Electric potential in the low-Mach number quasiperpendicular collisionless shock ramp revisited

M. Gedalin

Department of Physics, Ben-Gurion University, Beer-Sheva, Israel

M. Balikhin

ACSE, Sheffield University, Sheffield, UK

1. Introduction

The cross-shock electric field, together with the magnetic field, determine the ion and electron dynamics in quasiperpendicular collisionless shocks. Although turbulent fields cannot be neglected, it seems that at the scale of the shock transition (foot, ramp, and overshoot for a high-Mach number shock, and ramp only for a low-Mach number shock) dc magnetic and electric field are those which are mostly responsible for ion reflection and heating, and electron heating as well [*Sckopke et al.*, 1983; *Thomsen et al.*, 1985; *Lee et al.*, 1986, 1987; *Goodrich and Scudder*, 1984; *Schwartz et al.*, 1988; *Burgess et al.*, 1989; *Gedalin*, 1996a]. On the other hand, the fields are affected by the reflected/heated distributions [*Phillips and Robson*, 1972; *Woods*, 1971; *Leroy et al.*, 1982; *Leroy*, 1983], so that, for example, the relative strength of the cross-shock potential decreases with the Mach number increase [*Phillips and Robson*, 1972; *Formisano, V.*, 1982; *Thomsen et al.*, 1987; *Wygant et al.*, 1987]. In order to understand in detail the ion and electron dynamics inside quasiperpendicular shocks, one has to know in detail the distribution of the dc magnetic and electric field across the shock. While high-resolution magnetic field measurements became a routine, high-resolution electric field measurements (three-components of the vector) are still not always available, and it would be useful if such measurements could be compared (or completed) with other available data to make the obtained information more comprehensive. Theoretical modeling of the electric field distribution inside the shock would be of substantial help. Such modeling of the electric field across the whole quasiperpendicular shock front (including ramp, foot, and overshoot) would be a difficult problem, especially because of the sensitivity of ion reflection to the ion temperature and small-scale features [*Burgess et al.*, 1989; *Gedalin*, 1996a]. However, in most low-Mach number shocks the number of reflected ions is small [*Thomsen et al.*, 1985; *Sckopke et al.*, 1990], and there is no foot and overshoot. We can expect that the relation between the field and ion dynamics is more simple in such shocks consisting of a single ramp. We will show below that, indeed, this is the case, and rather general relations can be obtained with rather modest assumptions. Similar simplification can be achieved for high-Mach number shocks *inside the ramp* which is rather narrow and may be only a small part of the whole shock transition [*Newbury et al.*, 1998]. Most existing derivations of the cross-shock electric field are hydrodynamics based [see, e.g., *Gedalin*, 1998] and, as a rule, exploit polytropic state equations for ions and electrons as well, thus introducing additional assumptions with, at best, limited empirical support. Not only are such derivations model dependent, but they are also poorly justified. Indeed, in low-Mach number shocks only part of the ion heating occurs in the ramp, while the main heating is related to the downstream gyration of the ion distribution, as is seen from observations [*Thomsen et al.*, 1985; *Sckopke et al.*, 1990], numerical simulations [*Burgess et al.*, 1989], and analytical studies [*Gedalin*, 1997; *Zilbersher et al.*, 1998] as well. The widely used phenomenological polytropic state equations do not take into account the distinction between ion heating inside the ramp and the total shock heating. In high-Mach number shocks reflected ions also considerably distort the state equation in the ramp. Yet is has been shown that, unless initial ion temperature is high (high-β upstream plasma), ion motion across the ramp is intimately related to the cross-shock potential [*Gedalin*, 1996a, 1997]. In the present paper we show that the ion density can be approximately represented as a function of the potential in the ramp and propose that this relation may be used for indirect potential measurements. The derived functional dependence allows us to establish two more relations which promise to be useful for comparison with direct electric field measurements, or completing those if necessary. We discuss the applicability of the method to high-Mach number shocks.

