Restricted quantum-classical correspondence and counting statistics for a coherent transition

Maya Chuchem and Doron Cohen

Department of Physics, Ben-Gurion University, Beer-Sheva 84105, Israel

The conventional probabilistic point of view implies that if a particle has a probability p to make a transition from one site to another site, then the average transport should be $\langle Q \rangle = p$ with a variance $\mathrm{Var}(Q) = (1-p)p$. In the quantum mechanical context this observation becomes a non-trivial manifestation of restricted quantum-classical correspondence. We demonstrate this observation by considering the full counting statistics which is associated with a two level coherent transition in the context of a continuous quantum measurement process. In particular we test the possibility of getting a valid result for $\mathrm{Var}(Q)$ within the framework of the adiabatic picture, analyzing the simplest non-trivial example of a Landau-Zener crossing.

I. INTRODUCTION

The discussion of quantum-classical correspondence (QCC) in the mathematical physics literature is as old as the history of quantum mechanics. Traditionally one has in mind complicated semiclassical approximation schemes for calculating the propagation of wavepackets. A more elaborated analysis of the spreading process [1], leads to the distinction between robust *restricted QCC* and fragile *detailed QCC*. Restricted QCC means that $\langle A \rangle_{\mathrm{qm}} \approx \langle A \rangle_{\mathrm{cl}}$ holds only for a restricted set of observables, and survives even in the presence of diffraction, while detailed QCC means $\langle A \rangle_{\mathrm{qm}} \approx \langle A \rangle_{\mathrm{cl}}$ for all the well behaved observables, and requires smooth potentials as in the traditional formulation.

Controlling atoms in a few site system is state of the art [2, 3]. The prevailing studies are focused in restricted questions such as "what is the probability for a transition from one site to the other site" [4, 5]. But what about the associated noise [6, 7] and the full counting statistics (FCS) of a quantum transition? We argue below that the simplest example of a single particle in a closed two site system can be worked out exactly so as to illuminate the essence of restricted QCC in the context of FCS studies. This has the side benefit of shedding new light on the vague picture in the pioneering works [8, 9] about shot noise, where the discussion of FCS is immersed in complicated diagrammatic calculations involving a many-body system of Fermions in an open geometry, hence obscuring the simple physics that underlays the bottom line results. While most follow-up publications about FCS [10, 11, 12, 13, 14] are aimed in studying the distribution of transmission eigenvalues, our concern below is with some fundamental aspects that distinguish quantum dynamics from its classical stochastic analog.

We consider the simplest non-trivial example: the adiabatic crossing of a particle from one site to another site, which involves a two level Landau-Zener transition [15, 16, 17, 18]. Within the framework of a classical probabilistic point of view the particle has some probability p to make the transition. In any particular realization the particle either makes the transition or not. Accordingly the integrated current from the first to the second site is either $Q = 1$ or $Q = 0$ respectively. It follows that

$$\langle \mathcal{Q} \rangle = p \tag{1}$$

with the variance

$$\mathrm{Var}(\mathcal{Q}) = (1-p)p \tag{2}$$

This classical probabilistic point of view does not hold in the quantum mechanical reality. The characterization of the full counting statistics requires some care in the description of a continuous quantum measurement. Accordingly one has to distinguish between the naive mathematical definition [8] of the Q probability distribution $\mathrm{P}(Q)$, and the proper physical definition [20] of the quasi-probability distribution $\mathrm{P}(Q; x)$, where x signifies the strength of the interaction with the detector. It is important to realize that in general $\mathrm{P}(Q; x{=}0)$ is not the same as $\mathrm{P}(Q)$, but still they have the same first and second moments.

This paper has two parts. In the first part we establish restricted QCC for the counting statistics using general principles. In the second part we study whether the adiabatic approximation can be used in order to derive leading order results. The importance of the latter analysis becomes apparent once one tries to obtain results for more complicated multiple-path geometries where exact solution for $\mathrm{P}(Q)$ is out of reach. We further discuss this latter point in the concluding section.

