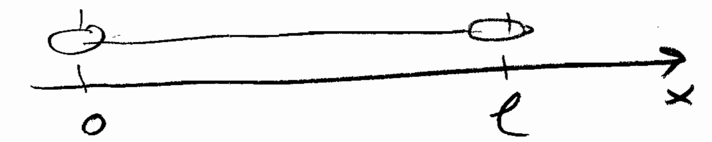
שאלה 1



א) $v = \sqrt{\frac{T}{\rho}}$

ב) $\left.\frac{\partial \psi}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi}{\partial x}\right|_{x=\ell} = 0$

ג) $\psi(x,t) = \cos(\omega t + \varphi)(A\cos kx + B\sin kx)$

$\frac{\partial \psi}{\partial x} = \cos(\omega t + \varphi)(-kA\sin kx + kB\cos kx)$

$\left.\frac{\partial \psi}{\partial x}\right|_{x=0} = 0 \Rightarrow B = 0$

$\left.\frac{\partial \psi}{\partial x}\right|_{x=\ell} = 0 \Rightarrow \sin k\ell = 0$

$k\ell = \pi n \qquad k_n = \frac{\pi}{\ell} n$

$\omega_n = v k_n = \sqrt{\frac{T}{\rho}}\,\frac{\pi}{\ell} n$

$\psi_n(x,t) = A_n \cos k_n x \cos(\omega_n t + \varphi_n)$

$\underline{\omega = 0}$

$\psi(x,t) = (At+B)(Cx+D)$

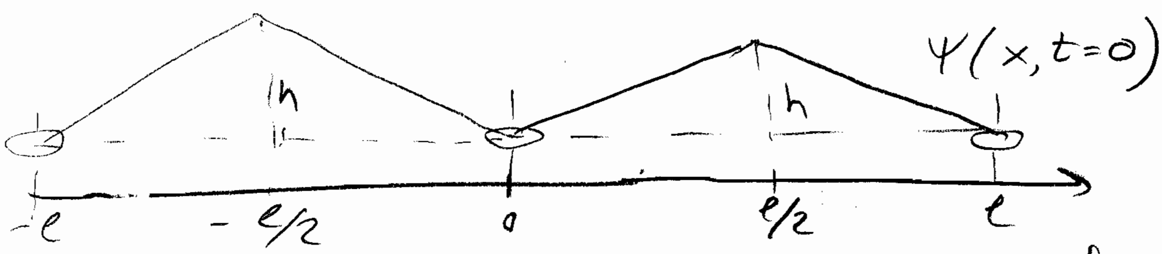
$\frac{\partial \psi}{\partial x} = C(At+B)$

$\left.\frac{\partial \psi}{\partial x}\right|_{x=0} = \left.\frac{\partial \psi}{\partial x}\right|_{x=\ell} = 0$

$C = 0$

$\psi(x,t) = A_0 t + B_0$

ד)



הפתרון הכללי: $\psi(x,t) = A_0 t + B_0 + \sum_{n=1}^{\infty} A_n \cos k_n x \cos(\omega_n t + \varphi_n)$

$\left.\frac{\partial \psi}{\partial t}\right|_{t=0} = A_0 + \sum_{n=1}^{\infty} A_n \cos(k_n x)\,\omega_n \sin\varphi_n = 0$

$\varphi_n = 0, \quad A_0 = 0$

$\psi(x,t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos k_n x \cos \omega_n t$

$$\Psi(x,t=0) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos k_n x = \begin{cases} \frac{2h}{\ell}x + 2h, & -\ell<x<-\ell/2 \\ -\frac{2h}{\ell}x, & -\ell/2<x<0 \\ \frac{2h}{\ell}x, & 0<x<\ell/2 \\ -\frac{2h}{\ell}x+2h, & \ell/2<x<\ell \end{cases}$$

$$A_n = \frac{1}{\ell}\int_{-\ell}^{\ell} \Psi(x,t=0)\cos\frac{\pi n}{\ell}x\,dx = \qquad (n\neq 0)$$

