Long-wavelength mirror modes in multispecies plasmas with arbitrary distributions

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1. Introduction

Mirror modes are observed almost everywhere where the plasma β and the temperature anisotropy ratio $r = \beta_\perp/\beta_\parallel$ are sufficiently high [*Kaufmann et al.*, 1970; *Tsurutani et al.*, 1982; *Russell et al.*, 1987; *Kivelson et al.*, 1996; *Russell et al.*, 1999; *Balikhin et al.*, 2000]. The mirror instability in the proton-electron plasma has been extensively studied in the long-wavelength limit [*Rose*, 1965; *Hasegawa*, 1975; *Gary*, 1992; *Southwood and Kivelson*, 1993; *Pantellini and Schwartz*, 1995; *Pokhotelov et al.*, 2000; *Gedalin et al.*, 2001]. Dispersion relation, instability criterion, and growth rate for weak instability were derived for bi-Maxwellian distributions. Multispecies plasma is less studied and almost no analytical results exist. It was shown that the presence of heavier ion species affects the competing ion-cyclotron instability [*Gary*, 1992; *Gary et al.*, 1993]. *Huddleston et al.* [1999] have shown that dominating heavy ions can cause mirror instability themselves. These results were obtained using direct numerical solving of the complete Vlasov dispersion equation. However, the existing solvers (like WHAMP) work efficiently only with combinations of (shifted) Maxwellian distributions and so far there is no software which would allow to analyze highly non-Maxwellian distributions observed in the collisionless space plasmas. This emphasizes the need in the analytical study (at least in the long-wavelength range) which would show the general dependence of the mirror instability condition and growth rate on the plasma parameters. So far no systematic comprehensive analysis has been performed yet of the mirror instability in a multispecies plasma. Although expressions for the instability threshold were derived earlier [*Pokhotelov and Pilipenko*, 1976; *Northrop and Schardt*, 1980] in the integral form, they are too general to be applicable directly to multispecies non-Maxwellian plasmas. *Leubner and Schupfer* [2000] analyzed the instability threshold in a single species plasma (electrons and heavy species not taken into account) for a certain class of non-Maxwellian distributions. *Gedalin et al.* [2001] studied the effect of the non-Maxwellian shape (in the parallel direction for separable distributions) on the threshold and growth rate of the mirror instability in electron-proton plasmas, taking into account the hot electron contribution. In the present paper we generalize the analysis of *Gedalin et al.* [2001] onto the case of multispecies plasma when the shapes of the distributions may be arbitrary (but gyrotropic). We provide simple expressions in terms of a limited number of distribution moments which can be used for the determination of the instability threshold and (approximately) growth rate for any observed distribution, without necessity to approximate the distributions by bi-Maxwellian.

2. General theory

In the analytical treatment of the long-wavelength mirror instability we closely follow *Gedalin et al.* [2001]. In what follows we assume that for each species s the distribution function $f_s = f_s(v_\perp, v_\parallel)$, where subscripts $\perp$ and $\parallel$ refer to the ambient magnetic field direction. We are interested in the limit $kv_{Ts}, \omega \ll \Omega_s$, $0 < \omega/kv_{Ts} < \infty$ for all species. Here $\Omega_s = q_s B_0/m_s c$ is the gyrofrequency and $v_{Ts} = \sqrt{T_s/m_s}$ is the thermal velocity for species s. The general dielectric tensor has the form $\epsilon_{ij} = \delta_{ij} + \sum_s \lambda_{ij,s}$, and in our limit the dominant terms are written as follows [*Gedalin et al.*, 2001]:

$$\begin{aligned}
\lambda_{12,s} &= i\frac{\omega_{ps}^2}{\Omega_s\omega}, \quad \lambda_{33,s} = \frac{\omega_{ps}^2}{k_\parallel^2 v_{T\parallel s}^2}\chi_s^{(0)}, \\
\lambda_{22,s} &= \frac{\omega_{ps}^2}{\Omega_s^2}\left[1 + \frac{k_\parallel^2(v_{T\parallel s}^2 - v_{T\perp s}^2)}{\omega^2}\right. \\
&\left. -\frac{2k_\perp^2 v_{T\perp s}^2}{\omega^2} + \frac{2k_\perp^2\alpha_s v_{T\perp s}^4}{\omega^2 v_{T\parallel s}^2}\chi_s^{(2)}\right], \\
\lambda_{23,s} &= i\frac{\omega_{ps}^2}{\Omega_s\omega}\tan\theta\frac{v_{T\perp s}^2}{v_{T\parallel s}^2}\chi_s^{(1)},
\end{aligned} \tag{1}$$