II. RESTRICTED QCC FOR MOMENTS OF $\mathcal{Q}$

For a general network that consists of several sites we can define a set of occupation operators $\mathcal{N}$ and current operators $\mathcal{I}$. In the Heisenberg picture the time derivative of any occupation operator equals a sum over the ingoing current operators. In order to demonstrate the general idea of restricted QCC we consider below the simplest example of one particle in a two-site system. The

model Hamiltonian in the position basis is

$$\mathcal{H} = \begin{pmatrix} \alpha t/2 & c \\ c & -\alpha t/2 \end{pmatrix}, \tag{3}$$

where c is the hopping amplitude. We define an occupation operator $\mathcal{N}$ and a current operator $\mathcal{I}$ as follows:

$$\mathcal{N} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{4}$$

$$\mathcal{I} = \begin{pmatrix} 0 & ic \\ -ic & 0 \end{pmatrix} \tag{5}$$

Using the Heisenberg picture the two are related by the equation

$$\frac{d}{dt}\mathcal{N}(t) = \mathcal{I}(t) \tag{6}$$

where $\mathcal{I}(t) = U(t)^\dagger \mathcal{I} U(t)$, and $U(t)$ is the evolution operator. Consequently it is useful to define a counting ("transported charge") operator:

$$\mathcal{Q} = \int_0^t \mathcal{I}(t')dt' \tag{7}$$

From the Heisenberg equation of motion Eq.(6) it follows that the occupation probability $\langle \mathcal{N}(t) \rangle$ and the probability current $\langle \mathcal{I}(t) \rangle$ are related by a *continuity equation*. But in fact we can derive a stronger statement. By integrating Eq.(6) over time one obtains

$$\mathcal{Q} = \mathcal{N}(t) - \mathcal{N}(0) \tag{8}$$

Assuming that the particle is initially in the left site, which is a zero eigenstate of $\mathcal{N}$, one concludes the non-trivial relation

$$\langle \mathcal{Q}^k \rangle = \langle \mathcal{N}^k \rangle_t \qquad \text{for } k = 1, 2. \tag{9}$$

This relation between the kth moment of the counting operator (as determined in the Heisenberg picture by an expectation value at the reference time $t=0$) and the kth moment of the occupation operator (as determined in the Schrodinger picture by an expectation value at time t) does not hold for $k > 2$, because the operators in $\langle 0|[\mathcal{N}(t) - \mathcal{N}(0)]^k|0\rangle$ are non-commuting.

Thus for a coherent quantum transition where initially $\langle \mathcal{N}^k \rangle_0 = 0$, we find at the end of the process $\langle \mathcal{N}^k \rangle_t = p$, and consequently it follows that $\langle \mathcal{Q}^k \rangle = p$ for $k = 1, 2$, leading to Eq.(1) and Eq.(2). But the higher moments would be different compared with the classical expectation, as further discussed in the next section, and therefore we have here a very simple example for *restricted* rather than detailed QCC.

III. THE FCS - NAIVE RESULTS

Both Eq.(1) and Eq.(2) can be derived on the basis of a classical probabilistic point of view. The classical reasoning is based on the following idea regarding the counting statistics:

$$P(Q) = \begin{cases} 1-p & \text{for } Q = 0 \\ p & \text{for } Q = 1 \end{cases} \tag{10}$$

The kth moment of this probability distribution is $\langle Q^k \rangle = p$, leading to Eq.(1) as well as to Eq.(2). But we are going to explain that the classical reasoning is wrong: It is wrong both according to the naive mathematical definition of $P(Q)$, and also according to the proper physical definition of $P(Q; x)$, which we review in Appendix A. Consequently in the present section we derive the mathematically correct results for $P(Q)$, while in the next section we work out the physically meaningful result for $P(Q; x)$.

