Noncoplanar magnetic field in the collisionless shock front

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1. 1. Introduction

Both observations and theory [*Goodrich and Scudder*, 1984; *Scudder et al.*, 1986a; *Thomsen et al.*, 1987; *Jones and Ellison*, 1987; *Gosling et al.*, 1988; *Friedman et al.*, 1990; *Jones and Ellison*, 1991; *Farris et al.*, 1993; *Scudder*, 1995] show that there is a substantial noncoplanar component of the magnetic field inside the shock front. In addition, a significant cross-shock potential electric field exists, which decelerates ions and accelerates electrons across the shock transition layer. This electric field is frame-dependent. The difference between the electric field in the the normal incidence frame (NIF)(where the upstream fluid velocity is directed along the shock normal) and in the de Hoffman-Teller frame (HTF)(where the fluid velocity is directed along the magnetic field at the both sides of the transition layer), is related to the noncoplanar magnetic field by the usual Lorentz transformations [*Goodrich and Scudder*, 1984]

$$E_x^{HT} = E_x^N + \frac{V_u}{c}\tan\theta B_y, \tag{1}$$

$$\varphi^{HT} = \varphi^N - \frac{V_u}{c}\tan\theta\int B_y\,dx, \tag{2}$$

where it is assumed that the shock normal is along x axis, and the noncoplanarity direction is along y axis, while θ is the angle between the shock normal and upstream magnetic field, and V_u is the upstream plasma velocity in NIF. The difference $\Delta\varphi = \varphi^N - \varphi^{HT}$ is found to be large, so that typically $\varphi^N/\varphi^{HT} \sim 2-6$ [*Thomsen et al.*, 1987] or even greater [*Scudder et al.*, 1986b].

An analytical expression for the spatially integrated noncoplanar magnetic field component

$$\int B_y\,dx \approx -\frac{B_x}{env}\int j_{yT}\,dx, \tag{3}$$

where $nv = n_uV_u = const$, and $j_{yT} = j_{y,e} + j_{i,e}$ is the total current in y direction, was proposed by *Jones and Ellison* [1987] in the assumption that $|j_{y,i}| \ll |j_{y,e}|$ due to the large mass ratio $m_i/m_e \gg 1$. Since $\mathbf{J}_T = (c/4\pi)\nabla\times\mathbf{B}$ (3) gives

$$\Delta\varphi \approx \frac{B_u\sin\theta\Delta B_z}{4\pi e n_u}, \tag{4}$$

The estimate given by (4) was found to be consistent with the values measured at low-Mach number low-beta shocks [*Friedman et al.*, 1990]. However, is strongly underestimates the spatial integral of the noncoplanar magnetic field in high-Mach number supercritical shocks, where actually observed values can be by an order of magnitude larger [*Gosling et al.*, 1988], than predicted by (4). This discrepancy was attributed to the substantial current of reflected ions within the shock transition layer [see *Gosling and Robson*, 1985, and references therein]. The ion current was shown to be large observationally [*Scudder et al.*, 1986a] and numerically [*Gosling et al.*, 1988]. On the basis of the data analysis, *Gosling et al.* [1988] proposed to substitute the electron current j_e for the total current j_T in (3). This phenomenological substitution appeared to be in agreement with observations, although no analytical justification nor validity domain analysis have been provided.

In the present paper we fill this gap by deriving general expressions for the noncoplanar magnetic field component on the basis of the stationary one-dimensional quasi-neutral hydrodynamics of two fluids with no additional assumptions. We analyze possible sources for deviations of observable noncoplanar magnetic fields from the predicted earlier by *Jones and Ellison* [1987] and (empirically) by *Gosling et al.* [1988].