where $\omega_{ps} = \sqrt{4\pi n_s q_s^2/m_s}$ is the plasma frequency for species s, and $v_{T\|,\perp}^2 = \int (v_{\|,\perp}^2) f dv_\| v_\perp dv_\perp$ (f is normalized on unity). The wavevector is chosen as $\mathbf{k} = (k_\perp, 0, k_\|) = k(\sin\theta, 0, \cos\theta)$. The functions $\chi_s^{(n)}$ are defined as follows:

$$\chi_s^{(n)} = \frac{1}{\langle u_\perp^{2n}\rangle} \int \frac{1}{Z_s - v} \frac{\partial f_s}{\partial v} u_\perp^{2n+1} du_\perp dv, \tag{2}$$

where $Z_s = \omega/k_\| v_{T\|s}$, $v = v_\|/v_{T\|s}$, $u_\perp = v_\perp/v_{T\perp s}$, $\langle\ldots\rangle = \int (\ldots) f u_\perp du_\perp dv$, and the integration is taken along the path below the singularity $v = Z_s$. The factor $\alpha_s = \langle v_{\perp s}^4\rangle / 2\langle v_{\perp s}^2\rangle^2$ describes the deviation of the distribution from the Maxwellian in the perpendicular direction, $\alpha = 1$ for the Maxwellian distribution and $\alpha = 1/2$ for the monoenergetic ring. The functions χ_s play the crucial role in the analysis. For separable distributions, $f = f_\perp(u_\perp) f_\|(v)$, one has

$$\chi_s^{(n)} = \chi_s = \int_{-\infty}^{\infty} \frac{1}{Z_s - v} \frac{\partial f_{\|,s}}{\partial v} dv. \tag{3}$$

It is now convenient to choose reference density n_0, mass m_0, and charge q_0. These reference parameters can be parameters of one of species, although this is not necessary. Otherwise they can be chosen arbitrarily from convenience arguments. We further define the normalized density $N_s = n_s/n_0$, mass $M_s = m_s/m_0$, and charge $Q_s = q_s/q_0$. The quasineutrality condition requires that $\sum_s N_s Q_s = 0$. We also define $Z = \omega/k_\| v_{T\|0}$, $R_s = T_{\|s}/T_{\|0}$, $r_s = T_{\perp s}/T_{\|s}$, and $\beta_0 = 2v_{T\|0}^2 \omega_{p0}^2/\Omega_0^2 c^2$, so that $Z_s = Z\sqrt{M_s/R_s}$.

For the dispersion relation one needs $D_{ij} = (k^2c^2/\omega^2 - 1)\delta_{ij} - k_i k_j c^2/\omega^2 - \sum_s \lambda_{ij,s}$, where $\mathbf{k} = (k_\perp, 0, k_\|) = k(\sin\theta, 0, \cos\theta)$. It is easy to see that $D_{12} \propto \sum_s N_s Q_s = 0$. Since $\lambda_{23} \propto \Omega_0/\omega$ and $\lambda_{33} \propto (\Omega_0/\omega)^2$, in the limit $\omega/\Omega_0 \to 0$ the dispersion relation takes the form $D_{22}D_{33} - D_{23}D_{32} = 0$. With all above taken into account and neglecting $1/\Delta \ll 1$ one eventually gets the dispersion relation in the following form

$$\begin{aligned}\Psi(Z) = \Bigg[& 2\beta_0^{-1} - Z^2\cos^2\theta \sum_s N_s M_s \\ & - \cos^2\theta \sum_s N_s R_s (1 - r_s) \\ & + 2\sin^2\theta \sum_s N_s R_s r_s \\ & -2\sin^2\theta \sum_s N_s R_s \alpha_s r_s^2 \chi_s^{(2)} \Bigg] \Big(\sum_s N_s Q_s^2 R_s^{-1} \chi_s^{(0)}\Big) \\ & + \sin^2\theta \Big(\sum_s N_s Q_s r_s \chi_s^{(1)}\Big)^2 = 0.\end{aligned} \tag{4}$$

Eq. (4) is the general dispersion relation for long-wavelength low-frequency mirror modes in a multispecies plasma. The species distributions are arbitrary with the only limitation that $f_s = f_s(u_\perp, v)$, that is, the distributions are gyrotropic.