Let us elaborate on the straightforward procedure for naive FCS calculation. The first step is to write the current operator in the Heisenberg picture:

$$\mathcal{I}(t)_{nm} = \langle n|U(t)^\dagger \mathcal{I} U(t)|m \rangle \tag{11}$$

It is important to realize that the current operator and hence the counting operator have a zero trace. Consequently integrating over time we get

$$\mathcal{Q}_{nm} \equiv \begin{pmatrix} +Q_\parallel & iQ_\perp \\ -iQ_\perp^* & -Q_\parallel \end{pmatrix} \tag{12}$$

The first two moments $\langle \mathcal{Q} \rangle$ and $\langle \mathcal{Q}^2 \rangle$ are obtained from this matrix, leading to the identifications:

$$\langle \mathcal{Q} \rangle = Q_\parallel \tag{13}$$
$$\text{Var}(\mathcal{Q}) = |Q_\perp|^2 \tag{14}$$

The zero trace property also implies that the eigenvalues of $\mathcal{Q}$ are opposite in sign:

$$Q_\pm = \pm\sqrt{(Q_\parallel)^2 + |Q_\perp|^2} \tag{15}$$

On the basis of the continuity equation we can argue that $\langle \mathcal{Q} \rangle_\psi \leq 1$ for any preparation. Therefore we must get $|Q_\pm| \leq 1$. In fact we can deduce much more on the basis of restricted QCC. Observing that Eq.(13) and Eq.(14) should be in agreement with Eq.(1) and Eq.(2) one deduces that

$$Q_\parallel = p \tag{16}$$
$$Q_\perp = \sqrt{(1-p)p} \times \text{PhaseFactor} \tag{17}$$

If we could regard $\mathcal{Q}$ as a conventional observable, then upon measurement its observed values would have the distribution

$$P(Q) = \begin{cases} p_- & \text{for } Q = Q_- \\ p_+ & \text{for } Q = Q_+ \end{cases} \tag{18}$$

This distribution is characterized by two parameters because $Q_\pm$ are opposite and sign, and $p_\pm$ sum up to unity. Restricted QCC provides the two equations $\langle \mathcal{Q} \rangle = p$ and $\langle \mathcal{Q}^2 \rangle = p$ that can be solved, leading to

$$Q_\pm = \pm\sqrt{p} \qquad (19)$$

$$p_\pm = \frac{1}{2}\left(1 \pm \sqrt{p}\right) \qquad (20)$$

where the expression for $Q_\pm$ is in agreement with Eqs.(15)-(17) of the previous paragraph. A look-alike result had been obtained for shot noise of Fermions using complicated diagrammatic techniques [8]. An equivalent way to express Eq.(18) is to say that the kth moment is

$$\langle \mathcal{Q}^k \rangle = p_+ Q_+^k + p_- Q_-^k = p^{\left\lfloor \frac{k+1}{2} \right\rfloor} \qquad (21)$$

where $k = 0, 1, 2, 3, ...$ and $\lfloor ... \rfloor$ stands for the integer part (i.e. rounded downwards). The corresponding classical result is p irrespective of k. One may say that we have encountered here the simplest example of *restricted* QCC.

IV.   THE FCS - PHYSICAL RESULTS

In a later publication [9] the naive mathematical definition of the full counting statistics has been criticized. The most illuminating approach [20] is to analyze the reduced dynamics of the detector. Using the standard von-Neumann pointer scheme [21] and transforming to the Wigner representation one deduces that the final state $\rho(q, x)$ of the pointer is the convolution of its initial state with a kernel $P(q-q'; x)$, as defined in Eq.(A5). It follows that

$$P(Q; x{=}0) = \qquad (22)$$
$$\frac{1}{2\pi} \int \left\langle \left[\mathcal{T} e^{-i(r/2)\mathcal{Q}} \right]^\dagger \left[\mathcal{T} e^{+i(r/2)\mathcal{Q}} \right] \right\rangle e^{-iQr} dr$$

can be regarded as a quasi-distribution that describes the full quantum statistics. It is of course a physically measurable object. The naive mathematical definition is obtained if we ignore time ordering:

$$P(Q) = \frac{1}{2\pi} \int \langle e^{+ir\mathcal{Q}} \rangle e^{-iQr} dr$$
$$= \langle \delta(Q - \mathcal{Q}) \rangle \qquad (23)$$