$$= \frac{1}{\ell}\Big(\int_{-\ell}^{-\ell/2}\Big(\frac{2h}{\ell}x+2h\Big)\underset{x'=-x}{\cos\frac{\pi n}{\ell}x\,dx} + \int_{-\ell/2}^{0}\Big(-\frac{2h}{\ell}x\Big)\underset{x'=-x}{\cos\frac{\pi n}{\ell}x\,dx} +$$

$$+ \int_0^{\ell/2}\frac{2h}{\ell}x\cos\frac{\pi n}{\ell}x\,dx + \int_{\ell/2}^{\ell}\Big(-\frac{2h}{\ell}x+2h\Big)\cos\frac{\pi n}{\ell}x\,dx\Big) =$$

$$\Big/ \int \underset{f}{x}\cos \underset{g'}{ax}\,dx = +\frac{1}{a}x\sin ax - \frac{1}{a}\int\sin ax\,dx = \frac{1}{a}x\sin ax + \frac{1}{a^2}\cos$$

$$= \frac{1}{\ell}\Big(-\int_{\ell}^{\ell/2}\Big(-\frac{2h}{\ell}x'+2h\Big)\cos\frac{\pi n}{\ell}x'\,dx' + (-1)\int_{\ell/2}^{0}\frac{2h}{\ell}x'\cos\frac{\pi n}{\ell}x'\,dx' +$$

$$+ \int_0^{\ell/2}\frac{2h}{\ell}x\cos\frac{\pi n}{\ell}x\,dx + \int_{\ell/2}^{\ell}\Big(-\frac{2h}{\ell}x+2h\Big)\cos\frac{\pi n}{\ell}x\,dx\Big) =$$

$$= \frac{2}{\ell}\Big(\int_0^{\ell/2}\frac{2h}{\ell}x\cos\frac{\pi n}{\ell}dx + \int_{\ell/2}^{\ell}\Big(-\frac{2h}{\ell}x+2h\Big)\cos\frac{\pi n}{\ell}x\,dx\Big) =$$

$$= \frac{2}{\ell}\cdot 2h\Big[\frac{1}{\ell}\Big(\frac{\ell}{\pi n}\Big(\frac{\ell}{2}\sin\frac{\pi n}{\ell}\frac{\ell}{2} + \frac{\ell}{\pi n}\cos\frac{\pi n}{\ell}\frac{\ell}{2} - 0 - \frac{\ell}{\pi n}\cdot 1\Big)\Big) -$$

$$- \frac{1}{\ell}\Big(\frac{\ell}{\pi n}\Big(\ell\sin\frac{\pi n}{\ell}\cdot\ell + \frac{\ell}{\pi n}\cos\frac{\pi n}{\ell}\ell - \frac{\ell}{2}\sin\frac{\pi n}{\ell}\frac{\ell}{2} - \frac{\ell}{\pi n}\cos\frac{\pi n}{\ell}\frac{\ell}{2}\Big)$$

$$+ \frac{\ell}{\pi n}\Big(\sin\frac{\pi n}{\ell}\cdot\ell - \sin\frac{\pi n}{\ell}\frac{\ell}{2}\Big)\Big)\Big] =$$

$$= \frac{4h}{\pi n\ell}\Big[\frac{\ell}{2}(-1)^{n+1}\frac{1}{\pi n} \cancel{+} \frac{\ell}{\pi n} - \cancel{\frac{\ell}{\pi n}(-1)^{n}} + \frac{\ell}{2}(-1)^{n+1}\frac{1}{\pi n} - \cancel{\frac{\ell}{\pi n}(-1)^{n+1}}\Big] =$$

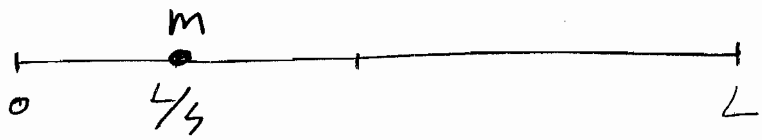
$$= \frac{4h}{\pi n}\Big((-1)^{n+1}\frac{1}{\pi n} - \frac{1}{\pi n}\Big) = -\frac{4h}{\pi^2 n^2}\big((-1)^n + 1\big)$$