3. Instability criterion

Assuming that there exists an unstable mode with $Z \to 0$ (nonpropagating limit) we expand (4) using the following expansion of $\chi_s^{(n)}$ [*Gedalin et al.*, 2001]:

$$\begin{aligned}\chi_s^{(n)} &= d_s^{(n)} + Z_s(a_s^{(n)} + i b_s^{(n)}), \\ d_s^{(n)} &= -\frac{1}{\langle u_\perp^{2n}\rangle} \int \frac{1}{v}\frac{\partial f_s}{\partial v} u_\perp^{2n+1} du_\perp dv, \\ a_s^{(n)} &= -\frac{1}{\langle u_\perp^{2n}\rangle} \int \frac{1}{v^2}\frac{\partial f_s}{\partial v} u_\perp^{2n+1} du_\perp dv, \\ b_s^{(n)} &= -\frac{\pi}{\langle u_\perp^{2n}\rangle} \int \left(\frac{1}{v}\frac{\partial f_s}{\partial v}\right)_{|v=0} u_\perp^{2n+1} du_\perp,\end{aligned} \tag{5}$$

where the integrals over v are the principal values integrals and convergence is required for the expansion. For $f_s = f_s(u_\perp, v^2)$ the parameters $a_s^{(n)} \equiv 0$. It is now straightforward to find

$$\begin{aligned}
&Z = -\Psi(0)/\Psi'(0),\\
&\Psi(0) = \Bigg[2/\beta_0 - \cos^2\theta \sum_s N_s R_s (1-r_s)\\
&+ 2\sin^2\theta \sum_s N_s R_s r_s\\
&-2\sin^2\theta \sum_s N_s R_s \alpha_s r_s^2 d_s^{(2)}\Bigg] \left(\sum_s N_s Q_s^2 R_s^{-1} d_s^{(0)}\right)\\
&+ \sin^2\theta (\sum_s N_s Q_s r_s d_s^{(1)})^2,\\
&\Psi'(0) = \Bigg[2/\beta_0 - \cos^2\theta \sum_s N_s R_s (1-r_s)\\
&+ 2\sin^2\theta \sum_s N_s R_s r_s\\
&-2\sin^2\theta \sum_s N_s R_s \alpha_s r_s^2 d_s^{(2)}\Bigg]\\
&\cdot \left(\sum_s \sqrt{M_s/R_s} N_s Q_s^2 R_s^{-1} C_s^{(0)}\right)\\
&- 2\sin_2\theta \left(\sum_s \sqrt{M_s/R_s} N_s R_s \alpha_s r_s^2 C_s^{(2)}\right)\\
&\cdot \left(\sum_s N_s Q_s^2 R_s^{-1} d_s^{(0)}\right)\\
&+ 2\sin^2\theta \left(\sum_s N_s Q_s r_s d_s^{(1)}\right)\\
&\cdot \left(\sum_s \sqrt{M_s/R_s} N_s Q_s r_s C_s^{(1)}\right),
\end{aligned} \tag{6}$$

where $C_s = a_s + ib_s$. This is correct only near the threshold where $|Z| \ll 1$, that is, $\Psi(0) \ll |\Psi'(0)|$, otherwise the above Taylor expansion is not justified. In general, the mirror mode has a real part (frequency) if the distribution functions are not symmetric, so that not all $a_s = \mathrm{Re}\, C_s = 0$. For usually considered symmetric distributions, $f_s = f_s(u_\perp, v^2)$, the instability is aperiodic.

The instability threshold at $Z = 0$ is found by solving $\Psi(0) = 0$ with respect to $\sin^2\theta$, and requiring $0 \le \sin^2\theta \le 1$, which gives

$$\begin{aligned}
&2\left(\sum_s N_s R_s \alpha_s r_s^2 d_s^{(2)}\right) - 2\left(\sum_s N_s R_s r_s\right)\\
&- \left(\sum_s N_s Q_s r_s d_s^{(1)}\right)^2 \left(\sum_s N_s Q_s^2 d_s^{(0)}/R_s\right)^{-1}\\
&- 2\beta_0^{-1} > 0,
\end{aligned} \tag{7}$$

provided that $\sum N_s R_s (r_s - 1) - 2/\beta_0 > 0$, which is always satisfied for $T_\perp/T_\parallel \ge 1$. For separable distributions $d_s^{(n)} = d_s = -\int v^{-1}(\partial f_{\parallel,s}/\partial v) dv$, provided that the last integral converges, that is, $(\partial f_{\parallel,s}/\partial v)|_{v=0} = 0$. If the last condition is not fulfilled the instability is not aperiodic.

It is worth noting that when $\sum N_s R_s (r_s - 1) + 2/\beta_0 < 0$ (which is possible only if at least one of the species has $r_s < 1$) and $D < 0$ the plasma is firehose unstable.