In general the calculation of $P(Q; x{=}0)$ is very complicated. But we can gain some insight by considering the simplest case of a Bloch transition, which is Eq.(3) with $\alpha = 0$. After time t the probability to find the particle in the second site is $p = [\sin(ct)]^2$. The expectation value in the integrand of Eq.(22) is

$$\tilde{P}(r) = 1 - \frac{r}{(r/2) + i} \left[\sin\left(ct\sqrt{1 + (r/2)^2} \right) \right]^2 \qquad (24)$$

[More conveniently one can use instead of Eq.(22) the equivalent expression Eq.(A5), where the Us can be interpreted as spin 1/2 rotation matrices]. In order to get

$P(Q; x{=}0)$ we have to Fourier transform (FT) the function $\tilde{P}(r)$. For the purpose of discussion let us assume long times ($ct \gg 1$) so as to have separation of scales. Then we can distinguish between a central part where $\tilde{P}(r) \approx 1 + ipr - (1/2)pr^2 + ...$ as implies by restricted QCC, and oscillatory far tails where $\tilde{P}(r) \approx \cos(ctr)$. The FT of the central part gives a non-singular exponential-like piece, while the FT of the tails contributes two delta functions $(1/2)\delta(Q \pm ct)$ which are screened by negative clouds as illustrated in Fig. 1. We can regard the singular delta functions at $Q = \pm ct$ as the remnants of those that are centered at $Q_\pm = \pm\sqrt{p}$ in the naive calculation. It is interesting that for long time their location reflects the eigenvalues of the current operator rather than the eigenvalues of the counting operator.

Fig. 2 is a caricature that illustrates the significance of the FCS results with regard to the outcome of a quantum measurement. We sketch the final probability distribution of the Von Neumann pointer as implied by different "conceptions". The upper panel is based on the classical expectation of having either $Q = 0$ or $Q = 1$ displacement. The middle panel is the naive quantum mechanical prediction that suggests $Q = Q_\pm$ displacements, while the lower panel is the actual quantum prediction based on a convolution with $P(Q; x)$. An actual numerical illustration for the outcome of this convolution is presented in Fig. 3. Needless to say that in principle one can measure not only the q distribution but the whole probability matrix so as to determine $P(Q; x)$ and in particular the quasi probability distribution $P(Q; x{=}0)$ via a deconvolution procedure. But looking in Fig. 3, it is nice to realize that the main features of the FCS are not smeared and pop to the eyes even without any deconvolution.

V.   FCS IN THE ADIABATIC APPROXIMATION

In the following sections we would like to examine the capabilities of the *leading order* adiabatic approximation in obtaining physically significant results for Var($\mathcal{Q}$). We consider the model system of Eq.(3). Initially (at $t{=}-\infty$) the particle is in the ground state, which is the left site. The probability at $t{=}\infty$ to find the particle in the right site is

$$p = 1 - P_{\mathrm{LZ}} \qquad (25)$$

where

$$P_{\mathrm{LZ}} = \exp\left[-2\pi\frac{c^2}{\alpha}\right] \equiv \exp\left[-\frac{\pi}{2}\gamma\right] \qquad (26)$$

is the Landau-Zener probability to make a transition from the lower to the upper energy level. The rate of the driving is characterized by the dimensionless parameter

$$\gamma = (2c)^2/\alpha. \qquad (27)$$

The straightforward way to calculate $\mathrm{Var}(Q)$ is also the most complicated one, because it requires an explicit evaluation of the evolution operator $U(t)$. For our two site system this can be done using parabolic cylinder functions [18]. But having in mind more complex systems [23], for which exact solution are not available, we would like to make the explicit calculation within the framework of the adiabatic approximation:

$$U(t) \approx \sum_n \left| n(t) \right\rangle \exp\left[-i \int_{t_0}^{t} E_n(t') dt' \right] \left\langle n(t_0) \right|$$