2. 2. Basic Equations and Derivation

We start with the two-fluid hydrodynamics for electrons e and ions i with the traditional assumptions that (1) the shock structure is time stationary, (2) the shock is one-dimensional, and (3) the flow is quasi-neutral $n_e = n_i = n$ (cf. for example, *Goodrich and Scudder* [1984] and *Scudder et al.* [1986a]). The first two conditions mean $\partial/\partial t = \partial/\partial y = \partial/\partial z = 0$. The last condition means that also $v_{xe} = v_{xi} = v$.

The hydrodynamical equations take the following form:

$$v\frac{dv}{dx} = \frac{e}{m_i}E_x + \frac{e}{m_i c}\hat{\mathbf{n}}\cdot(\mathbf{U}_i \times \mathbf{B}_\perp) - \frac{1}{nm_i}\frac{d}{dx}P_{xx}^{(i)}, \tag{5}$$

$$v\frac{dv}{dx} = -\frac{e}{m_e}E_x - \frac{e}{m_e c}\hat{\mathbf{n}}\cdot(\mathbf{U}_e \times \mathbf{B}_\perp) - \frac{1}{nm_e}\frac{d}{dx}P_{xx}^{(e)}, \tag{6}$$

$$v\frac{d\mathbf{U}_i}{dx} = \frac{e}{m_i}\mathbf{E}_\perp + \frac{e}{m_i c}v\hat{\mathbf{n}}\times\mathbf{B}_\perp + \frac{e}{m_i c}B_x\mathbf{U}_i\times\hat{\mathbf{n}} - \frac{1}{nm_i}\frac{d}{dx}\Pi^{(i)}, \tag{7}$$

$$v\frac{d\mathbf{U}_e}{dx} = -\frac{e}{m_e}\mathbf{E}_\perp - \frac{e}{m_e c}v\hat{\mathbf{n}}\times\mathbf{B}_\perp - \frac{e}{m_e c}B_x\mathbf{U}_e\times\hat{\mathbf{n}} - \frac{1}{nm_e}\frac{d}{dx}\mathbf{\Pi}^{(e)}, \tag{8}$$

$$\hat{\mathbf{n}}\times\frac{d\mathbf{B}_\perp}{dx} = \frac{4\pi}{c}ne(\mathbf{U}_i - \mathbf{U}_e), \tag{9}$$

$$nv = n_u V_u = const, \tag{10}$$

where $\hat{\mathbf{n}} = (1,0,0)$ is the unit vector in the shock normal direction, subscript $\perp$ refers to the shock normal direction, that is $\mathbf{B}_\perp \perp \hat{\mathbf{n}}$, $\mathbf{U} \perp \hat{\mathbf{n}}$, $\mathbf{E}_\perp \perp \hat{\mathbf{n}}$, P_{ij} is the pressure tensor, and we use the following notation:

$$\mathbf{\Pi} = (0, P_{xy}, P_{xz}), \tag{11}$$

Because of the above assumptions $\mathbf{E}_\perp = const$.

For the perpendicular components of the velocity one has

$$\frac{d}{dx}(m_i\mathbf{U}_i + m_e\mathbf{U}_e) = \frac{B_x}{4\pi nv}\frac{d}{dx}\mathbf{B}_\perp - \frac{1}{nv}\frac{d}{dx}\mathbf{\Pi}^{(t)}, \tag{12}$$

where $\mathbf{\Pi}^{(t)} = \mathbf{\Pi}^{(e)} + \mathbf{\Pi}^{(i)}$. In the NIF, the boundary conditions read as follows: $\mathbf{U}_e, \mathbf{U}_i \to 0$ and $\mathbf{B}_\perp \to \mathbf{B}_{\perp 0}$ at $x \to -\infty$, and one has

$$\mathbf{U}_i + \mu\mathbf{U}_e = \frac{B_x}{4\pi nm_i v}(\mathbf{B}_\perp - \mathbf{B}_{\perp 0}) - \frac{1}{nm_i v}(\mathbf{\Pi}^{(t)} - \mathbf{\Pi}_0^{(t)}), \tag{13}$$

where $\mu = m_e/m_i$, and we assumed also $\mathbf{\Pi}^{(t)} \to \mathbf{\Pi}_0^{(t)}$ at $x \to -\infty$ (see below).