2. Ion motion in the ramp and relation to the potential

We start with the analysis of the ion motion in the ramp assuming that the magnetic and electric field are stationary (time-independent) and one-dimensional (depend only on the single coordinate along the shock normal). The two assumptions may be satisfied only approximately: the fields should not change much during the ramp crossing time, and the typical scale of the shock inhomogeneity along the shock front should be substantially larger than the perpendicular ion displacement during the ramp crossing. Fortunately, these conditions are met in most low-Mach number shocks, since they are nearly stationary and one-dimensional [*Zilbersher et al.*, 1998]. Such conditions should be also often met in high-Mach number shocks with narrow ramp [*Newbury et al.*, 1998].

Let x be the coordinate along the shock normal, and y be the noncoplanarity direction, so that $B_x = \text{const}$, $E_y = \text{const}$, and B_y is present only inside the ramp. We analyze the ion motion within the single particle equations of motion, which read:

$$m_i \dot{\mathbf{v}} = e\mathbf{E} + e\mathbf{v} \times \mathbf{B}. \tag{1}$$

Eq. (1) is always correct, even when the fields are varying and/or turbulent. In our case it is assumed that both $\mathbf{E}$ and $\mathbf{B}$ depend only on x. As is said above, this is an approximation, and the below derivation remains valid even if there is weak dependence on y, z and t.

We proceed in the normal incidence frame where an ion enters the ramp with the initial velocity $\mathbf{v}_{in} = (V_u + u_x, u_y, u_z)$, where $\mathbf{u}$ is the random (thermal) velocity, typically $u \sim \sqrt{\beta_i}/M$. Here, as usual, $\beta_i = 2\mu_0 n_u T_{iu}/B_u^2$, n_u is the upstream plasma density, B_u is the upstream magnetic field, T_{iu} is the upstream ion temperature, and M is the Alfven Mach number. The upstream magnetic field B_u, the upstream plasma velocity V_u, the motional electric field E_y and the angle between the shock normal and upstream magnetic field θ are related as follows:

$$E_y = V_u B_u \sin\theta. \tag{2}$$

Of course, physics is frame independent. Calculations, however, are easier and more convenient in the normal incidence frame, where inside the ramp $|v_y, v_z| \ll v_x$, as we shall see below. Indeed, for typical parameters of the moderately warm upstream plasma, $(u/V_u)^2 \sim \beta_i/M^2 \ll 1$. The cross-ramp potential with the magnetic field do not stop ions inside the ramp [*Burgess et al.*, 1989; *Gedalin*, 1997], so that $v_x > 0$ throughout the ramp and we may substitute $d/dt = v_x(d/dx)$ along the particle trajectory. Now the formal exact solution of (1) can be written as follows:

$$v_x^2 = (V_u + u_x)^2 - \frac{2e\phi}{m_i} + \frac{e}{m_i}\int_0^x (v_y B_z - v_z B_y)dx', \tag{3}$$

$$v_y = u_y + \int_0^x \frac{eE_y}{m_i v_x}dx' - \frac{e}{m_i}\int_0^x B_z dx' + \frac{e}{m_i}\int_0^x \frac{v_z B_x}{v_x}dx', \tag{4}$$

$$v_z = u + z + \frac{e}{m_i}\int_0^x B_y dx' - \frac{e}{m_i}\int_0^x \frac{v_y B_x}{v_x}dx'. \tag{5}$$

General solution of (3)-(5) is not available. However, in the case of a thin ramp these equations can be solved approximately [*Gedalin*, 1997]. Eq. (4) shows that

$$\begin{aligned} v_y &\sim \frac{eB_u \sin\theta}{m_i}\int\left(\frac{V_u}{v_x} - \frac{B_z}{B_u\sin\theta}\right)dx \\ &\lesssim \frac{eB_u\sin\theta}{m_i}L, \end{aligned} \tag{6}$$