In this expression $|n(t)\rangle$ and $E_n(t)$ are the so-called adiabatic states and the adiabatic energies [16]. The associated expression for the matrix elements of the time dependent current operator in the Heisenberg picture is:

$$
\begin{aligned}
\mathcal{I}(t)_{nm} &= \langle n|U(t)^\dagger \mathcal{I} U(t)|m\rangle \\
&\approx \langle n(t)|\mathcal{I}|m(t)\rangle \times \exp\left[i \int_{t_0}^{t} E_{nm}(t') dt' \right] \\
&= \begin{pmatrix} \ldots & ice^{i\Phi(t)} \\ -ice^{-i\Phi(t)} & \ldots \end{pmatrix}
\end{aligned}
\tag{28}
$$

where

$$\Phi(t) = \int^{t} \sqrt{(\alpha t)^2 + (2c)^2}\, dt' \tag{29}$$

Using the zero order ($\mathcal{O}(\alpha^0)$) adiabatic states we get for the off diagonal terms $Q_\perp \approx Q_{\mathrm{LZ}}$ where

$$
\begin{aligned}
Q_{\mathrm{LZ}} &\equiv \frac{\gamma}{2} \int_{-\infty}^{\infty} e^{i\Phi(\tau)} d\tau \\
&= \frac{\gamma}{2} \int_{-\infty}^{\infty} \cosh(z)\, e^{i\Phi(z)} dz
\end{aligned}
\tag{30}
$$

Above we use the substitution $t = (2c/\alpha)\tau$ followed by $\tau = \sinh(z)$.

Before we go on with the calculation of Q_{LZ} we would like to comment on the calculation of the diagonal terms in Eq.(28). This terms are trivially related via Eq.(16) to the Landau-Zener transition probability P_{LZ} of Eq.(26). The leading order estimate of this probability is based on a conventional time dependent treatment [17], leading to

$$
\begin{aligned}
P_{\mathrm{LZ}} &\approx \left| \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{\tau^2 + 1} e^{i\Phi(\tau)} d\tau \right|^2 \\
&= \left| \frac{1}{2} \int_{-\infty}^{\infty} \frac{1}{\cosh(z)} e^{i\Phi(z)} dz \right|^2
\end{aligned}
\tag{31}
$$

It is important to realize that in the zero order adiabatic approximation we get *zero* for the diagonal terms of Eq.(28), because the zero order adiabatic states are time-reversal symmetric and hence do not support non-zero average current. If we use the first order ($\mathcal{O}(\alpha^1)$) adiabatic states we get for the diagonal terms ± 1 but still miss the non-adiabatic P_{LZ} correction. It is therefore non-trivial and requires verification that $Q_\perp \approx Q_{\mathrm{LZ}}$ is a valid approximation.

VI. THE CALCULATION OF Q_{LZ}

In order to evaluate Q_{LZ} of Eq.(30) we use the contour integration method as in Ref.[17]. The explicit expression for the phase Φ as a function of $z = x + iy$ is

$$
\begin{aligned}
\Phi(t) &= \frac{\gamma}{2} \left(z + \frac{1}{2} \sinh(2z) \right) \\
&= \frac{\gamma}{2} \left[\left(x + \frac{1}{2} \sinh(2x) \cos(2y) \right) \right. \\
&\quad \left. + i \left(y + \frac{1}{2} \cosh(2x) \sin(2y) \right) \right]
\end{aligned}
\tag{32}
$$

The contour of integration in Eq.(30) is $y = 0$, but we would like to deform it into the complex plane so as to get rid of the rapid oscillations of the phase factor, and have instead a smooth monotonic variation. The deformed contour is displayed in Fig. 4. The phase is pure imaginary along the curves C_- and C_+. At $z_0 = 0 + i(\pi/2)$ we have $\Phi = i(\pi/4)\gamma$, while $\cosh z \approx i(z - z_0)$. Consequently in the P_{LZ} integral of Eq.(31) we have a pole leading to the standard LZ results (disregarding the prefactor which requires higher orders). But in the case of the Q_{LZ} integral Eq.(30) we do not have a pole: rather we have to consider the non singular part that comes from the integration along the $C_\pm$ curves. One observes that