With the help of (9) one finds

$$\mathbf{U}_i = \frac{\mu}{1+\mu}\frac{c}{4\pi en}\hat{\mathbf{n}}\times\frac{d}{dx}\mathbf{B}_\perp + \frac{B_x}{4\pi nvm_i(1+\mu)}(\mathbf{B}_\perp - \mathbf{B}_{\perp 0}) - \frac{1}{nm_i v(1+\mu)}(\mathbf{\Pi}^{(t)} - \mathbf{\Pi}_0^{(t)}), \tag{14}$$

$$\mathbf{U}_e = -\frac{1}{1+\mu}\frac{c}{4\pi en}\hat{\mathbf{n}}\times\frac{d}{dx}\mathbf{B}_\perp + \frac{B_x}{4\pi nvm_i(1+\mu)}(\mathbf{B}_\perp - \mathbf{B}_{\perp 0}) - \frac{1}{nm_i v(1+\mu)}(\mathbf{\Pi}^{(t)} - \mathbf{\Pi}_0^{(t)}). \tag{15}$$

Substituting (14) into (7), one obtains the following equation for the magnetic field

$$\begin{aligned}
&\hat{\mathbf{n}} \times \mathbf{B}_\perp (1 - \frac{B_x^2}{4\pi n m_i v^2 (1+\mu)}) \\
&- \hat{\mathbf{n}} \times \mathbf{B}_{\perp 0} (\frac{V_u}{v} - \frac{B_x^2}{4\pi n m_i v^2 (1+\mu)}) \\
&= \frac{c B_x (1-\mu)}{4\pi n v e (1+\mu)} \frac{d}{dx} \mathbf{B}_\perp \\
&+ \frac{\mu c^2 m_i}{4\pi e^2 (1+\mu)} \frac{d}{dx} \frac{1}{n} \frac{d}{dx} \hat{\mathbf{n}} \times \mathbf{B}_\perp \\
&+ \frac{\mu c}{n v e (1+\mu)} \frac{d}{dx} \mathbf{\Pi}^{(i)} \\
&- \frac{c}{n v e (1+\mu)} \frac{d}{dx} \mathbf{\Pi}^{(e)} \\
&+ \frac{B_x}{n m_i v^2 (1+\mu)} (\mathbf{\Pi}^{(t)} - \mathbf{\Pi}_0^{(t)}) \times \hat{\mathbf{n}}
\end{aligned} \tag{16}$$

(16) is exact since no additional assumptions are used. It is completed with the asymptotic condition $\mathbf{E}_\perp = -(V_u/c)\hat{\mathbf{n}} \times \mathbf{B}_{\perp 0}$, where $v \to V_u$ at $x \to -\infty$.

In what follows, we shall use the widely accepted approximation $m_e = 0$. In this case (16) gives the following expression for the noncoplanar component of the magnetic field:

$$\begin{aligned}
&B_y (1 - \frac{B_x^2}{4\pi n m_i v^2}) = \frac{c B_x}{4\pi n v e} \frac{d}{dx} B_z \\
&- \frac{c}{n v e} \frac{d}{dx} P_{xz}^{(e)} + \frac{B_x}{n m_i v^2} P_{xy}^{(t)}
\end{aligned} \tag{17}$$

where we assumed that the pressure is gyrotropic $P_{xy} = 0$ in the asymptotic upstream region where $B_y = 0$.

