

**ON THE EXISTENCE OF INERTIAL MANIFOLDS AND
GLOBAL ATTRACTORS IN INFINITE DIMENSIONAL
SYSTEMS; IMPLICATIONS FOR THE STUDY OF
SPACE PLASMA TURBULENCE USING NONLINEAR
DYNAMICS APPROACHES.**

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Numerous nonstationary processes in space plasma can be modelled in terms of infinite-dimensional nonlinear partial differential equations. Fortunately, many classes of infinite dimensional systems that exhibit complex (turbulent) dynamics can be characterized in terms of a finite number of degrees of freedom. This behaviour has been related to the existence of a finite-dimensional global attractor for the infinite dimensional flow, which can be fractal like a Cantor set. The attractor provides the best mathematical characterization of the observed nonstationary flow. The global attractor however is in principle complex and difficult to compute. A more practical concept, originally introduced by Foias, Sell, & Temam (1988), is that of inertial manifold. The inertial manifold is a smooth, low-dimensional invariant manifold which contains the global attractor and attracts exponentially all the trajectories. Furthermore, the dynamics of the system, when restricted on the inertial manifold, is described by a system of ordinary differential equations, which are known as the inertial form of the corresponding infinite dimensional system. The main advantage of the inertial form is that it can be used to accurately simulate, analyse and characterise the infinite dimensional system at much lower computational cost than one which is usually associated with standard approximating methods. Many computationally prohibitive problems associated with infinite dimensional systems can become tractable within the inertial manifold framework. Inertial manifolds provide a useful framework in which to perform analysis and characterize plasma turbulence using reduced order models.