where L is the ramp width, which is typically smaller than the ion inertial length $\lesssim c/\omega_{pi}$, $\omega_{pi}^2 = e^2 n/\epsilon_0 m_i$ [*Mellott and* , 1984; *Farris et al.*, 1993; *Newbury et al.*, 1998]. From this one can easily see that $v_y \ll V_u$, while $v_x \sim V_u$, so that $v_y \ll v_x$ throughout the ramp. Similarly, $v_z \ll v_x$. Note, that these conclusions hold even for substantial B_y [*Thomsen et al.*, 1987; *Farris et al.*, 1993], because of the small width of the ramp. This is in the complete agreement with general understanding of the charged particle motion in the inhomogeneous magnetic field: the particle becomes demagnetized when the inhomogeneity scale is of the order or smaller than the particle gyroradius. The situation inside the ramp is opposite to what happens in the foot and behind the ramp [*Leroy et al.*, 1982; *Leroy*, 1983; *Burgess et al.*, 1989; *Gedalin*, 1996a]. In the foot magnetic forces dominate and cause deflection, although ions are not magnetized. In the downstream, behind the ramp, ions start to gyrate because of magnetization.

Thomsen et al. [1987] argue that the noncoplanar magnetic field plays an important role in the ion deceleration in the ramp, in particular when viewed in the de Hoffman-Teller frame. The de Hoffman-Teller frames moves along B_z with the velocity $V_u/\cos\theta$ relative to the normal incidence frame. As a result, the electrostatic field E_x is reduced by $V_u B_y/\cos\theta$ in the de Hoffman-Teller frame, while the magnetic force $F_x = ev_y B_z - ev_z B_y \approx eV_u B_y/\cos\theta$ is large. When in the normal incidence frame, the Lorentz transformation of the field brings $V_u B_y/\cos\theta$ back into the electrostatic field, reducing the magnetic deceleration force down to $-euB_y$, making it unimportant. The combine effect is, of course, the same in both frames (physics is frame independent), but the role distribution between the electrostatic and magnetic force is quite different in different frames.

Taking into account that v_y and v_z remain small throughout the ramp it is easy to see that the contribution of last term in (3) is also small relative to the first term, where we used $E_x = -(d\phi/dx)$. In the first term $u_x \sim v_T \ll V_u$ can be neglected unless $V_u^2 - 2e\phi/m_i \approx 0$. Thus, the ion motion in the ramp is mostly affected by the potential and only weakly by the magnetic forces. This conclusion does not require any assumption about the magnitude of the noncoplanar magnetic field, and is the result if the small ramp width. It is worthwhile to note that magnetic deceleration and/or deflection is important in wider parts of the shock (foot and overshoot) where it has to be taken into account to properly describe the ion reflection [*Leroy*, 1983; *Gedalin*, 1996a]. *Thomsen et al.* [1987] found numerically that the magnetic forces due to the noncoplanar magnetic field component may contribute a substantial part of the ion decelerating force even in the normal incidence frame. Their findings are due to the substantial overestimate of the ramp width in hybrid simulations: several c/ω_{pi} in the simulations versus a fraction of c/ω_{pi} found in observations of high-Mach number shocks [*Scudder et al.*, 1986; *Newbury et al.*, 1998]. In low-Mach number shocks the ramp width is of the order of c/ω_{pi} [*Mellott and* , 1984; *Farris et al.*, 1993] but the noncoplanar magnetic field component is substantially lower.

In the lowest order approximation for low-β_i shocks, where $\beta_i = 2\mu_0 n T_i/B_u^2$, we arrive at the following simple relation

$$v_x = \sqrt{V_u^2 - 2e\phi/m_i}. \tag{7}$$

Since the upstream thermal spread does not appear here, the velocity v_x is at the same time the mean velocity of the ion flow inside the ramp, and the flux conservation gives immediately

$$n = n_u V_u/v_x = n_u(1-s)^{-1/2}, \tag{8}$$

where $s = 2e\phi/m_i V_u^2$ is the normalized potential. Thus, the transmitted ion density is directly related to the potential, and measuring n one can immediately find s.

Now the relation to the magnetic field variations can be found directly from the pressure balance condition (which is model independent and must be satisfied both in hydrodynamics and kinetics)

$$n m_i v_x^2 + p_i + p_e + \frac{B^2}{2\mu_0} = \text{const.} \tag{9}$$

Using (7) and $nv_x = n_u V_u$ one immediately has

$$n_u m_i V_u^2 \sqrt{1-s} + p_i + p_e + \frac{B^2}{2\mu_0} = \text{const,} \tag{10}$$

where the last three terms can be measured directly and independently from the electric field measurements. Thus, (10) allows indirect measurement of the potential inside the ramp even if the electric field measurements are impossible or incomplete. This relation can be further simplified in the low-β plasma.