Closer inspection of (15) and (17) shows that the last equation can be written in the following form:

$$B_y = -\frac{B_x}{n v} j_y^{(e)} - \frac{c}{n v e} \frac{d}{dx} P_{xz}^{(e)}, \tag{18}$$

where $\mathbf{j}^{(e)} = -ne\mathbf{U}_e$. When the electron pressure anisotropy is negligible $P_{xz}^{(e)} = 0$ and (18) reduces $B_y \propto j_y^{(e)}$, which proposed by *Gosling et al.* [1988]. It should be mentioned thay *Gosling et al.* [1988] obtained their expression in the one-dimensional hybrid numerical simulations with isotropic electron pressure, that is, when $P_{xz}^{(e)} = 0$. Precision of the approximation $B_y \propto j_y^{(e)}$ depends on the degree of the electron pressure anisotropy.

In the high-Mach number quasi-perpendicular shocks $B_x^2/4\pi n m_i v^2 = (\cos^2\theta/M^2)(n/n_u) \ll 1$. Assuming also that the electron pressure is gyrotropic everywhere, that is,

$$P_{ij}^{(e)} = P_\perp^{(e)} \delta_{ij} + (P_\parallel^{(e)} - P_\perp^{(e)}) \frac{B_i B_j}{B^2}, \tag{19}$$

(this assumption is reasonable due to the small electron gyroradius [*Scudder et al.*, 1986a]), one has

$$\begin{aligned}
B_y &= l_W \frac{d}{dx} (1 + \frac{\beta_\perp^{(e)} - \beta_\parallel^{(e)}}{2}) B_z \\
&- \cos\theta \, \frac{n}{n_u} \frac{P_{xy}^{(t)}}{n_u m_i V_u^2} B_u,
\end{aligned} \tag{20}$$

$$\begin{aligned}
\Delta\phi &= \frac{e\Delta\varphi}{\epsilon_i} = \Delta\phi_{diag} \\
&- \frac{2\sin\theta}{M(c/\omega_{pi})} \int \frac{n}{n_u} \frac{P_{xy}^{(t)}}{n_u m_i V_u^2} dx,
\end{aligned} \tag{21}$$

$$\begin{aligned}
\Delta\phi_{diag} &= \frac{2\sin^2\theta}{M^2} \Delta[(1 \\
&+ \frac{\beta_\perp^{(e)} - \beta_\parallel^{(e)}}{2}) \frac{B_z}{B_u \sin\theta}],
\end{aligned} \tag{22}$$

where $\beta_{\perp,\parallel}^{(e)} = 8\pi P_{\perp,\parallel}^{(e)}/B^2$, $l_W = c\cos\theta/M\omega_{pi}$, and $\epsilon_i = m_i V_u^2/2$ is the incident ion energy.

Anisotropy of the electron pressure results in the modification (22) of the expression proposed by *Jones and Ellison* [1987]. This part of the potential depends only on the initial and final values of B_z and $\beta^{(e)}_{\perp,\parallel}$. Another part of the potential difference is due to the off-diagonal (in the shock coordinates) component of the total pressure, which is in turn, mostly due to the existence of gyrophase-bunched ions [*Gurgiolo et al.*, 1981; *Sckopke et al.*, 1983, 1990; *Li et al.*, 1995]. This part is essentially nonlocal and depends crucially on the spatial dependence of the ion distribution function.

The potential distribution across the shock is of importance for particle dynamics in the shock front. In particular, the potential drop at the ramp is believed to determine the ion reflection [*Leroy*, 1983; *Schwartz et al.*, 1983; *Wilkinson and Schwartz*, 1990] and electron heating [*Feldman*, 1985; *Scudder*, 1995; *Gedalin et al.*, 1995]. Assuming $|\beta^{(e)}_\perp - \beta^{(e)}_\parallel| \lesssim 1$ we estimate the relative importance of the off-diagonal pressure component in the ramp as

$$\frac{B_{y,off}}{B_{y,diag}} \sim \cos\theta \frac{l_B}{l_W}\frac{|P_{xy}|}{n_u m_i V_u^2}, \tag{23}$$