The expressions (6) and (7) are the most general expressions for weak mirror instability growth rate and instability condition. These expressions are correct for any gyrotropic distribution and any number of species and generalize the expressions found earlier [*Southwood and Kivelson*, 1993; *Pantellini and Schwartz*, 1995; *Pokhotelov et al.*, 2000; *Gedalin et al.*, 2001].

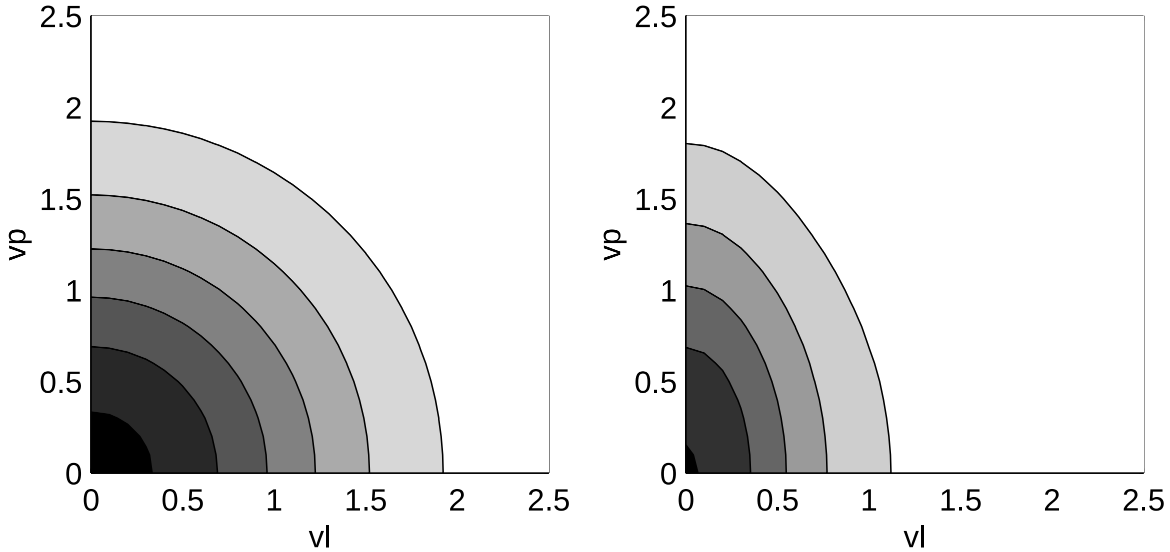
Let us consider the simplest case where all species have the same temperatures and temperature anisotropies, $\forall s: r_s = r$, $R_s = 1$, $\alpha_s = \alpha$, $d_s^{(n)} = d^{(n)}$. One immediately finds the instability criterion in the following form

$$r^2 d^{(2)} \alpha - r - 1/\beta_0 > 0, \tag{8}$$

which emphasizes the equally adiabatic behavior of all species in the $k, \gamma \to 0$ limit: the condition does not depend on the species mass, charge, or relative density. On the other hand, the dependence on d and α is very important. For the bi-Maxwellian distribution $\alpha = 1$ and $d = 1$, so that we arrive at the usual instability criterion [*Hasegawa*, 1975; *Southwood and Kivelson*, 1993; *Pantellini and Schwartz*, 1995; *Pokhotelov et al.*, 2000; *Gedalin et al.*, 2001]. For the monoenergetic ring distribution $f_\perp(v_\perp) = \delta(v_\perp^2 - v_{\perp 0}^2)$ the parameter $\alpha = 1/2$ and the instability is suppressed. On the other hand, let us consider the distribution which is Maxwellian in the perpendicular direction, $\alpha = 1$, and generalized Lorentzian,

$$f_\parallel(v_\parallel) = \frac{2^{n-1} v_0^{2n-1}}{\pi(2n-3)!!} \frac{1}{(v_\parallel^2 + v_0^2)^n}, \tag{9}$$

$n \geq 2$, in the parallel direction. It is easy to find that $v_{T\parallel}^2 = v_0^2/(2n-3)$, and $d = (2n-1)/(2n-3) > 1$. Thus, one can expect that the generalized Lorentzian distributions would be more unstable than the Maxwellian of the same temperature ratio $r = T_\perp/T_\parallel$. One interesting conclusion is that for $\beta_0 > 1/(d-1)$ no temperature anisotropy, $r \neq 1$, is required to excite mirror modes. For example, a generalized Lorentzian distribution with $T_\perp = T_\parallel$ is mirror unstable if $\beta_0 > (2n-3)/2$. For the lowest possible value $n = 2$ this gives $\beta_0 > 0.5$. It should be emphasized that equality of the temperatures (second moments of the distribution) in the parallel and perpendicular direction, $T_\perp = T_\parallel$, does not mean distribution isotropy as is illustrated in Figure 1 where filled contour plots are shown for the (a) Maxwellian $f = (2\pi)^{-3/2} \exp(-v_\perp^2/2 - v_\parallel^2/2)$