The lowest order approximation used above neglects thermal effects completely. This is not necessary, and calculation of p_i requires taking into account the broadening of the ion distribution inside the ramp [*Thomsen et al.*, 1985; *Sckopke et al.*, 1990]. The comprehensive analysis of the equations of motion (3)-(5) with the initial temperature of the incident ions taken into aacount has been done by *Gedalin* [1997], and we refer the reader to this paper for details, providing here only the relevant results (in the normalized form):

$$n/n_u = (1-s)^{-1/2}\left[1 + \frac{3s}{2(1-s)^2}\frac{\beta_i}{2M^2}\right], \tag{11}$$

$$\frac{p_i}{n_u m_i V_u^2} = \frac{\beta_i}{2M^2}(1-s)^{-3/2}, \tag{12}$$

and $v = 1/n$. It is easy to see that the correction to n remains small provided $\beta_i \ll M^2$, so that (8) is a quite good approximation for a wide range of shock parameters. The relation (8) or the equivalent $v = \sqrt{1-s}$ provides a method of indirect measurement of the potential by measuring the density of the transmitted ions. A good alternative would be measuring v_x at the maximum of the ion distribution throughout the ramp. It is worth noting that the ion velocity at the top of the ramp, as provided by this expression, $V_f = \sqrt{1-s_f}$ (where s_f is the total potential drop across the ramp) does *not* coincide with the downstream plasma velocity V_d, which appears in the Rankine-Hugoniot relations and is often used for estimates of the ion *flow* kinetic energy loss [*Wygant et al.*, 1987]. The difference $V_f - V_d$ determines the *gyration* velocity of the ion distribution behind the ramp and, therefore, the downstream ion temperature $T_d \sim m_i(V_f - V_d)^2/2$ [*Gedalin*, 1997]. Thus, the potential drop *must* obey the condition $e\phi < m_i(V_u^2 - V_d^2)/2$, otherwise the ion heating would be weak. Reformulating the above, the ion deceleration inside the ramp is almost solely due to the electrostatic field, while the eventual ion *flow* deceleration is completed with the magnetic deceleration *behind* the ramp.

Yet another method of indirect measurements of the electrostatic potential is to use (10) with the density given by (8) and ion pressure given by (12) (in the absence of reflected ions) or measured independently. The magnetic field measurements are most reliable, while the electron pressure can be measured too. If the particle measurements are difficult we need a model for the electron pressure. Electrons cannot be treated in simple kinetics. Indeed, electrons are accelerated by the cross-shock electric field, and if they are crossing the shock collisionlessly from upstream to downstream, their density decreases [see, e.g., *Gedalin*, 1999]. At microscales substantial deviations from quasineutrality can be expected accompanied with electric field spikes [*Bale et al.*, 1998]. However, at the scales under consideration, $\gtrsim c/\omega_{pe}$, quasineutrality cannot be violated in any noticeable way, which requires $n_e = n_i$. In order to reconcile the quasineutrality requirements with electron dynamics in the ramp we have to conclude that there is another electron population [*Scudder*, 1995] (trapped or leaked from downstream) or there are small scale electron instabilities causing effective collisions. The second option seems to be inevitable especially in view of strong two-stream instability developing in the ramp [*Gedalin*, 1999]. Necessity of collisions becomes quite clear in the case of a perpendicular shock where trapping or leakage are not possible. In any case, typical electron times are much smaller than the ramp crossing time, and electrons easily pass to the hydrodynamic stage. In the absence of a good quantitative model of mixed (collisionless+turbulent) electron heating in the ramp, we have to treat them phenomenologically, assuming some state equation. For the estimate we shall use the empirical polytropic law $p_e \propto n^\gamma$, where γ was found observationally to be between 5/3 and 2, from the upstream and downstream electron pressure comparison. Normalizing (9) on $n_u m_i V_u^2$ one finds