$$
\begin{aligned}
Q_{\mathrm{LZ}} &= \int_{C_-+C_+} \ldots dz = 2\Re \int_{C_+} \ldots dz \\
&= \int_0^{\infty} f(x)\, e^{i\Phi(x)} dx
\end{aligned}
$$

where

$$
\begin{aligned}
i\Phi(x) = -\frac{\gamma}{4} & \left[\pi - \arccos\left(\frac{2x}{\sinh(2x)} \right) \right. \\
& \left. + \left[1 - \left(\frac{2x}{\sinh(2x)} \right)^2 \right]^{1/2} \cosh(2x) \right]
\end{aligned}
$$

and

$$f(x) = \frac{\gamma}{\sqrt{2}} \left[1 - \frac{2x}{\sinh(2x)} \right]^{-1/2} \left(\frac{\sinh(4x) - 4x}{\cosh(4x) - 1} \right) \sinh(x)$$

Deep in the adiabatic regime ($\gamma \gg 1$) the integration is dominated by the small x interval where $f(x) \propto \gamma x$ and $[\Phi(x) - \Phi(0)] \propto \gamma x^3$. Accordingly

$$Q_{\mathrm{LZ}} \sim \gamma^{1/3} \exp\left[-\frac{\pi}{4} \gamma \right] \tag{33}$$

Inspired by the calculation procedure of P_{LZ} in Ref.[16], our speculation is that also here the pre-exponential term would be renormalized to unity by higher orders, which would imply consistency with the expected (exact) result $|Q_\perp| = \sqrt{(1 - P_{\mathrm{LZ}})P_{\mathrm{LZ}}}$ that follows from Eq.(17).

VII. DISCUSSION

We have demonstrated how restricted QCC can be established and utilized in order to determine the counting statistics of a quantum coherent transition. It is important to realize that this procedure has some limitations. Namely, if we had considered not a two site system but a more complex system with multiple path geometry, then the same considerations would not allow to deduce the variance $\text{Var}(\mathcal{Q})$ of the integrated current. In particular one wonders what happens if a particle has *two* optional paths available to get from one site to another site [22]. Within the framework of the classical probabilistic theory, a splitter would imply a noisy outgoing current. But in Ref.[23] we argue that the coherent splitting of a wavepaket is not noisy, and furthermore that the whole study of fluctuations in quantum stirring devices requires to go beyond QCC considerations. Consequently the problem that has been raised in Ref.[23] has motivated us to study the capabilities of the leading order adiabatic approximation for the purpose of calculating $\text{Var}(\mathcal{Q})$.

The calculation of the Landau-Zener transition probability P_{LZ} and the associated dispersion Q_{LZ} in the leading order adiabatic approximation yields a contour integral in the complex plane. While the P_{LZ} integral is dominated by a pole, the Q_{LZ} integration is related to the corresponding principal part. Restricted QCC implies that P_{LZ} and Q_{LZ} are related. Our analysis has demonstrated that the dominant exponential term is correctly reproduced, but not the pre-exponential term which apparently requires infinite order.

The traditional QCC principle is based on the semiclassical approximation and implies *detailed* QCC for all the moments up to the quantum resolution limit. In contrast to that *restricted* QCC is very robust, and survives even in the presence of diffraction [1]. The Landau-Zener problem is possibly the simplest non-trivial example where this idea can be demonstrated. For some more complicated (chaotic) systems restricted QCC can be established as an approximation in the long time limit, on the basis of the short time perturbative analysis which is extrapolated using the central limit theorem. It would be interesting to explore the implications of coupling to the environment in this context [24].

APPENDIX A: THE VON-NEUMANN MEASUREMENT SCHEME

In this section we present a short simple derivation of the main result of Ref.[20] regarding the measurement of the full quantum statistics. The original derivation has been based on an over-complicated path integral approach.