(a) (b)

Figure 1. Filled contour plots for the (a) Maxwellian and (b) Maxwellian-Lorentzian distributions for $T_\perp = T_\parallel$.

and (b) Maxwellian-Lorentzian $f = (1/\pi^2) \exp(-v_\perp^2/2)(v_\parallel^2 + 1)^{-2}$ distributions (for both velocities are normalized on v_T). The Maxwellian distribution is obviously isotropic, $f = f(v_\perp^2 + v_\parallel^2)$, while the Maxwellian-Lorentzian is not, despite the fact that in both cases $T_\perp = T_\parallel$. Thus, the instability is sensitive not only to the temperature anisotropy but also to the anisotropy of the distribution shape.

For the above case of identical temperature anisotropies for all species the plasma can be firehose unstable only when $\beta_0 > 2$ and $r < \min(1 - \frac{2}{\beta_0}, r_0)$, where $r_0 = (1 + \sqrt{1 + 4d\alpha/\beta_0})/2d\alpha$.

4. Examples

The expression (4) is the general long-wavelength dispersion for any number of species and for arbitrary (gyrotropic) distribution functions. In what follows we shall consider the behavior of the mirror instability assuming the distributions separable, so that there is no necessity to distinguish $\chi^{(j)}$ for different j. For the parallel distribution function we shall use the Lorentzian shape with $n = 2$. We shall assume also that the instability is moderate so that the long-wavelength low-frequency approximation qualifies. We believe, however, that the behavior of the instability in the range $kv_T, \gamma \sim \Omega$ can be understood, at least qualitatively, using (4) (see below).

To begin with let us consider a three-component plasma consisting of electrons, protons, and He^{++}. For the graphical presentation of the growth rate we put $\beta_0 = 8\pi n_e T_{e\parallel}/B^2 = 0.15$. We shall assume equal parallel temperatures for all species. Electrons and protons are assumed Maxwellian in the perpendicular direction, $\alpha_e = \alpha_p = 1$, while a ring distribution is used for He^{++}, so that $\alpha_h = 0.5$.

Figure 2 shows the growth rate of the mirror instability as a function of the propagation angle for different sets of

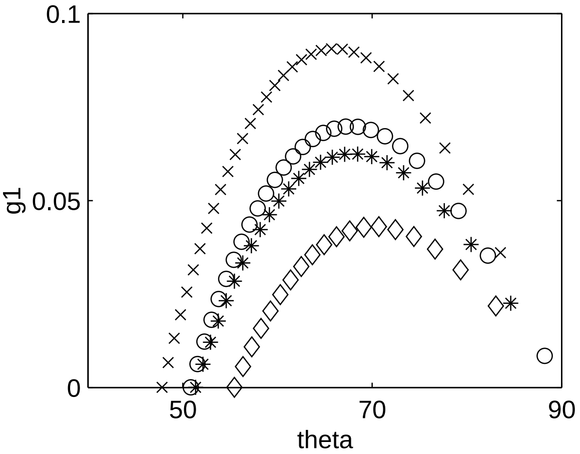


Figure 2. Growth rate of the mirror instability as a function of the propagation angle for the following parameter sets: (a) $n_h = 0$, $r_e = 1$, and $r_p = 2$ (stars), (b) $n_h = 0.1n_e$, $r_e = 1$, $r_p = 2$, and $r_h = 3$ (diamonds), (c) $n_h = 0.1n_e$, $r_e = 1.2$, $r_p = 2$, and $r_h = 3$ (circles), and (d) $n_h = 0$, $r_e = 1.2$, and $r_p = 2$ (crosses). Here $r_s = T_{s\perp}/T_{s\parallel}$ is the temperature anisotropy ratio for species s.

the plasma parameter. Stars correspond to the electron-proton plasma with isotropic electrons and $r_p = 2$. Diamonds correspond to a small admixture of α-particles, $n_h = 0.1n_e$, with the temperature anisotropy ratio $r_h = 3$. Circles stand for the three-component plasma when electrons are also anisotropic with $r_e = 1.2$, while crosses show the effects of the electron temperature anisotropy in the absence of α-particles. The presence of heavier ions (which replace a part of the protons) reduces the growth rate (in agreement with what was found earlier for bi-Maxwellian plasmas [*Gary*, 1992; *Gary et al.*, 1993]) but the difference is small and not especially significant unless the fraction of heavy ions is substantial and/or Q/M is significantly small (it is 1 for protons and 0.5 for α particles). On the other hand, heavy species can substantially suppress the ion-cyclotron instability [*Gary et al.*, 1993], so that the relative importance of the mirror instability increases.