$$v = 1 - \frac{b^2-1}{2M^2} - \frac{\beta_i}{2M^2}(v^{-3}-1) - \frac{\beta_e}{2M^2}(v^{-\gamma}-1), \tag{13}$$

where $v \equiv v_x/V_u = n_u/n$, $b = B/B_u$, and $M^2 = \mu_0 n_u m_i V_u^2/B_u^2$. Since $v = \sqrt{1-s}$ equation (13) establishes the relation between the potential and magnetic field variations across the ramp. For sufficiently low-β shocks this relation is independent of the upstream parameters except Mach number. It should be understood that (13) is written for a low-Mach number shock and is not applicable for high-Mach number shocks where ion reflection occurs. It should be also understood that (13) is much more limited than (8) and (10) since it uses additional assumptions regarding electron behavior. Far from the ramp edges, $b-1 \sim 1$, and in the low-β limit the ratio of thermal contributions to the magnetic contribution into (13) is $\sim \beta \ll 1$, so that further simplification is possible:

$$\sqrt{1-s} = 1 - (b^2-1)/2M^2. \tag{14}$$

The precision of this relation should be quite good except at the downstream edge of the ramp, where gyrating ions begin to alter ion distribution significantly changing n and p_i. It should be also noted that the actual ion pressure may differ from the derived expression for the transmitted ions, because of the reflected ion population which may contribute noticeably into the ion heating inside the ramp even for low-Mach number shocks [*Thomsen et al.*, 1985; *Sckopke et al.*, 1990; *Zilbersher et al.*, 1998]. In this case one would have to use (10) with measured pressures. The reflected ion contribution into the ion density inside the ramp of low-Mach number shocks remains at the level of a few percent *Thomsen et al.* [1985]; *Sckopke et al.* [1990] and can be neglected in the derivation of n. The ion reflection itself is very sensitive to the ion temperature even for low β_i [*Sckopke et al.*, 1983; *Burgess et al.*, 1989; *Gedalin*, 1996a] since it occurs in the tail of the ion distribution.

3. Role of electrons

It is of interest to analyze the electron contribution into the current

$$\hat{\mathbf{x}} \times \frac{d\mathbf{B}_\perp}{dx} = \mu_0 n e(\mathbf{v}_{i\perp} - \mathbf{v}_{e\perp}), \tag{15}$$

where $\perp$ refers to y and z components, and $\hat{\mathbf{x}}$ is the unit vector in the shock normal direction. The ion contribution is small since the velocity $\mathbf{v}_{i\perp}$ is small. The perpendicular electron velocity can be estimated from hydrodynamical equations with the electron mass neglected (unless in perpendicular shocks), which simply gives

$$E_y + v_{ez}B_x - v_{ex}B_z = 0, \tag{16}$$

$$v_{ex}B_y - v_{ey}B_x = 0. \tag{17}$$

With $v_{ex} = v_x$ one gets

$$v_{ey} = v_x B_y/B_x, \qquad v_{ez} = (v_x B_z - E_y)/B_x. \tag{18}$$

Substituting into (15) one gets

$$\frac{dB_y}{dx} = -\frac{\mu_0 n e(v_x B_z - E_y)}{B_x} = -\frac{\mu_0 n_u e V_u \sin\theta}{\cos\theta}\left(\frac{B_z}{B_u \sin\theta} - \frac{V_u}{v_x}\right),$$

$$\frac{dB_z}{dx} = \frac{\mu_0 n e v_x B_y}{B_u \cos\theta} = \frac{\mu_0 e n_u V_u}{B_u \cos\theta} B_y,$$

where we have used $nv_x = n_u V_u$, $E_y = V_u B_u \sin\theta$, and $B_x = B_u \cos\theta$ (θ begin the angle between the shock normal and the upstream magnetic field. Further differentiation of the equation for dB_z/dx and substitution of dB_y/dx result in the following equation

$$\frac{c^2\cos^2\theta}{M^2\omega_{pi}^2}\frac{d^2 b_z}{dx^2} = N - b_z, \tag{19}$$

where $b_z = B_z/B_u \sin\theta$, and $N = n/n_u = 1/v$ is given by (13). One can see that $L \equiv c\cos\theta/M\omega_{pi}$ is the main scale parameter in the shock front (cf. *Mellott and* [1984]; *Farris et al.* [1993]; *Gedalin* [1998]). Equation (19) is valid for the magnetic field in the ramp but cannot be used for the description of the whole shock transition since downstream ion behavior (gyration) is quite different from their behavior in the ramp, and the dependence of v_x on B is no longer given by (13).