$n = 0$

$$A_0 = \frac{1}{l}\int_{-l}^{l} \psi(x, t=0)\,dx = h$$

$$\psi(x,t) = \frac{h}{2} - \frac{4h}{\pi^2}\sum_{n=1}^{\infty} \frac{1+(-1)^n}{n^2}\cos\left(\frac{\pi n}{l}x\right)\cos\left(\sqrt{\frac{T}{\rho}}\,\frac{\pi n}{l}t\right)$$

שאלה 2



א) המסה במנוחה.

$$\Psi(x,t) = \cos(\omega t + \varphi)(A\sin kx + B\cos kx)$$

$$\Psi(x=0) = 0 \Rightarrow B = 0$$

$$\Psi(x=L) = 0 \Rightarrow \sin kL = 0 \Rightarrow k_n = \frac{\pi n}{L}$$

אופני התנודה ללא המסה:

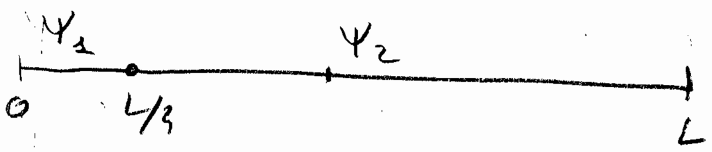
$$\Psi_n(x,t) = A_n \sin\frac{\pi n}{L}x \cos(\omega_n t + \varphi)$$

הנקודה $x = L/4$ נמצאת במנוחה כאשר

$$\sin\left(\frac{\pi n}{L}\cdot\frac{L}{4}\right) = 0$$

זה קורה עבור $n = 4, 8, 12, \ldots$

ב) תנאי השפה באופן כללי:



$$\Psi_1(0) = 0 \qquad \Psi_1(L/4) = \Psi_2(L/4)$$

$$\Psi_2(L) = 0 \qquad T\left(\left.\frac{\partial\Psi_1}{\partial x}\right|_{x=L/4} - \left.\frac{\partial\Psi_2}{\partial x}\right|_{x=L/4}\right) = m\left.\frac{\partial^2\Psi}{\partial t^2}\right|_{x=L/4}$$

$$\Psi_1 = \cos(\omega t + \varphi_1)(A\sin kx + B\cos kx)$$

$$\Psi_2 = \cos(\omega t + \varphi)(C\sin kx + D\cos kx)$$

$$\psi_1(0)=0 \Rightarrow B=0$$

$$\psi_2(L)=0=C\sin kL+D\cos kL=0 \Rightarrow D=-C\tan kL \quad (1)$$

$$\psi_1(L/4)=\psi_2(L/4)$$

$$\Rightarrow A\sin\frac{kL}{4}=C\sin\frac{kL}{4}-C\tan kL\cos\frac{kL}{4}$$

$$A=C\left(1-\tan kL\cot\frac{kL}{4}\right) \quad (2)$$

$$T\left(\frac{\partial\psi_1}{\partial x}\bigg|_{L/4}-\frac{\partial\psi_2}{\partial x}\bigg|_{L/4}\right)=m\frac{\partial^2\psi}{\partial t^2}$$

$$T\left(Ak\cos\frac{kL}{4}-Ck\cos\frac{kL}{4}+Dk\sin\frac{kL}{4}\right)=-m\omega^2A\sin\frac{kL}{4} \quad (3)$$

$$(1),(2),(3) \Rightarrow$$

$$k\left(\cancel{\cos\frac{kL}{4}}-\tan kL\cot\frac{kL}{4}\cos\frac{kL}{4}-\cancel{\cos\frac{kL}{4}}\right)-k\tan kL\sin\frac{kL}{4}=$$

$$=-\frac{m\omega^2}{T}\left(\sin\frac{kL}{4}-\tan kL\cos\frac{kL}{4}\right)$$

$$k\tan kL\left(\frac{\cos^2\frac{kL}{4}}{\sin\frac{kL}{4}}+\sin\frac{kL}{4}\right)=\frac{m\omega^2}{T}\left(\sin\frac{kL}{4}-\tan kL\cos\frac{kL}{4}\right)$$

$$k\tan kL=\frac{m\omega^2}{T}\left(\sin^2\frac{kL}{4}-\tan kL\cos\frac{kL}{4}\sin\frac{kL}{4}\right)$$

$$\omega=vk$$

$$\tan kL=\frac{mv^2}{T}k\sin^2\frac{kL}{4}\left(1-\tan kL\cot\frac{kL}{4}\right)$$