It is worth noting that the instability range (minimum instability angle) also changes. If r_s and α_s were equal for all species this angle would not depend on the heavy ion density.

The suppressing effect of heavy ions is because of their reduced mobility (larger mass) relative to protons, which is crucial for the mirror instability [*Gedalin et al.*, 2001]. On the other hand, electrons are much more mobile, so that the strong effect of the electron anisotropy on the growth rate could be expected (see also *Pokhotelov et al.* [2000]). Indeed, the instability growth rate depends on the efficiency of particle removal from the magnetic enhancement region [*Southwood and Kivelson*, 1993], which means that a particle should move by a distance $l_\parallel \sim 1/k_\parallel$ during the time substantially less than the perturbation growth time $1/\gamma$. For the bulk of the distribution that means that $\gamma \lesssim k_\parallel v_{T\parallel}$. The last inequality is much better fulfilled for electrons. Anisotropic electron distributions with $T_{e\perp} > T_{e\parallel}$ are whistler unstable. Quasilinear relaxation does not result in complete isotropization and the post-saturation distributions remain anisotropic with $T_{e\perp}/T_{e\parallel} - 1 \approx \beta_{e\parallel}^{-0.5}$ [*Gary and Wang*, 1996]. Therefore, some electron anisotropy is not something extraordinary. Yet, their relative contribution is more substantial than that of ions, so that even modest electron anisotropies should be taken into account.

One of the situations where heavy ions may result in the mirror mode excitation is encountered on the edge of the Io wake, where mirror waves have been detected by the Galileo magnetometer [*Kivelson et al.*, 1996]. While the background torus plasma is nearly isotropic and stable to the mirror mode growth, highly anisotropic pickup ions are added at a high rate in the flow passing close to Io [*Russell et al.*, 1999]. Figure 3 shows an example of a series of mirror waves observed as Galileo moved out of the Io wake inward to Jupiter on Dec 7, 1995. These structures are those labelled 7, 8, 9, and 10 in *Russell et al.* [1999]. The magnetic field depressions have amplitudes that are a large fraction (up to 20 %) of the background field and are spaced aperiodically. The magnetic field between the depressions seems to be undisturbed, so that we expect that the local analysis of the background plasma parameters would indicate on instability. The growth rate of the mirror instability for the background plasma parameters was calculated numerically by *Huddleston et al.* [1999] within the approximation of the distributions as bi-Maxwellian (shifted bi-Maxwellian for ring distributions). Here we present the results of the analysis

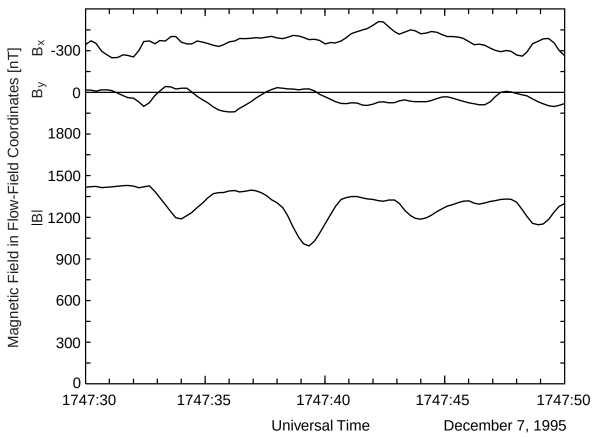


Figure 3. The magnetic field measured by Galileo as it passed out of the Io wake on Dec 7, 1995. The coordinate system is arranged so that the Z direction is antiparallel to the model field direction through the center of Io. The X direction is along the flow perpendicular to $\mathbf{B}$, and Y direction is toward Jupiter perpendicular to $\mathbf{B}$. The Z component is not shown because of the similarity to the total field.

of the same plasma using (4) and assuming Lorentzian distribution in the parallel direction. The below analysis should not be considered as a direct comparison with observations, since distribution shapes are not known, and the mirror modes have, probably, already grown to nonlinear amplitudes. Rather this should be considered as an example of the application of the developed theory to the problem of stability of the background plasma. *Russell et al.* [1999] and *Huddleston et al.* [1999] estimate the width of the mirror structures as several SO_2^+ gyroradii, and the angle of maximum growth rate as $\sim 60°$, so that our approach is expected to provide a rather good approximation.

