Comment on "Quantum Suppression of Irregularity in the Spectral Properties of the Kicked Rotator"

The statistical properties of the spectrum of a quantum system with small Planck's constant $\hbar$ can be correlated with the regularity of the behavior of the classical system at $\hbar = 0$. For example, the nearest-neighbor level spacing distribution $P(x)$ (for an energy-conserving system) is known to be of the Wigner type if the classical system is ergodic, but it is Poissonian if the classical limit is regular. In a recent Letter, Frahm and Mikeska[1] have discussed the example of the quasienergy level spacing distribution of the kicked rotator, where they observe a transition from Wigner to Poisson distribution as $\hbar$ is increased. It is the purpose of this note to point out that this type of transition has been previously discussed and is already well understood.[2] A secondary purpose is to counter the suggestion of Ref. 1 that a new concept, "the degree of regularity of the quantum system," is needed and that our previous work lacks the ability to explain their results. Finally, we wish to mention that part of the work of Izrailev[3] is very similar to that of Ref. 1.

It is by now commonly accepted that the quasienergy eigenstates of the kicked rotator are localized in the momentum space for generically irrational values of $\hbar$.[4] This is due to the same mechanism as the Anderson localization of an electron on a one-dimensional infinite lattice with random potential. Moreover, it has been rigorously proven that the energy levels for the Anderson problem have Poisson statistics.[5] Since the degree of level repulsion is proportional to the overlap of the eigenfunctions, this result is rather obvious. In an infinite system almost all the eigenstates overlap less than $O(\epsilon)$ for any finite ϵ. Therefore, pairs of levels which repel each other are dominated by pairs which are uncoupled to arbitrary precision. Although there are no rigorous results for a system of finite size N, the Poissonian statistics will certainly persist if the system is much bigger than the localization length ξ ($N \gg \xi$). Furthermore, one expects to find Wigner statistics when $N \ll \xi$ and a crossover for $N \approx \xi$. Because of its equivalence to the Anderson problem, we expect the same conclusions to hold for the kicked rotator. Indeed, we have numerically found a crossover from Wigner to Poisson statistics for constant nonlinearity k as N was gradually changed from $N \ll \xi$ to $N \gg \xi$.[2] Since assigning a rational value M/N to $\hbar$ in the kicked rotator is equivalent to considering an Anderson problem on a finite-size lattice of length N, we believe that the same crossover observed in Ref. 1 is also due to the localization mechanism described above.

Using the relation[6] between k, $\hbar$, and ξ, $\xi \approx \alpha(k) \times (4\hbar^2)^{-1}(k/2\pi)^2$ where $\alpha(10) \cong 0.5$, we can quantitatively check this claim. On the one hand, for $\hbar = \frac{1}{1944}$ and $k = 10$, $\xi \approx 1.2\times 10^6 \gg N = 1944$. Accordingly, we predict Wigner-type statistics, and this is confirmed by Fig. 1(a) of Ref. 1. Moreover, if for the same k, the case $\hbar = \frac{625}{1944}$ is considered, then $\xi \approx 3 \ll N = 1944$ and again, our prediction for this regime (Poissonian statistics) is supported by Fig. 1(b) of Ref. 1. We can easily estimate the value of $\hbar$ for which the crossover occurs by setting $\xi = N$. For $k = 10$ a value of $\hbar \approx 1.3\times 10^{-2}$ is obtained which is in good agreement with Fig. 2 of Ref. 1. Therefore, because of the relation between k, $\hbar$, and ξ, the results of Ref. 1 are equivalent to the ones that we have obtained in Ref. 2 once the k and $\hbar$ parameters are replaced with k and ξ.

In conclusion, we have shown that the localization of eigenstates in angular momentum space accounts both qualitatively and quantitatively for the Wigner to Poisson transition observed in Ref. 1. An appropriate parameter for "degree of regularity" is simply the ratio of the localization length to the system size. We would not call this a "regularity" parameter, but that is a matter of taste.

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