The coupling of the system to a Von-Nuemann pointer [21] whose canonical coordinates are $(\hat{x}, \hat{q})$ is described by the Hamiltonian

$$\mathcal{H}_{\text{total}} = \mathcal{H}(t) - \mathcal{I}\hat{x} \tag{A1}$$

The states of system can be expanded in some basis $|n\rangle$, and accordingly for the system with the detector we can use the basis $|n, x\rangle$. The representation of the evolution operator is

$$U(n, x|n_0, x_0) = U[x]_{n,n_0} \, \delta(x - x_0) \tag{A2}$$

where $U[x]$ is a system operator that depends on the constant parameter x. We formally write its explicit expression both in the Schrodinger picture and also in the interaction picture using time ordered exponentiation:

$$
\begin{aligned}
U[x] &= \mathcal{T} \exp\left[-i \int_0^t (\mathcal{H} - x\mathcal{I}) dt'\right] \\
&= U[0] \, \mathcal{T} \exp\left[ix \int_0^t \mathcal{I}(t') dt'\right] \tag{A3}
\end{aligned}
$$

The time evolution of the detector is described by its reduced probability matrix

$$
\begin{aligned}
\rho_t(x'', x') &= \sum_{n, n_0', n_0''} \int dx_0' dx_0'' \\
&\quad U(n, x''|n_0'', x_0'') U(n, x'|n_0', x_0')^* \, \rho_{n_0'', n_0'}^{\text{sys}} \rho_0(x_0'', x_0') \\
&= \left[\sum_{n, n_0', n_0''} U[x'']_{n, n_0''} U[x']_{n, n_0'}^* \, \rho_{n_0'', n_0'}^{\text{sys}}\right] \rho_0(x'', x')
\end{aligned}
$$

where $\rho^{\text{sys}} = |\psi\rangle\langle\psi|$ is the initial state of the system, and $\rho_0(x_0'', x_0')$ is the initial preparation of the detector. Transforming to the Wigner function representation we get the convolution

$$\rho_t(q, x) = \int \text{P}(q - q'; x) \rho_0(q', x) dq' \tag{A4}$$

where

$$
\begin{aligned}
\text{P}(Q; x) &= \tag{A5} \\
&\frac{1}{2\pi} \int \left\langle \psi \left| U[x - (r/2)]^\dagger U[x + (r/2)] \right| \psi \right\rangle e^{-iQr} dr
\end{aligned}
$$

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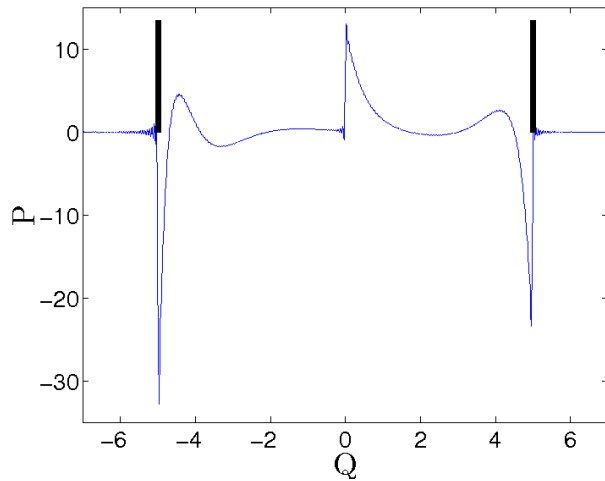
Fig.1: Numerical calculation of the full counting statistics kernel $P(Q; x=0)$ for a coherent Bloch transition from one site to another site. The evolution is governed by the Hamiltonian of Eq.(3) with $\alpha = 0$ and $ct = 5$. The thick bars represent delta functions. See the text for further explanations.
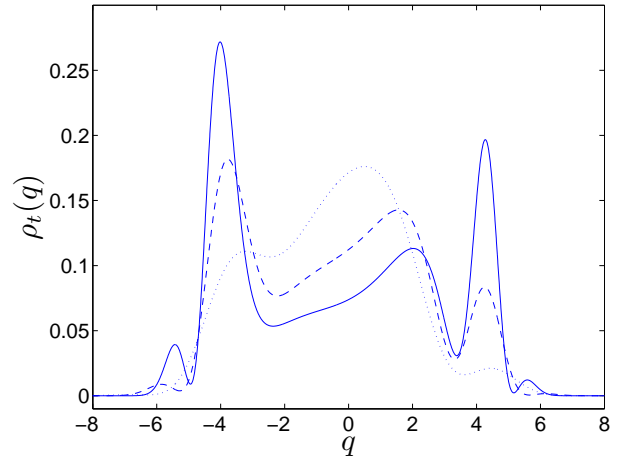