For the illustration we use the parameters suggested by *Huddleston et al.* [1999] (see their Table 1 for the case A). The plasma composition is as follows: $n(e^-) = 10,000\text{cm}^{-3}$, $n(\text{H}^+) = 500\text{cm}^{-3}$, $n(\text{S}^+) = 2,000\text{cm}^{-3}$, $n(\text{S}^{++}) = 1,000\text{cm}^{-3}$, $n(\text{O}^+) = 5,000\text{cm}^{-3}$, and $n(\text{SO}_2^+) = 500\text{cm}^{-3}$. The parallel temperatures of all ion species are the same and equal $T_\parallel = 100\text{eV}$, which is taken as the "gauge" temperature T_0. The electron parallel temperature is 5eV. Electrons and protons are isotropic. The temperature anisotropies of other species are as follows: $r(\text{S}^+) = 4$, $r(\text{S}^{++}) = 3$, $r(\text{O}^+) = 3$, and the magnetic field $B = 1200\text{nT}$. The heavy species perpendicular distributions consist of background (Maxwellian) and ring components, and the parameters α (which we do not give here) are calculated by integration over these distributions. Figure 4 shows the growth rate of the mirror instability as a function of θ for the above parameters and various $r(\text{SO}_2^+) = 1$

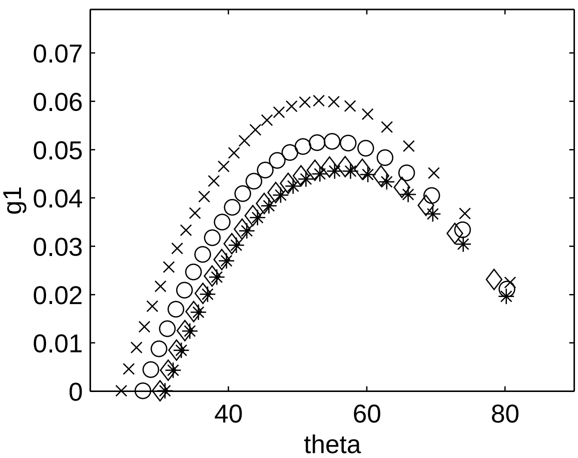


Figure 4. Growth rate for the mirror instability with heavy ions for various temperature anisotropy ratios $r(\text{SO}_2^+) = 1$ (stars), 4 (diamonds), 8 (circles), and 12 (crosses). Other parameters see in the text.

(stars), 4 (diamonds), 8 (circles), and 12 (crosses). It is easily seen that the effect of SO_2^+ on the mirror instability is negligible. At the same time *Huddleston et al.* [1999] find that the ion-cyclotron instability is extremely sensitive to the

SO_2^+ anisotropy. Thus, reduction the of SO_2^+ anisotropy might substantially suppress the ion-cyclotron instability while not affecting noticeably the mirror instability.

5. Discussion and conclusions

Throughout the present paper we used the long-wavelength low-frequency approximation where $\gamma, kv_{Ts} \ll \Omega_s$ for all species. From the previous numerical analyses [*Gary*, 1992; *Pantellini and Schwartz*, 1995] it is known that this is correct for weak instabilities, when the anisotropies are not too high and the growth rate is only a small fraction of the gyrofrequency. For higher anisotropies and stronger mirror instability the maximum growth rate occurs when $kv_{T\parallel}$ constitutes a substantial fraction of Ω. *Gary* [1992] finds that at $\beta_{p\parallel} = 1$ and $r_p = 2 - 2.5$ the ratio $k_\perp v_{Tp\perp}/\Omega_p \sim 0.5$, while $k_\parallel v_{Tp\parallel}/\Omega_p \sim 0.2$. The set of parameters used by *Pantellini and Schwartz* [1995] results in smaller ratios. In this range of parameters the use of the long-wavelength expressions is still a quite good approximation. There are two kinds of modifications at higher anisotropies or temperatures. The increase of $k_\perp v_{T\perp}/\Omega$ requires to use the complete Bessel functions instead of the lowest order expansion. Such modification should not affect qualitatively the behavior of the growth rate but may result in some numerical factor of the order of unity. The second kind of possible error is related to the denominators $\omega - k_\parallel v_\parallel - j\Omega$, which in the low-frequency limit are substituted by $-j\Omega$, when $j \neq 0$ [*Gedalin et al.*, 2001]. At the first sight presence of such denominators may cause significant effects which cannot be predicted by the derived expressions. However, because of the aperiodic nature of the mirror instability, $\omega = i\gamma$, $\gamma > 0$, the denominator never approaches zero but remain of the order of Ω, which also may result in a numerical factor but hardly change qualitatively the behavior of the growth rate. This conclusion is more or less confirmed by previous numerical analyses of bi-Maxwellian distributions. We, therefore, expect that the derived expressions would provide a satisfactory approximation even for a strong mirror instability. In any case, the proposed dispersion relation is more transparent and easy to use than the direct application of the WHAMP software, which can treat limited number of distribution shapes.