3 שאלה

נשתמש בנוסחא הבאה:

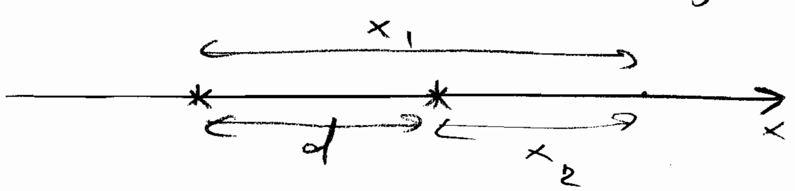
$$a\cos\alpha + b\cos\beta =$$

$$= \frac{1}{2}(a+b)(\cos\alpha + \cos\beta) + \frac{1}{2}(a-b)(\cos\alpha - \cos\beta) =$$

$$= (a+b)\cos\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta) - (a-b)\sin\tfrac{1}{2}(\alpha+\beta)\sin\tfrac{1}{2}(\alpha-\beta)$$

$$\Psi(r,t) = A(r)\cos(\omega t - kr)$$ כל גל נכתב כמו

א) $A(r)$ - פונקציה יורדת מונוטונית



$$d = x_2 - x_1$$

$$\Psi_1 + \Psi_2 = A(x_1)\cos(\omega t - kx_1) + A(x_2)\cos(\omega t - kx_2) =$$

$$= \left(A(x_1) + A(x_2)\right)\cos\left(\omega t - k\frac{x_1+x_2}{2}\right)\cos\left(\frac{k(x_2-x_1)}{2}\right) -$$

$$-\left(A(x_1) - A(x_2)\right)\sin\left(\omega t - k\frac{x_1+x_2}{2}\right)\sin\left(\frac{k(x_2-x_1)}{2}\right)$$

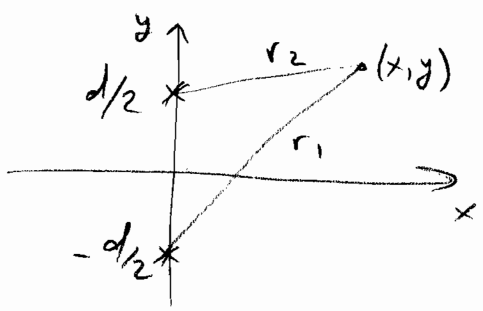
$$I = \langle(\Psi_1+\Psi_2)^2\rangle = \frac{1}{2}\left(A(x_1)+A(x_2)\right)^2\cos^2\frac{kd}{2} +$$

$$+ \frac{1}{2}\left(A(x_1)-A(x_2)\right)^2\sin^2\frac{kd}{2} =$$

$$= \frac{1}{2}\left[A^2(x_1) + A^2(x_2) + 2A(x_1)A(x_2)\cos kd\right]$$

התוצאה יורדת עם המרחק.

ב)



$$r_1^2 = x^2 + \left(y+\frac{d}{2}\right)^2$$

$$r_2^2 = x^2 + \left(y-\frac{d}{2}\right)^2$$

$$r_1, r_2 \gg d \qquad R \equiv \frac{r_1+r_2}{2}$$

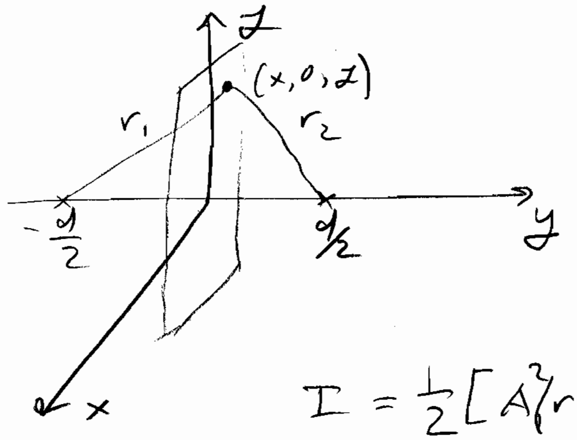
$$r_1^2 - r_2^2 = \left(y+\frac{d}{2}\right)^2 - \left(y-\frac{d}{2}\right)^2 = 2yd = (r_1-r_2)(r_1+r_2) =$$

$$= 2R(r_1-r_2)$$

$$I \approx \frac{1}{2}\left[A^2(r_1) + A^2(r_2) + 2A(r_1)A(r_2)\cos\left(k\frac{y}{R}d\right)\right]$$