When the electron current dominates, using (15) with $\mathbf{v}_i$ neglected in the electron equation of motion

$$m_e\left(\frac{\partial v_x}{\partial t} + v_x\frac{\partial v_x}{\partial x}\right) = -eE_x - \frac{1}{n}\frac{dp_e}{dx} - e\hat{\mathbf{x}}\cdot(\mathbf{v}_e\times\mathbf{B})$$

and neglecting the electron mass, one obtains the following frequently used relation [*Gedalin*, 1996b]

$$eE_x = -\frac{1}{n}\frac{dp_e}{dx} - \frac{1}{n}\frac{d}{dx}\frac{B^2}{2\mu_0}. \tag{20}$$

Direct comparison of this relation with observations requires calculation of spatial derivatives or invoking model relations like $n/B \approx$ const and $p_e \propto \gamma$. Such model assumptions were used by *Schwartz et al.* [1988] for the determination of the de Hoffman-Teller potential. In our case the integration is more simple and does not require additional assumptions. Having established the relation (8) between the density and potential, (20) can be now directly integrated to

$$n_u V_u m_i\sqrt{V_u^2 - 2e\phi/m_i} + p_e + \frac{B^2}{2\mu_0} = \text{const}, \tag{21}$$

which is much easier to apply to observations since p_e and B can be directly measured and no spatial derivatives or phenomenological models need to be invoked (thus additional errors related to these two are also excluded). The first term in (21) is written for the lowest order approximation. In one wishes to take into account the nonzero ion temperature in the ramp, (11) has to be used when integrating (20), which would introduce relative corrections $\sim \beta_i/M^2(1-s)$. Unfortunately, the expression (20) cannot be used directly in high-Mach number shocks since the ion current is not negligible.

4. Discussion and conclusions

The only assumption used in the above derivation of the relation between the potential and ion density inside the ramp is that the incident ion $\beta_i \ll M^2$. We have also exploited the narrowness of the ramp, $L \lesssim c/\omega_{pi}$, which ensured that the main ion deceleration is produced by the electric field. Thus, the derived relation (8) (or the more precise version (11)) should be valid for virtually all low-Mach number shocks, unless the upstream temperature is too high. It can be used for the determination of the potential from the measurements of the transmitted ion density of by just following the maximum of the ion distribution. The relation (10) is also applicable in most cases and can be used for the determination of the potential from the direct measurements of the electron and ion pressure and magnetic field. The accompanying expression (13) is less universal since the electron pressure behavior cannot be theoretically predicted at this stage, and some empirical models should be applied. Yet for sufficiently low electron β_e the electron contribution is only a small correction and (13) with electron term omitted may be a good approximation. The last expression (21) is valid as long as the ion current is much smaller than the electron current, which is typical for low-Mach number shocks.

We also found that the ramp widths of oblique low-Mach number shocks should always correlate with $L = c\cos\theta/M\omega_{pi}$. The only reason for that is that almost massless, hydrodynamically behaving, electrons provide almost all current needed for the magnetic field increase. It should be noted that (19) can describe the magnetic field behavior inside the ramp but fails at the upstream and downstream edges, thus not allowing to derive the shock profile. The reason of the failure is the same as the reason for the failure of polytropic hydrodynamics: the ion behavior at the entry to the ramp and just behind the ramp is essentially kinetic, and the polytropic state equation is inappropriate.