It can seem that inclusion of heavier species makes the approximation invalid, since kv_T/Ω scales as $\sqrt{M}/Q$. However, in the case when only a small admixture of heavy ions is included, their role is actually to reduce the number of protons participating in the instability, and their own contribution is unimportant. If heavy ions dominate, the above arguments hold if we substitute everywhere the word "protons" to "heavy particles". Thus, the derived expressions are expected to be useful not only for weak and moderate instability but also in the strong instability regime, if they are understood more qualitatively than quantitatively. Eq. (4) is not limited by any distribution shape, provided it is gyrotropic.

It is worth noting that the global instability criterion (7) is exact since it describes the instability near the threshold where $kv_T, \gamma \to 0$.

It is known [*Price et al.*, 1986; *McKean et al.*, 1992; *Gary et al.*, 1993] that the ion-cyclotron instability usually dominates over the mirror instability unless the former is suppressed by, for example, damping produced by heavies species. This question has been studied earlier numerically for bi-Maxwellian distributions and is beyond of the scope of the present paper. It seems that in the presence of a single ion species the ion cyclotron mode for that species will generally dominate over the mirror mode. However, if there are multiple ion species present, there are multiple ion cyclotron modes possible (perhaps no one is particularly dominant over the others) and yet at the same time, all of these species might still contribute to the mirror mode (to varying degrees). In such circumstances the mirror mode may dominate in multispecies plasmas.

The developed theory is linear while in most cases observed mirror modes are of large amplitude. Complete analysis of the instability and relative importance of ion-cyclotron and mirror modes in the final states, as well as comparison with observations, requires development of a nonlinear theory of saturation. One-dimensional hybrid simulations [*Price et al.*, 1986] have shown that in the electron-proton anisotropic plasma ion-cyclotron waves dominate after quasilinear relaxation (see also *Yoon* [1992]), while presence of heavier species can reverse the situation in favor of mirror modes. *McKean et al.* [1995] conclude that mirror modes are excited at the shock and further convected deeper into magnetosheath. In this case the local instability is not related to the developed structures. At the same time in some cases mirror modes seem to be excited locally. After quasilinear relaxation the system would be expected to be in a marginally stable state so that local instability analysis would misleadingly show that the growth rate is too low. However, in the case of a strong instability the mirror mode amplitude may quickly grow so that the plasma becomes strongly inhomogeneous and intermittent, with strong magnetic depressions or elevations neighboring almost undisturbed plasma regions. In this case anisotropy does not have to be removed in the undisturbed regions and the ambient plasma may remain locally unstable. The last argument shows that it is reasonable to check the instability criterion in the quiet plasma regions between the large aperiodically spaced magnetic depressions, as is seen in a number of observations (see, e.g. *Russell et al.* [1999]). If the instability is weaker and magnetic depressions and elevations fill the space more or less evenly, such instability test would be misleading.

In conclusion, we derived the most general dispersion relation and mirror instability condition in the long wavelength low-frequency limit, for a multispecies plasma with arbitrarily non-Maxwellian distributions of all species (the distributions are assumed gyrotropic though). The dispersion relation describes properly the weak instability and can be used as an approximation for moderate and even strong instability (which occurs in the range of the wavelengths comparable to the ion gyroradius), when numerical analysis (using WHAMP) is difficult because of the non-Maxwellian distribution shape. The dispersion is determined by a number of specially constructed integral characteristics of the distribution functions, and not only β. We derived an exact general mirror instability criterion, which is valid for arbitrary distributions.

We found that when replacing a part of the protons by heavier ions, the maximum growth rate of the instability reduces. The reduction is weak which, along with the substantial suppression of the ion-cyclotron instability [*Gary et al.*, 1993] may make mirror instability more favorable. When highly anisotropic heavy species dominate they themselves may be responsible for the mirror instability. Anisotropic electrons are more efficient in producing the instability than ions and their contribution may be important.

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