Fig.3: Numerical calculation that illustrates the final probability distribution of the Von Neumann pointer q corresponding to the lower panel in Fig. 2. The parameters are the same as in Fig. 1. The initial preparation is a Gaussian wavepacket centered at $q = x = 0$ of width $\sigma_x = 1.2$ (solid line), and $\sigma_x = 0.8$ (dashed line), and $\sigma_x = 0.5$ (dotted line). Its evolution has been calculated using Eq.(A4) with Eq.(A5).
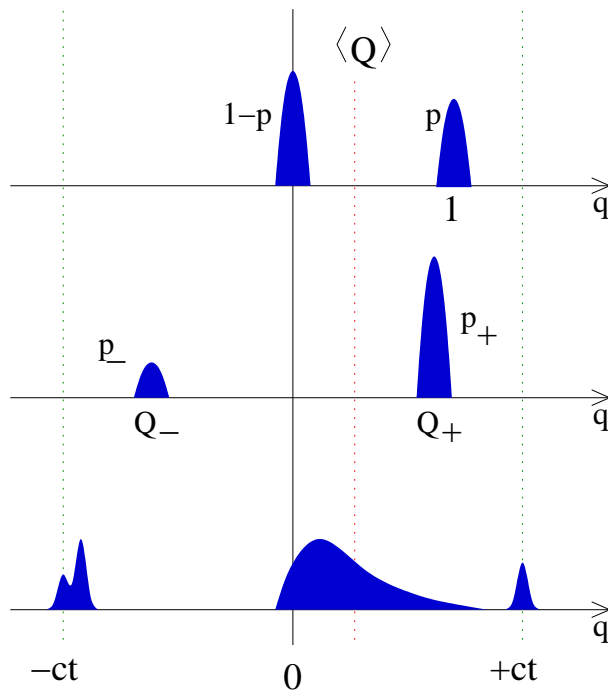

Fig.2: A caricature that illustrates the final probability distribution of the Von Neumann pointer q. It is assumed that the initial state is a wavepacked that is concentrated at $q \sim 0$. The upper panel is based on the classical expectation of having either $Q = 0$ or $Q = 1$ displacement. The middle panel is the naive quantum mechanical prediction that suggests $Q = Q_{\pm}$ displacements, while the lower panel is the actual quantum prediction based on a convolution with $P(Q; x)$.
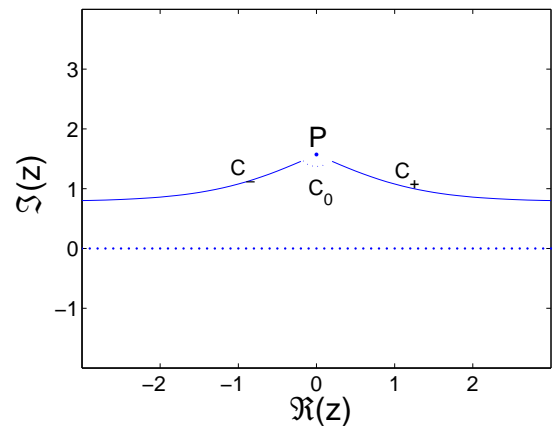

Fig.4: The calculation of P_{LZ} and Q_{LZ} is done by deforming the integration contour into the complex plane. The integration is along the curves $C_{\pm}$ where the integrand is non-oscillatory, and along the arc C_0 that encircles the pole P at $z_0 = 0 + i(\pi/2)$. The contributions of the (non-displayed) vertical segments that connect C_- and C_+ to the real axis at infinity can be neglected.