מינימום:

$$\cos\left(k\frac{y}{R}d\right) = -1 \qquad \frac{y}{R} = \frac{(2n+1)\pi}{kd}$$



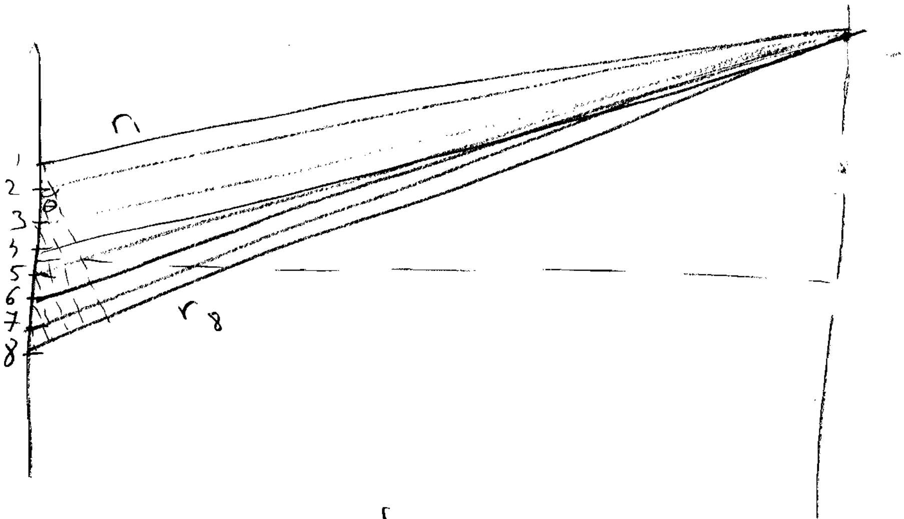
d

$$r_1^2 = x^2 + \left(\frac{d}{2}\right)^2$$
$$r_2^2 = x^2 + \left(\frac{d}{2}\right)^2$$
$$r_1 - r_2 = 0$$

$$I = \frac{1}{2}\left[A^2(r_1) + A^2(r_2) + 2A(r_1)A(r_2)\cdot 1\right] =$$
$$= \frac{1}{2}\left(A(r_1) + A(r_2)\right)^2$$

העוצמה יורדת עם המרחק.

שאלה 4



(א)

כיוון שהמסך רחוק הזווית בין כל קרן לישר המחבר את המקורות שווה.

הפרש הדרכים האופטיות בין כל שתי קרניים סמוכות $\Delta = d\sin\theta$

והפרש הדרכים ביחס לקרן הראשונה הוא $\Delta_n = (n-1)d\sin\theta$

כאשר n- מספר המקור.

מכל מקור n יוצא גל

$$\Psi_n = Ae^{i(\omega t - kr_n + \varphi_n)}$$

כאשר $r_n = r_1 + \Delta_n = r_1 + (n-1)d\sin\theta$

$$\varphi_1 = \varphi_3 = \varphi_5 = \varphi_7 = 0$$
$$\varphi_2 = \varphi_4 = \varphi_6 = \varphi_8 = \delta$$

$$\Psi = \sum_{n=1}^{8}\Psi_n = Ae^{i\omega t}e^{-ikr_1}\left(1 + e^{-i2\Delta k} + e^{-i4\Delta k} + e^{-i6\Delta k} + e^{i\delta}\left(e^{-i\Delta k} + e^{-i3\Delta k} + e^{-i5\Delta k} + \dots\right.\right.$$

$$= Ae^{i(\omega t - kr_1)}\left(\frac{e^{-ik2\Delta\cdot 4} - 1}{e^{-ik2\Delta} - 1} + e^{i\delta}e^{-i\Delta}\frac{e^{-ik2\Delta\cdot 4} - 1}{e^{-ik2\Delta} - 1}\right) =$$