In the low-Mach number case above the behavior of the ion velocity inside the shock was determined by the cross-shock potential and the fact that the ion motion is almost completely demagnetized in the thin ramp [*Gedalin*, 1997; *Zilbersher et al.*, 1998]. In high-Mach number shocks the *transmitted* ion behavior remains essentially the same [*Burgess et al.*, 1989; *Gedalin*, 1996a], since the ramp is thin [*Balikhin et al.*, 1995; *Newbury et al.*, 1998]. It has been shown that some ions gyrate back to the ramp from downstream and cross it once again backwards [*Gedalin*, 1996a]. Making another loop, almost half-gyration, just ahead of the ramp, these ions cross the ramp forward once again and finally drift away downstream. The number of the reflected ions correlates with the shock strength, B_d/B_u, and anti-correlates with the cross-shock potential. The ion reflection depends on the details of the magnetic and electric field distribution in the shock structure (foot, ramp, and overshoot), and there is no simple analytical estimate of the number of reflected ions and their distribution. It seems, at the first sight, that one could use the specular reflection approximation, where the back-gyrating ions leave the ramp with the velocity $\mathbf{v} = (-V_u, 0, 0)$ and enter the ramp again as an almost cold beam. In that case three fluid hydrodynamics would be appropriate for the ion description in the shock, and all we need would be the fraction of reflected ions. However, it has been shown [*Sckopke et al.*, 1983; *Gedalin*, 1996a] that the dispersion of the reflected ions is substantial already at the first return to the ramp from the downstream, so that the above used approximation is no longer applicable and fully kinetic description is needed. Since the ion reflection process is very sensitive to the details of the shock front and initial ion parameters, we cannot expect any pronounced similarity in the structure of the ramp of high-Mach number shocks, in contrast with what happens at low-Mach number shocks, so that our predictive ability is greatly reduced. Yet the relation between the velocity of the *incident* (transmitted) ions and the potential in the ramp is maintained here too, and, probably, even with greater precision because of the smaller thickness of the ramp and even weaker effects of the Lorentz force inside it. The ratio β_i/M^2 also decreases with the increase of the Mach number, so that zeroth-order approximation may appear to be even more efficient in the ramp of a high-Mach number shock than in low-Mach number shocks. Thus, one can measure the potential indirectly by measuring the velocity v_x of the maximum of the core (transmitted ions) throughout the ramp. Unfortunately, (10) and (21) are no longer useful, since reflected ions contribute significantly to the ion density and current, thus making n and electron velocities unusable for our objectives. It is worth mentioning that the incident ion velocity at the upstream edge of the ramp is no longer the upstream velocity V_u but the velocity of the ion flow decelerated in the foot [*Woods*, 1971; *Leroy et al.*, 1982; *Leroy*, 1983].

Direct measurements of the electrostatic potential even in low Mach number shocks are a very delicate task. Complications in such measurements are results of both physical and instrumental effects. In the ideal theoretical planar shock the motional component of the electric field related to the $\mathbf{V} \times \mathbf{B}$ drift must be constant across the shock front. However, nonstationarity of the shock front, errors in the determination of the shock normal, and possible nonplanar geometry of the front do not allow precise separation of the electrostatic electric field component, that results from the cross shock potential, from the motional electric field. That hampers the experimental determination of the cross shock potential. Instrumental effects can lead to the different offsets in the solar wind and in the magnetosheath that as well undermines determination of the electric field in the shock front transition region. The relation derived in the present paper allows to use density measurements to determine cross shock potential for low Mach number shock. Temporal resolution of particle instruments is often too low to provide complete information for subsequent determination of the cross shock potential, especially in narrow high-Mach number shocks. However, wave instruments, such as WHISPER instrument on board of Cluster spacecraft, can be used to determine plasma frequency (i.e. density as well) with resolution high enough to provide detailed profile of the electrostatic potential inside the ramp of most observed low-Mach number shocks and at least a part of high-Mach number shocks. It should be noted, however, that the shock front of a high-Mach number shock can be very structured and have no a single monotonic ramp. Since the above analysis is valid only in the narrow transition layers, the indirect determination of the potential distribution in a structured shock may be still a difficult observational problem requiring proper determination of the regions where the proposed method is applicable.

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