where we have used also $(n/n_u) \approx (B_z/B_u)$ [*Scudder et al.*, 1986a] and $\sin\theta \approx 1$ for quasi-perpendicular shocks. The typical scale of the magnetic field variation
$l_B \sim B_z/(dB_z/dx)$ in the ramp $l_B \sim \pi l_W$ [*Mellott and Greendstadt*, 1984; *Scudder et al.*, 1986a; *Farris et al.*, 1993]. Estimating $|P_{xy}|/n_u m_i V_u^2 \sim (n_r/n_u)$, where n_r is the reflected ion fraction [*Gedalin and Zilbersher*, 1995], one finds that the ratio in (23) is ~ 0.1. Thus there is no large error in using the expression of *Jones and Ellison* [1987] for the potential at the ramp. The smaller is the scale of the magnetic field variation the less is the relative contribution of the off-diagonal terms.

On the other hand, similar estimate in the foot and downstream, where $l_B \sim V_u/\Omega_u$ [*Woods*, 1971; *Leroy*, 1983; *Scudder et al.*, 1986a] shows that the noncoplanar magnetic fields (and therefore potential difference) in these regions may be completely determined by the off-diagonal pressure P_{xy}. The corresponding

$$\Delta\phi_{off} \sim -(\frac{\langle nP_{xy}\rangle}{n_u^2 m_i V_u^2}), \tag{24}$$

where $\langle\cdot\rangle$ denotes average and the integration is carried out along the length $\sim V_u/\Omega_u$ [cf. *Scudder et al.*, 1986a]. This value may be large and $\sim m_i V_u^2/2$ due to possible correlation of $-P_{xy}$ with n.

These conclusions are in agreement with the observation that $j_T \propto dB_z/dx$ and rapidly decreases with the increase of the typical scale. *Gosling et al.* [1988] found small j_T and especially large ratio j_e/j_T in the transition layer is because in hybrid simulations ramp is wide ($\sim 2(c/\omega_{pi})$). Observed shocks [*Scudder et al.*, 1986a] exhibit similar jumps of the magnetic field $B/B_u \sim 5-6$ at the ramp but much smaller ramp width $\sim 0.2(c/\omega_{pi})$.

3. 3. Conclusions

We have derived general expressions for the noncoplanar magnetic field which relate this magnetic field component to the pressure tensor. Strong deviations from the regime of *Jones and Ellison* [1987] are shown to be due to large off-diagonal components of the ion pressure, which in turn is a direct manifestation of the presence of gyrophase-bunched ions. In this way we explained the success and the domain of validity of the approach by *Gosling et al.* [1988]. We have shown also that the noncoplanar magnetic field and the potential electric field in the thin ramp are only weakly modified due to the pressure. However, the off-diagonal pressure induced field may dominate in the extended foot and downstream region. These extended regions may also contribute largely in the overall spatial integral, probably even increasing it by an order of magnitude [cf. *Gosling et al.*, 1988] relative to the laminar value predicted by *Jones and Ellison* [1987].

It can be, of course, that the underlying assumptions (one-dimensionality, stationarity, and quasi-neutrality) are violated in the supercritical shock front. Strong fluctuations of the normal component of the magnetic field are observed in the shock front [see *Farris et al.*, 1993], when the Mach number exceeds the critical Mach number, which implies that the shock is not exactly one-dimensional and/or stationary. *Scudder et al.* [1986a] also found that the shock is not exactly one-dimensional but the typical scale of the magnetic field variation along the shock front is always substantially greater than the corresponding scale along the shock normal. Consideration of how nonstationarity and non-one-dimensionality can modify the results of *Jones and Ellison* [1987]; *Gosling et al.* [1988], and ours is beyond the scope of the present report. It should be noted, however, that (14), (15), and (20) (in a more general form (16)) provide yet another tool for testing the shock one-dimensionality and stationarity, since magnetic field and ion and electron distributions are directly measured at the shock front.

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