$$= Ae^{i(\omega t - kr_1)}\frac{e^{-ik4\Delta}\left(e^{-ik4\Delta} - e^{i4\Delta k}\right)}{e^{-ik\Delta}\left(e^{-i\Delta k} - e^{i\Delta k}\right)}\left(1 + e^{i(\delta - \Delta k)}\right) =$$

$$= Ae^{i(\omega t - kr_1)}e^{-i3k\Delta}\frac{\sin 4k\Delta}{\sin k\Delta}e^{\frac{1}{2}i(\delta - k\Delta)}2\cos\left(\tfrac{1}{2}(\delta - k\Delta)\right)$$

$$\mathrm{Re}\,\Psi = 2A\cos\left(\omega t - kr_1 - 4k\Delta + \tfrac{1}{2}\delta - \tfrac{1}{2}k\Delta\right)\frac{\sin 4k\Delta}{\sin k\Delta}\cos\left(\tfrac{1}{2}(\delta - k\Delta\right.$$

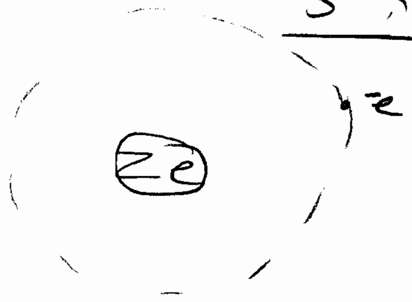
$$I = I_0\frac{\sin^2 4k\Delta}{\sin^2 k\Delta}\cos^2\left(\tfrac{1}{2}(\delta - k\Delta)\right) \qquad \Delta = d\sin\theta$$

(ב)

$$\delta = \pi$$

$$I = I_0\frac{\sin^2 4k\Delta}{\sin^2 k\Delta}\sin^2\tfrac{1}{2}k\Delta = \tfrac{1}{4}I_0\frac{\sin^2 4k\Delta}{\cos^2\frac{1}{2}k\Delta} = I_0\left(\frac{\sin 4k\Delta}{2\cos\frac{1}{2}k\Delta}\right)^2$$

שאלה 5



Bohr quantization

$$L = n\hbar$$

$$L = mvr = n\hbar$$

$$K\frac{Ze^2}{r^2} = m\frac{v^2}{r}$$ — חוק שני

$$v^2 = \frac{kZe^2}{mr} = \frac{kZe^2}{m\frac{n\hbar}{mv}} = \frac{kZe^2}{n\hbar}v$$

$$v_n = \frac{kZe^2}{n\hbar}$$

$$r_n = \frac{n\hbar}{mv} = \frac{n^2\hbar^2}{mkZe^2}$$

$$E_n = \frac{1}{2}mv^2 - k\frac{Ze^2}{r} = \frac{1}{2}m\frac{k^2Z^2e^4}{n^2\hbar^2} - k\frac{Ze^2\, kmZe^2}{n^2\hbar^2} =$$

$$= -\frac{mk^2Z^2e^4}{2n^2\hbar^2}$$

de Broglie

$$2\pi r = n\lambda \qquad \lambda = \frac{h}{p} = \frac{2\pi\hbar}{p} \Rightarrow r = \frac{n\hbar}{mv}$$

שקול ל- $L = mvr = n\hbar$

$$E_n = -\frac{Z^2 \cdot E_1^{(0)}}{n^2} \qquad E_1^{(0)} = 13.6\,\text{eV}$$

$$\lambda = \frac{h}{p} = \frac{hc}{E} = \frac{hc}{E_n - E_m} = \frac{hc}{-Z^2E_1^{(0)}\left(\frac{1}{n^2} - \frac{1}{m^2}\right)}$$

$$-\frac{1}{n^2} + \frac{1}{m^2} = \frac{hc}{Z^2E_1^{(0)}}\frac{1}{\lambda} \qquad * \qquad 400\,\text{nm} < \lambda < 700\,\text{nm}$$

$$0.03 \lesssim \frac{1}{m^2} - \frac{1}{n^2} \lesssim 0.06 \qquad Z = 2$$

The allowed transitions are $4 \to 3$, $n \to 4$ $(n > 5)$ (for $Z = 2$)