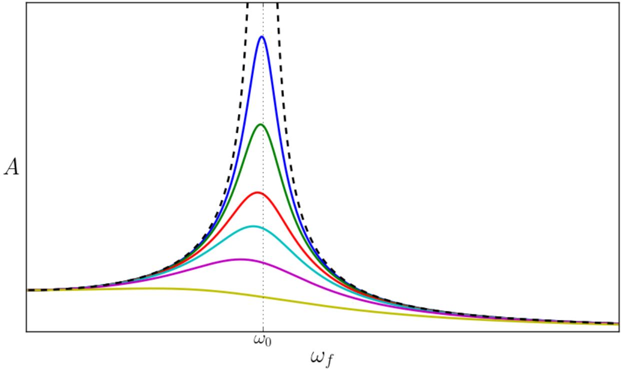
The damped harmonic oscillator

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\zeta\omega_0 \frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = 0$$



Sinusoidal driving force

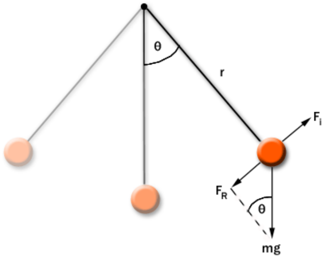
$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\zeta\omega_0 \frac{\mathrm{d}x}{\mathrm{d}t} + \omega_0^2 x = \frac{1}{m} F_0 \sin(\omega_f t)$$



Simple pendulum

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{\ell}\sin\theta = 0$$

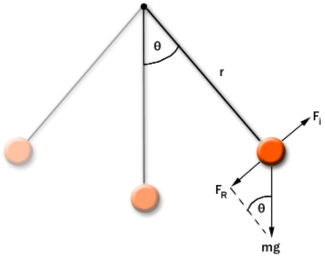
$$\omega_0 = \sqrt{\frac{g}{\ell}}$$



Simple pendulum

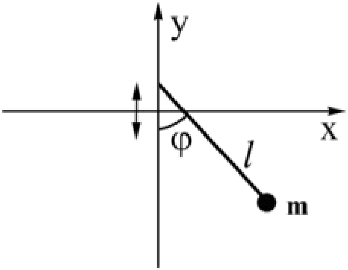
$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g}{\ell}\sin\theta = 0$$

$$\omega_0 = \sqrt{\frac{g}{\ell}}$$



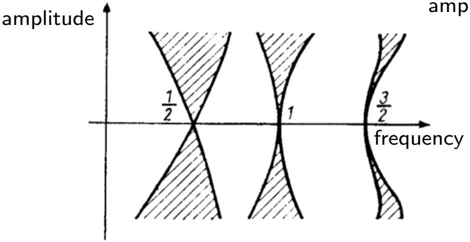
Kapitza's pendulum

$$\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + \frac{g + a\cos(\omega_f t)}{\ell}\sin\theta = 0$$

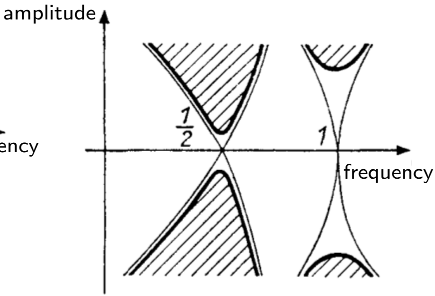


Parametric resonance - Arnold tongues

No damping



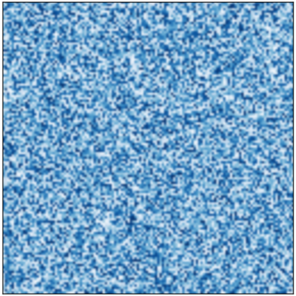
With damping



Ordinary Differential Equations - Vladimir I. Arnol'd

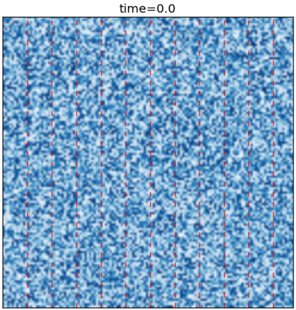
The simplest PDE that forms periodic patterns

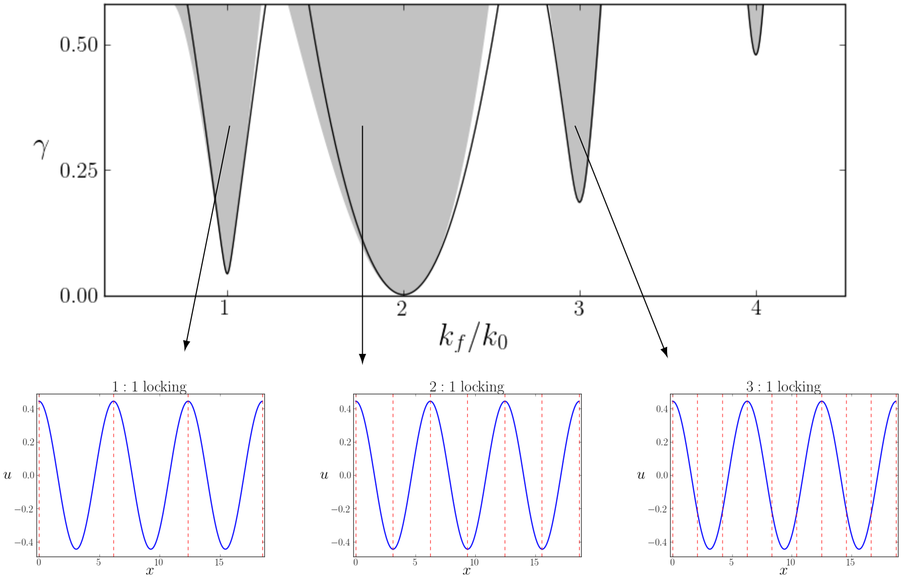
$$\frac{\mathrm{d}u}{\mathrm{d}t} = \epsilon u - u^3 - \left(\nabla^2 + k_0^2\right)^2 u$$



The same PDE with periodic forcing in space

$$\frac{\mathrm{d}u}{\mathrm{d}t} = \Big(\epsilon + \gamma\cos(k_f x)\Big)u - u^3 - \left(\nabla^2 + k_0^2\right)^2 u$$




0.50
0.25
0.00
γ
1
2
3
4
k_f/k_0
1 : 1 locking
2 : 1 locking
3 : 1 locking
u
x